Phase transitions of strongly interacting matter within non-local PNJL model

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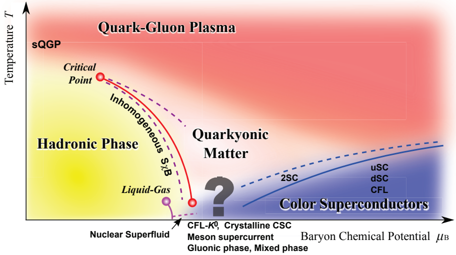
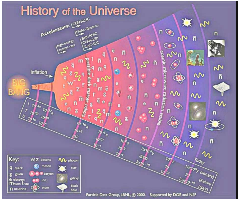
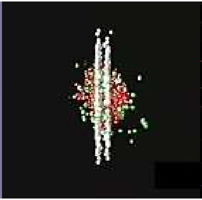
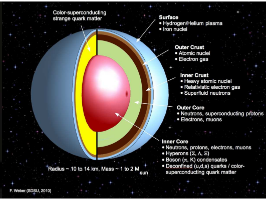
Motivation


Temperature T
Quark-Gluon Plasma
sQGP
Critical Point
Inhomogeneous SχB
Hadronic Phase
Quarkyonic Matter
Liquid-Gas
?
2SC
uSC
dSC
CFL
Color Superconductors
Nuclear Superfluid
CFL-K0, Crystalline CSC
Meson supercurrent
Gluonic phase, Mixed phase
Baryon Chemical Potential μB


History of the Universe
BIG BANG
Inflation
Key:




Color-superconducting strange quark matter
Surface
• Hydrogen/Helium plasma
• Iron nuclei
Outer Crust
• Atomic nuclei
• Electron gas
Inner Crust
• Heavy atomic nuclei
• Relativistic electron gas
• Superfluid neutrons
Outer Core
• Neutrons, superconducting protons
• Electrons, muons
Inner Core
• Neutrons, protons, electrons, muons
• Hyperons (Σ, Λ, Ξ)
• Boson (π, K) condensates
• Deconfined (u,d,s) quarks / color-superconducting quark matter
Radius ~ 10 to 14 km, Mass ~ 1 to 2 M sun
F. Weber (SDSU, 2010)

QCD at finite temperature: phase transitions

Chiral symmetry $m_{u,d} = 0$

$$\mathcal{L}_{QCD} = \mathcal{L}_{quark} + \mathcal{L}_{gauge}$$

invariant under:

$U(2)_V : \psi \to \exp(i\tau_s \alpha_V^s)\psi$ isospin and barionic N°

$U(2)_A : \psi \to \exp(i\gamma_5 \tau_s \alpha_A^s)\psi$ change parity

- $SU(2)_A$ spontaneously broken $\to$ pions Goldstone bosons
- Restaured at high temperature and/or density
- Order parameters: $\langle \bar{q}q \rangle$, f_π

Z_3 center symmetry $m_q \to \infty$

$$\mathcal{L}_{QCD} = \mathcal{L}_{gauge}$$

invariant under:
$SU(3)_c : \Omega_c = e^{i\omega_a(x)\lambda_a} \to Z_3$

- Z_3 spontaneously broken at high T

$$\Phi(\vec{x}) = \frac{1}{3}\mathrm{Tr}\left[\mathcal{P}\exp\left(ig\int_0^\beta A_0(\vec{x},\tau)\,d\tau\right)\right]$$

$\langle \Phi \rangle = e^{-\beta F} \Rightarrow$

- confined quarks: $\langle \Phi \rangle = 0$
- deconfined quarks $\langle \Phi \rangle \neq 0$

Non-local PNJL + WFR

$$S_E = \int \mathrm{d}^4 x \left\{ \bar{\psi}(x)(-i\gamma_\mu D_\mu + \hat{m}) - \frac{G_S}{2}\left[j_a(x) j_a(x) + j_P(x) j_P(x) \right] + \mathcal{U}\left(\Phi[A(x)]\right) \right\}$$

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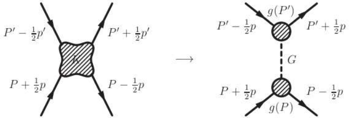
$$j_a(x) = \int d^4z\, \mathcal{G}(z)\bar{\psi}\left(x + \frac{z}{2}\right)\boldsymbol{\Gamma}_a \psi\left(x - \frac{z}{2}\right)$$

$$j_P(x) = \int d^4z\, \mathcal{F}(z)\bar{\psi}\left(x + \frac{z}{2}\right)\frac{i\overleftrightarrow{\not\partial}}{2\kappa_p}\psi\left(x - \frac{z}{2}\right)$$



$\boldsymbol{\Gamma}_a = (\mathbf{1}, i\gamma_5\vec{\tau})$, and $u(x')\overleftrightarrow{\partial}v(x) = u(x')\partial_x v(x) - \partial_x u(x')v(x)$

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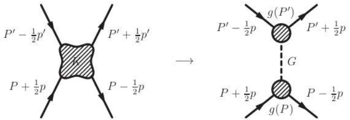

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Constant background color field: $\phi = iA_0 = ig\,\delta_{\mu 0}\, G^\mu_a\, \lambda^a/2$ Minimal coupling: $D_\mu = \partial_\mu - iA_\mu$

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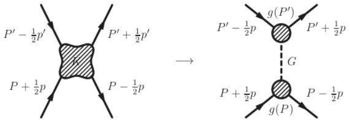

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Bosonization: physical degrees of freedom at low energy $\sigma_{1,2}$, π

$$S^{bos} = S^{MFA} + S^{quad} + \ldots$$

$$\sigma_1(x) = \bar{\sigma}_1 + \delta\sigma_1$$
$$\sigma_2(x) = \kappa_p \bar{\sigma}_2 + \delta\sigma_2$$
$$\vec{\pi}(x) = \delta\vec{\pi}(x)$$

Parameterizations

- Exponential form factors

$$g(p) = \exp(-p^2/\Lambda_0^2)$$

$$f(p) = \exp(-p^2/\Lambda_1^2)$$

without WFR (Set A), and with WFR (Set B).

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$$f(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} \frac{\alpha_m f_m(p) - m_c \alpha_z f_z(p)}{\alpha_m - m_c \alpha_z}$$

$$f_m(p) = \left[1 + (p^2/\Lambda_0^2)^{3/2}\right]^{-1}$$

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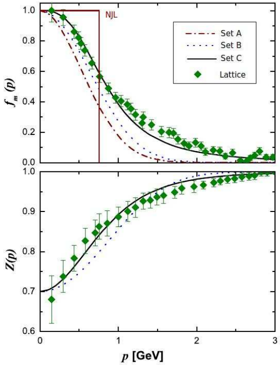
(Set C)

		Set A	Set B	Set C
m_c	MeV	5.78	5.70	2.37
$G_s\Lambda_0^2$		20.65	32.03	20.818
Λ_0	MeV	752.2	814.42	850.0
Λ_1	MeV	...	1034.5	1400.
κ_p	GeV	...	4.180	6.034



S. Noguera, N. N. Scoccola, PRD **78**, 114002 (2008)

Mean field approximation

$$\Omega^{MFA} = -\frac{4T}{\pi^2}\sum_{n,c}\int d\vec{p}\,\ln\left[\frac{(\rho^c_{n,\vec{p}})^2 + M^2(\rho^c_{n,\vec{p}})}{Z^2(\rho^c_{n,\vec{p}})}\right] + \frac{\sigma_1^2 + \kappa_p^2\sigma_2^2}{2G_S} + \mathcal{U}(\Phi,\Phi^*,T)$$

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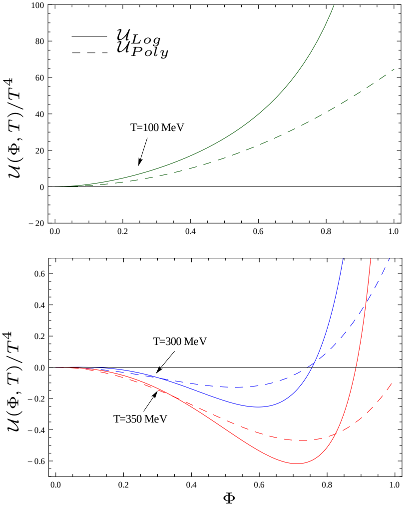
ϕ_3 and ϕ_8 parameterize the traced PL according to

$$\Phi = \frac{1}{3}\left[\exp\left(-\frac{2i}{\sqrt{3}}\frac{\phi_8}{T}\right) + 2\exp\left(\frac{i}{\sqrt{3}}\frac{\phi_8}{T}\right)\cos(\phi_3/T)\right].$$

Polyakov Loop thermodynamics



Parameters fitted to pure gauge LQCD data (eos and Φ)

LQCD *quenched limit*: $T_0 = 270$ MeV

$T < T_0$: $\Phi = 0$ confinement

$T > T_0$: $\Phi \neq 0$ deconfinement, Z_3 broken

Guinzburg-Landau - Ratti *et al.*, PRD **73**, 014019 (2006)

$$\frac{\mathcal{U}_{Poly}}{T^4} = -\frac{b_2(T)}{2}\Phi\Phi^* - \frac{b_3}{3}(\Phi^3 + \Phi^{*3}) + \frac{b_4}{4}(\Phi\Phi^*)^2$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3$$

$SU(3)_c$ Haar Measure - Roessner *et al.*, PRD **75**, 034007 (2007)

$$\frac{\mathcal{U}_{Log}}{T^4} = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\ln(1 - 6\Phi\Phi^* - 3(\Phi\Phi^*)^2 + 4(\Phi^3 + \Phi^{*3}))$$

$$a(T) = a_o + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 \quad \text{y} \quad b(T) = b_3(\frac{T_0}{T})^3$$

MFA: Gap equations

$$\Omega_{reg}^{MFA} = \Omega_{E}^{MFA} - \Omega^{free} + \Omega_{reg}^{free} + \Omega_0$$

$$\frac{\partial\Omega_{reg}^{MFA}}{\partial\sigma_1} = \frac{\partial\Omega_{reg}^{MFA}}{\partial\sigma_2} = \frac{\partial\Omega_{reg}^{MFA}}{\partial\phi_8} = \frac{\partial\Omega_{reg}^{MFA}}{\partial\phi_8} = 0$$

order parameters :

$$\langle\bar{q}q\rangle = \partial\Omega_{reg}^{MFA}/\partial m$$

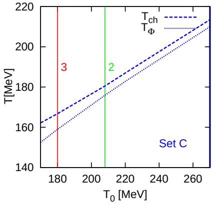
$$|\Phi|$$

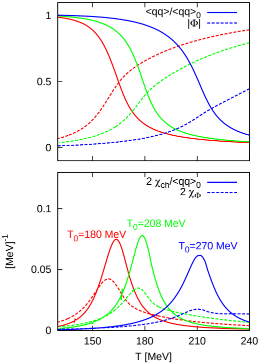
susceptibilities :

$$\begin{aligned} \chi_{ch} &= \partial\langle\bar{q}q\rangle/\partial m \\ \chi_{\Phi} &= d|\Phi|/dT \end{aligned}$$

$\mu = 0$: Critical temperature as a function of T_0

$\mathcal{U}_{Poly}$: smooth transitions - $\Delta T_c \leq 10$ MeV
(~ 40 MeV for local models)





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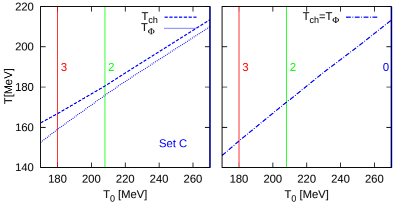
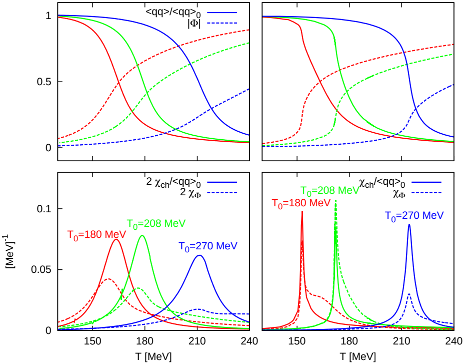
$T_{ch} = T_\Phi$ for a wide range of T_0





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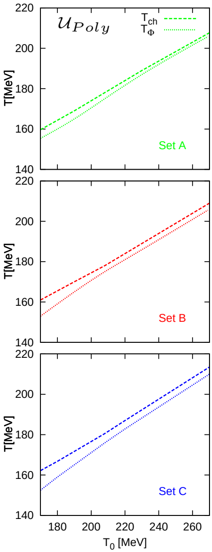

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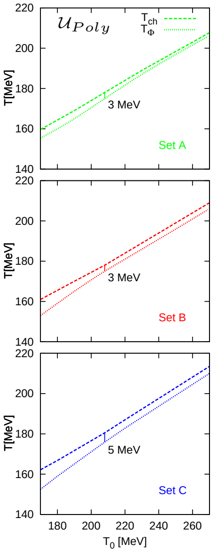

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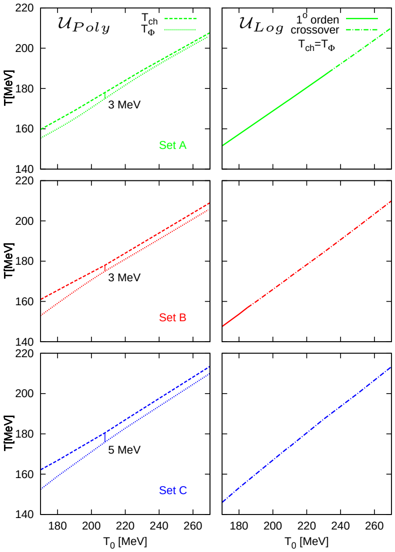

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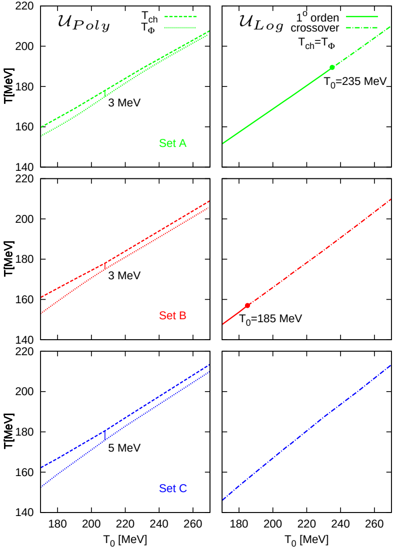

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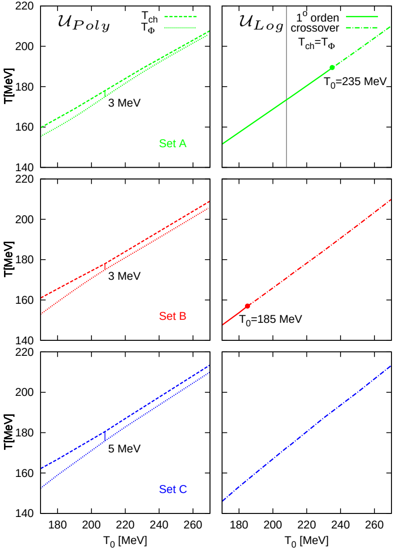

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Set A not realistic

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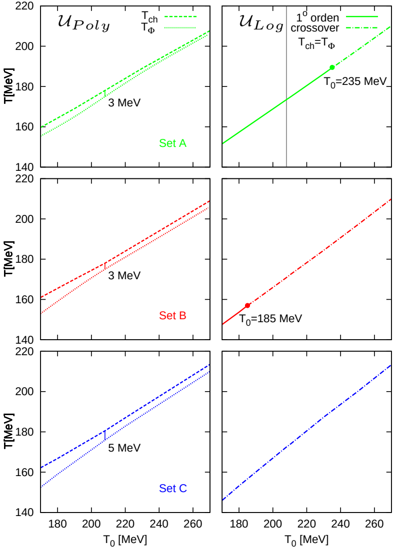

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↬ For 2 light flavors, $T_0 = 208$ MeV:

T_{ch} [MeV]	Set A	Set B	Set C	LQCD [*]
$\mathcal{U}_{Log}$	178	178	180	173(8)
$\mathcal{U}_{Poly}$	173	171	173	173(8)

[*]F. Karsch and E. Laermann, *Quark gluon plasma*, ed. Hwa, R.C.

The model favors a crossover-like transition ↫

$\mu = i\theta T$: Extended Z_3 symmetry

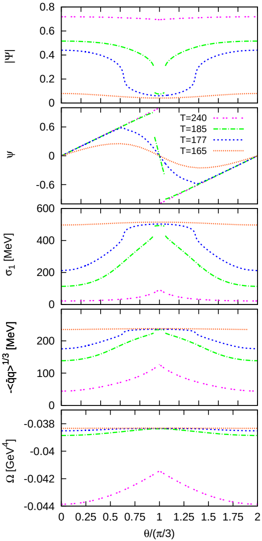
- *Roberge-Weiss*: $\mathcal{L}_{QCD}$ periodic function of θ, $2\pi/3$
- $T > T_{RW}$: 3 vacua of Z_3 - PL phase: ϕ, $\phi + 2\pi/3$, $\phi + 4\pi/3$
- RW 1^{st} order transition at $\theta = \pi/3$ mod $2\pi/3$

nlPNJL Z_3 extended symmetry

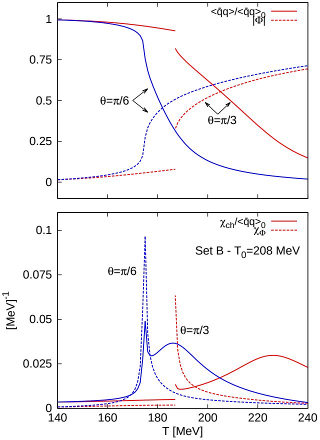
$$\begin{aligned} \Phi(\theta) &\rightarrow \Phi(\theta)\exp(-i\,2\,k\,\pi/3) \\ \Phi^*(\theta) &\rightarrow \Phi^*(\theta)\exp(i\,2\,k\,\pi/3) \\ \theta &\rightarrow \theta + 2\,k\,\pi/3 \end{aligned}$$

Extended PL: $\Psi = e^{i\theta}\Phi$, phase ψ: RW order parameter

Set B $T_0 = 208$ MeV: $T_{RW} = 188$ MeV



Phase diagram: imaginary μ



$\theta = \pi/6$: Smooth transitions at the same T

$\theta = \pi/3$: Beyond the discontinuity χ_{ch} has a peak

Deconfinement: change of character, T_Φ

Chiral transition: crossover for $T_{ch} > T_\Phi$

Phase diagram: imaginary μ

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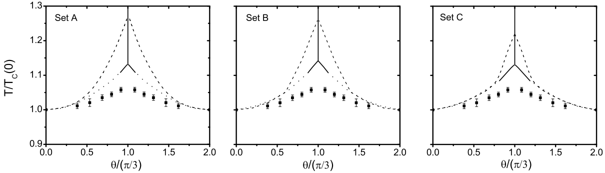

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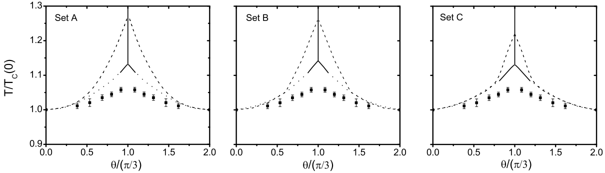
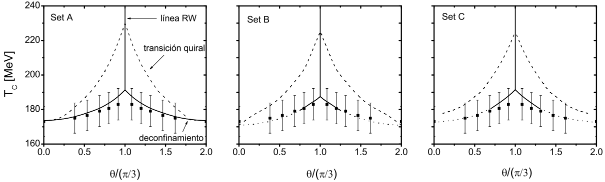
$T_0 = 270$ MeV



$T_0 = 208$ MeV: Sets B,C: $\theta_{CEP} \sim 0,7 \times \pi/3$

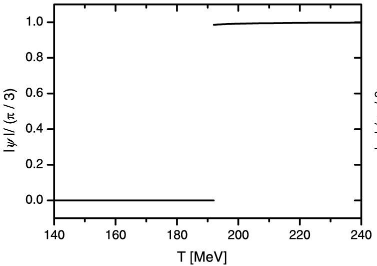


	Set A	Set B	Set C	LQCD [*]
T_{RW} [MeV]	191	188	191	185(9)

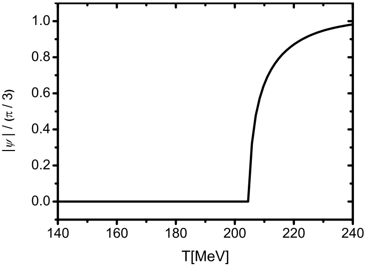
[*] L.-K. Wu *et al.* Phys. Rev. D **76**, 034505 (2007)

Extended PL

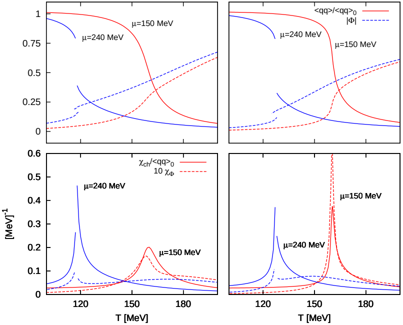
$\mathcal{U}_{Log}$: RW 1^{st} order

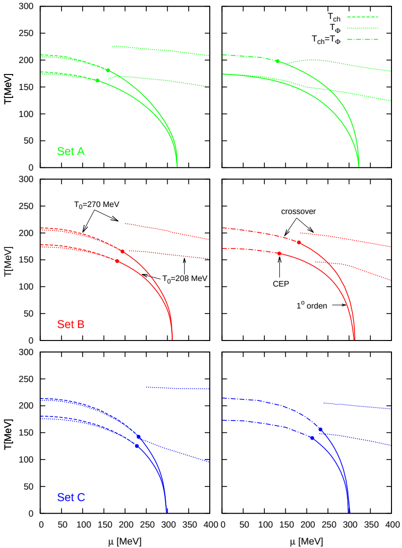


$\mathcal{U}_{Poly}$: RW 2^{nd} order



Phase Diagram: real μ

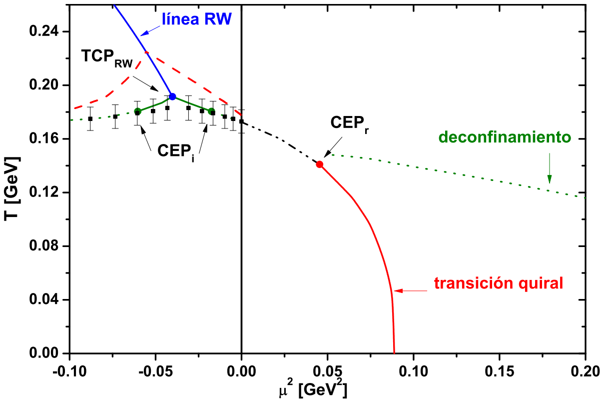




T_{ch} [MeV]	Set A	Set B	Set C	LQCD [*]
$\mathcal{U}_{Log}$	178	178	180	173(8)
$\mathcal{U}_{Poly}$	173	171	173	173(8)

[*]F. Karsch and E. Laermann, *Quark gluon plasma*, ed. Hwa, R.C.

Phase diagram in the $\mu^2 - T$ plane



Set C - $\mathcal{U}_{Log}$ - $T_0 = 208$ MeV

Conclusions

Real μ

- Chiral restoration and deconfinement transitions are strongly entangled whenever they occur as a smooth crossover.
- For $\mu > \mu_{CEP}$ we find a first order chiral restoration transition and a crossover-like deconfinement at a higher value of T: *quarkyonic matter*.
- The values of T_c predicted for $\mu = 0$ is in good agreement with LQCD results for all scenarios considered.

Imaginary μ

- When T_0 is set to the corresponding value of T_0 for $N_f = 2$ the deconfinement temperature agrees with LQCD data.
- Also does T_{RW}, being the transition first order there as predicted by LQCD calculations. The main drawback is that in this case both transition are disentangled.

* * *