Dynamical instability of white dwarfs and breaking of spherical symmetry under the presence of extreme magnetic fields

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The Sixth International Workshop on Astronomy and Relativistic Astrophysics (IWARA 2013)

Rio de Janeiro, September 30 to October 03

Sumário

Introduction

- It has been proposed that the light curves of some peculiar superluminous Ia supernovae could be explained by white dwarf progenitors whose masses are larger than the traditional Chandrasekhar limit $M_{\rm Ch} = 1.44\ M_{\odot}$
- Upasana & Banibrata (2013) recently purported that the effects of a quantizing strong and uniform magnetic field on the equation of state of a white dwarf, would increase its critical mass up to a new value $M_{\rm max} \approx 2.58\ M_{\odot}$

PRL **110,** 071102 (2013) PHYSICAL REVIEW LETTERS week ending 15 FEBRUARY 2013

New Mass Limit for White Dwarfs: Super-Chandrasekhar Type Ia Supernova as a New Standard Candle

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(Received 21 November 2012; revised manuscript received 8 January 2013; published 11 February 2013)

Ultramagnetized white dwarfs

- The equation of state of a degenerate electron gas in presence of a magnetic field B directed along the z-axis, in the limit $B \rightarrow \infty$ when all electrons are constrained to the lowest Landau level, obeys a polytrope-like for $P = K_m \rho^2$, where

$$K_m = \frac{m_e c^2 \pi^2 \lambda_e^3}{(\mu_e m_H)^2 B_D}, \tag{1}$$

with λ_e the electron Compton wavelength, and $B_D = B/B_c$ the magnetic field in units of the critical field $B_c = m_e^2 c^3/(e\hbar) = 4.41 \times 10^{13}$ G. For obtaining the above expression, in Upasana & Banibrata the density of the system was assumed to be given by $\rho = \mu_e m_H n_e$, so determined only by the nuclei component, where n_e is the electron number density.

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Then, Lane-Emden solution of Newtonian self-gravitating polytropes of index $n = 1$ was used to obtain the mass of an ultramagnetized white dwarf

$$M = 4\pi^2 \rho_c \left(\frac{K_m}{2\pi G} \right)^{3/2}, \tag{2}$$

and the corresponding radius

$$R = \sqrt{\frac{\pi K_m}{2G}}, \tag{3}$$

where ρ_c is the central density.

In the present limit of one Landau level with high electron Fermi energies E_e^F, $E_e^F = E_{\max}^F \gg m_e c^2$, with

$$E_{\max}^F = m_e c^2 \sqrt{1 + 2B_D} \approx m_e c^2 \sqrt{2B_D} \tag{4}$$

Ultramagnetized white dwarfs

the maximum possible value of E_e^F, ρ_c becomes

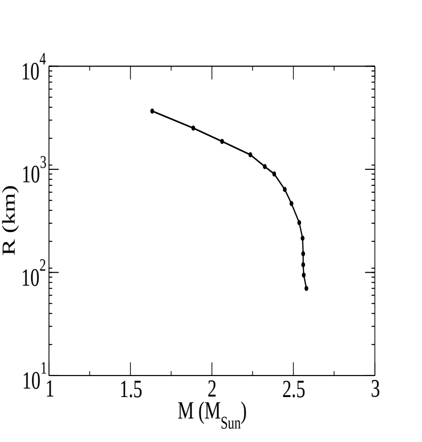
$$\rho_c = \frac{\pi M}{4R^3} = \frac{\mu_e m_H}{\sqrt{2}\pi^2 \lambda_e^3} B_D^{3/2}. \quad (5)$$

Introducing Eq. (5) into Eq. (2), Upasana & Banibrata obtained the mass limit of ultramagnetized white dwarfs

$$M_{\rm max} = \pi^{3/2} \frac{m_{\rm Pl}^3}{(\mu_e m_H)^2} \approx 2.58\ M_\odot, \quad (6)$$

when $\rho_c \to \infty$ and $R \to 0$, and Planck mass $m_{\rm Pl} = (\hbar c/G)^{1/2}$. This upper bound is larger than the canonical Chandrasekhar limit.

Ultramagnetized white dwarfs



1. We reproduce the evolutionary track of the white dwarf proposed. The magnetic field along the curve is increasing as a consequence of accretion of matter onto the star. It can be seen in the plot how the star reaches the maximum mass limit while reducing its radius.

Ultramagnetized white dwarfs

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Already at this point it is possible to identify some of the assumptions in the model of Upasana & Banibrata that led to the above results, and which we show below are incorrect and/or unjustified, invalidating their final conclusions.

- the equation of state assumed in the limit of very intense magnetic fields, $B \to \infty$;
- a uniform magnetic field is adopted;
- the huge magnetic fields and the obtained mass-radius relation explicitly violate even the absolute upper limit to the magnetic field imposed by the Virial theorem;
- dynamical instabilities due to quadrupole deformation are not taken into account either;

Ultramagnetized white dwarfs

- spherical symmetry is assumed for all values of the magnetic field;
- the role of the magnetic field in the hydrostatic equilibrium equations is neglected;
- general relativistic effects are ignored even if the final configuration is almost as compact as a neutron star and the magnetic energy is larger than the matter energy-density;
- microphysical effects such as inverse β decay and pycnonuclear fusion reactions, important in a regime where the electrons are highly relativistic, $E_e^F \gg m_e c^2$, are neglected;

Equation of state and virial theorem violation

The limiting field can be computed following the argument by Chandrasekhar & Fermi (ApJ 118, 116c, 1953). There exists a magnetic field limit ($B_{\rm max}$) above which an equilibrium configuration is impossible because the electromagnetic energy (W_B) exceeds the gravitational energy (W_G) therefore becoming gravitationally unbound. If one includes the forces derived from the magnetic field, one can write the virial scalar relation for an equilibrium configuration

$$3\Pi + W_B + W_G = 0, \tag{7}$$

where $\Pi = \int PdV$, with P the pressure of the system, W_B the positive magnetic energy, and W_G the negative gravitational potential energy. The quantity Π satisfies $\Pi = (\gamma - 1)U$ for a polytrope, $P = K\rho^{\gamma}$, where U is the total kinetic energy of particles.

Equation of state and virial theorem violation

$$E = -\frac{3\gamma - 4}{3(\gamma - 1)}(|W_G| - W_B), \tag{8}$$

and therefore the necessary condition for the stability of the star, $E < 0$, is given by

$$(3\gamma - 4)|W_G|\left(1 - \frac{W_B}{|W_G|}\right) > 0. \tag{9}$$

From this expression we can recover, in absence of magnetic field ($W_B = 0$), the known condition for bound unmagnetized polytropes $\gamma < 4/3$, or n< 3 in terms of the polytrope index n defined by $\gamma = 1 + 1/n$.

Equation of state and virial theorem violation

The presence of a magnetic field weakens the stability, and as shown in Eq. (9), no matter the value of γ, the star becomes gravitationally unbound when the magnetic energy exceeds the gravitational one; i.e. $W_B > |W_G|$. This condition clearly implies an upper bound for the magnetic field, obtained for $W_B = |W_G|$. In order to determine such limit we first obtain an expression for the magnetic energy of the star, which considering a constant magnetic field can be written as

$$W_B = \frac{B^2}{8\pi}\frac{4\pi R^3}{3} = \frac{B^2 R^3}{6}. \qquad (10)$$

Equation of state and virial theorem violation

As we discussed above, the equation of state assumes a polytrope-like for with $\gamma = 2$ or $n = 1$ under extreme magnetic fields, such that only one Landau level is populated and $E_F >> m_e c^2$. Thus, the gravitational energy density of the spherical star configuration is

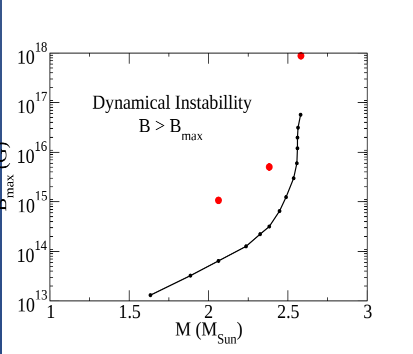
$$W_G = -\frac{3}{5-n}\frac{GM^2}{R} = -\frac{3}{4}\frac{GM^2}{R}, \tag{11}$$

where M and R are the mass and star radius, respectively, and G is the Newton gravitational constant. Using Eqs. (10) and (11), and expressing M and R in units of solar mass and solar radius, we find that the maximum value of magnetic field $B_{\rm max}$ is given by

$$B_{\rm max} = 2.24 \times 10^8 \frac{M}{M_\odot}\left(\frac{R_\odot}{R}\right)^2 . \tag{12}$$

Equation of state and virial theorem violation

- In the case of a Chandrasekhar white dwarf with the maximum mass $M = 1.44M_\odot$ and a radius of $3000\ km$, consistent with the recent calculation of massive white dwarfs, we obtain $B_{\rm max} \sim 1.7 \times 10^{13}$ G. This value is clearly lower than the critical field $B_c = 4.4 \times 10^{13}\ G$.



1. In order to quantify how strong is the violation of the virial theorem produced by the magnetic fields used in Upasana & Banibrata, we choose three star configurations whose values of M and R lie in the region of high mass configuration, $M > 2M_\odot$.

- In this table we show also for these configurations the magnetic energy W_B given by Eq. (10), and the magnitude of the gravitational energy $|W_G|$. These results indicate that the magnetic field obtained in Upasana & Banibrata, are at least one order of magnitude larger than the maximum magnetic field allowed, B_{max}.
- As a consequence, for the three star configurations, $W_B/|W_G| \sim 250$ well above the stability condition which requires $W_B/|W_G| \sim 1$. Thus, these white dwarf are unstable and unbound.
- One of the main consequences of the increasing magnetic field is that even a small ratio of magnetic to gravitational energy will produce an appreciable increase in the radii of magnetized white dwarfs.

TABLE I. Table with the mass-radius configurations of magnetized white dwarfs of Ref. [7] with the correspondent magnetic field B, the maximum virial magnetic field B_{max}, magnetic energy W_B and gravitational W_G, the ratio of them $W_B/|W_G|$, the magnetic mass in units of solar mass m_B, and the values of eccentricity in units of the spherical star radius ϵ/R.

M ($M_\odot$)	R (km)	B (G)	B_{max} (G)	W_B ($\times 10^{51}$ erg)	$\lvert W_G \rvert$ ($\times 10^{51}$ erg)	$W_B/\lvert W_G \rvert$	m_B ($M_\odot$)	ϵ/R
2.58	7.02×10^1	8.80×10^{17}	5.67×10^{16}	4.43×10^4	1.88×10^2	235	24.71	-2008.34
2.38	9.60×10^2	4.44×10^{15}	2.79×10^{14}	2.90×10^3	1.17×10^1	248	1.62	-2119.34
2.06	1.86×10^3	1.07×10^{15}	6.42×10^{13}	1.23×10^3	4.52	273	0.69	-2333.49

- very compact magnetized white dwarf configuration obtained in Upasana & Banibrata, in which large magnetic field implies large mass and *small* radius, are possible only because the effect of the repulsive magnetic force (Lorentz force) has not been properly considered.

- the limiting magnetic field values are clearly obtained with the radii much smaller than the self-consistent solution of the equilibrium equations would give. Since the maximum magnetic field depends on R^{-2}, see Eq. (12), the real maximum possible field would actually be smaller than the one computed here.

Breaking of spherical symmetry

It was shown by Chandrasekhar & Fermi that the figure of equilibrium of an incompressible fluid sphere with an internal uniform magnetic field that matches an external dipole field, is not represented by a sphere. The star becomes oblate by contracting along the axis of symmetry, namely along the direction of the magnetic field. Thus, we consider the fluid sphere to be deformed in such a way that the equation of the bounding surface is given by

$$r(\mu) = R + \epsilon P_l(\mu), \tag{13}$$

where $\mu = \cos\theta$, with θ the polar angle, and $P_l(\mu)$ denotes the Legendre polynomial of order l.

Breaking of spherical symmetry

- The quantity ϵ satisfies $\epsilon << R$ and measures the deviations from a spherical configuration. The polar and equatorial radii are $R_p = R + \epsilon P_l(1)$ and $R_{\rm eq} = R + \epsilon P_l(0)$ respectively, thus $\epsilon = -(2/3)(R_{\rm eq} - R_p)$ and therefore $\epsilon/R = -(2/3)(R_{\rm eq} - R_p)/R$, for the axisymmetric deformed configuration with $l = 2$.
- It was shown in Chandrasekhar & Fermi that such an axisymmetrically deformed object is favorable energetically with respect to the spherical star. Thus, the star becomes unstable and proceeds to collapse along the magnetic field axis, turning into an oblate spheroidal shape with $\epsilon < 0$. The contraction continues until the configuration reaches a value of ϵ/R given by

$$\frac{\epsilon}{R} = -\frac{15}{8}\frac{B^2 R^4}{GM^2}. \tag{14}$$

Breaking of spherical symmetry

Using the expression for $B_{\rm max}$ given by Eq. (12), one obtains

$$\frac{\epsilon}{R} = -\frac{135}{16}\left(\frac{B}{B_{\rm max}}\right)^2 \simeq -8.4\left(\frac{B}{B_{\rm max}}\right)^2 . \qquad (15)$$

Therefore, when the internal magnetic field is close to the limit set by the virial theorem, the star deviates to a highly oblate shape.
The results show that $|\epsilon/R| \gtrsim 2\times10^3$, which implies that the star has a highly oblate shape and thus the spherical symmetry is strongly broken.
Therefore, in order to account for the deformation caused by the presence of a magnetic field, a more consistent calculation considering cylindrical symmetry is mandatory.

Using the approximation of Eq. (5), we obtain the corresponding constant magnetic field B of these stars configurations. We compare these values of B with the maximum value, $B_{\rm max}$, allowed by the virial theorem (Eq. 12). We show that such extreme magnetic fields with $B > B_{\rm max}$ and the magnetized white dwarfs of Table are in the instability region, violating the virial theorem.

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M ($M_\odot$)	R (km)	B (G)	$B_{\rm max}$ (G)	W_B ($\times 10^{51}$erg)	$\lvert W_G\rvert$ ($\times 10^{51}$erg)	$W_B/\lvert W_G\rvert$	m_B ($M_\odot$)	ϵ/R
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General relativistic effects

- $m_B = W_B/c^2 \approx 24.7\ M_\odot$ is approximately one order of magnitude larger than the mass computed in Upasana & Banibrata, which implies a total star mass of $\approx 27.3\ M_\odot$, instead of $2.6\ M_\odot$.
- a large B cannot be reached in the star; thus the real configurations of equilibrium likely have a magnetic field energy-density much smaller than the matter energy-density, implying that the unmagnetized maximum mass, the Chandrasekhar mass still applies.

General relativistic effects

- the maximum mass configuration would have a radius $R \approx 70$ km and thus $\rho_c \approx 1.2 \times 10^{13}\ g\ cm^{-3}$.

- these values imply that the mass, radius, and density of the ultramagnetized objects considered in Upasana & Banibrata are much more similar to the parameters of neutron star rather than to the ones of WDs. Thus, it is natural to ask whether the compactness of the star, $\mathcal{C} = GM/(c^2R)$, is such to require a full general relativistic treatment. For the above star parameters close to the maximum mass configuration, we see that $\mathcal{C} \approx 0.05$, a value in clear contrast with a Newtonian treatment of the equilibrium equations.

Conclusions

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- We have shown that the ultramagnetized, $B \gtrsim 10^{15}$ G, massive, $M \gtrsim 2M_{\odot}$, white dwarfs introduced in Upasana & Banibrata are unlikely to exist in nature since their are subjected to several macro and micro instabilities which would make a white dwarf either to collapse or to explode much prior to the reaching of such a hypothetical structure. The construction of equilibrium configurations of a magnetized compact star needs the inclusion of several effects not accounted for in Upasana & Banibrata, and therefore the acceptance of such ultramagnetized white dwarfs as possible astrophysical objects has to be considered with most caution.

Microscopic instabilities

- It is known that at sufficiently high densities in the interior of white dwarfs, the inverse β decay or electron capture process becomes energetically favorable, and therefore a nucleus (Z, A) transforms into a different nucleus $(Z-1, A)$ by capturing energetic electrons.
- Such a process destabilizes the star since the electrons are the main responsible for the pressure in a white dwarf.
- Using Eq. (4), it can be seen that the electron capture process limits the magnetic field to values lower than

$$B_D^\beta = \frac{1}{2}\left(\frac{\epsilon_Z^\beta}{m_e c^2}\right)^2 \approx 812.6,\ 342.3,\ 207.9,\ 26.2, \tag{16}$$

or $B \approx 3.6 \times 10^{16}$, 1.5×10^{16}, 9.1×10^{15}, and $1.1 \times 10^{15}\ G$, where we have used the previously mentioned values of ϵ_Z^β for helium, carbon, oxygen, and iron, respectively.

Microscopic instabilities

- The electron capture in this case is shown to occur even at critical central densities lower than in the unmagnetized case. The values of the critical densities are $\rho^{\beta}_{\rm crit} \approx 9.6 \times 10^{10}$, 2.6×10^{10}, 1.2×10^{10}, and $5.6 \times 10^{8}\ g\ cm^{-3}$, respectively for helium, carbon, oxygen, and iron.
- configurations approaching the maximum mass $M \sim 2.58 M_{\odot}$ have magnetic fields with $B \gtrsim 10^{17}\ G$ and threfore $\rho_c \gtrsim 4 \times 10^{12}\ g\ cm^{-3}$, higher than neutron drip density $\rho_{\rm drip} \simeq 4.3 \times 10^{11}\ g\ cm^{-3}$, at which the less bound neutrons in nuclei start to drip out forming a Fermi gas.
- moreover, the pycnonuclear fusion reactions, important in a regime where the electrons are highly relativistic, $E^F_e \gg m_e c^2$, is neglected.
- Pycnonuclear fusion reactions might establish a more stringent limit with respect to the inverse β decay in an ultramagnetized WD.

Microscopic instabilities

- Carbon fusion leads to ^{24}Mg, which undergoes electron capture, thus inverse β decay instability, at a density of approximately $\rho^{\beta}_{\rm crit,Mg} \approx 3 \times 10^9\ g\ cm^{-3}$.
- Boshkayev, Rueda, Ruffini & Siutsou (ApJ 2013, 762,117) recently obtained the pycnonuclear carbon fusion in white dwarfs. C+C fusion occurs at a timescale of 0.1 Myr at a density $\rho^{\rm C+C}_{\rm pyc} \approx 1.6 \times 10^{10}$ g cm^{-3}. We infer that such a density is reached for a magnetic field $B_D \approx 246.6$, or $B \approx 1.1 \times 10^{16}\ G$. Longer reaction times implies lower densities and thus lower magnetic fields.
- the above limits to the magnetic field obtained from microscopic instability processes are, however, still higher than the maximal values allowed by the virial theorem. Thus, the macroscopic dynamical instabilities sets in before both electron captures and pycnonuclear reactions.

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