NEUTRINO OSCILLATIONS IN EXTERNAL FIELDS IN CURVED SPACE-TIME

Maxim Dvornikov
University of São Paulo, Brazil
IZMIRAN, Russia

Outline

- Brief review of the neutrino properties
- Neutrino interaction with matter
- Neutrino spin precession in Minkowski space
- Generalized equation for the neutrino spin evolution in matter and electromagnetic field in curved space-time
- Spin oscillations in Schwarzschild and Kerr metrics
- Oscillations of UHE neutrinos in realistic accretion disk
- Summary

References

- M. Dvornikov, *"Neutrino spin oscillations in matter under the influence of gravitational and electromagnetic fields"*, JCAP 06 (2013) 015, arXiv:1306.2659 [hep-ph].
- M. Dvornikov, *"Neutrino spin oscillations in gravitational fields"*, Int. J. Mod. Phys. D **15**, 1017 (2006), hep-ph/0601095.
- M. Dvornikov and A. Studenikin, *"Neutrino spin evolution in presence of general external fields"*, JHEP 09 (2002) 016, hep-ph/0202113.

Some facts about neutrinos

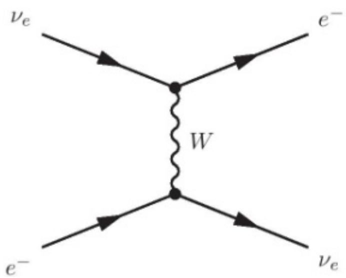
- Neutrinos are massive and mixed particles (Daya Bay 2012, Double Chooz 2012, RENO 2012)
- The absolute values of the neutrino masses are unknown (Troitsk 2011: $\boldsymbol{m_\nu < 2.2}$ **eV**)
- It is unclear whether neutrinos are Dirac of Majorana particles (GERDA 2013, EXO 2012)
- Although neutrinos are electrically neutral particles, they can interact with an electromagnetic field owing to their magnetic moments (GEMMA 2012: $\boldsymbol{\mu_\nu < 2.9 \times 10^{-11}\ \mu_B}$)

Why do we need to study the neutrino spin evolution (spin oscillations)?

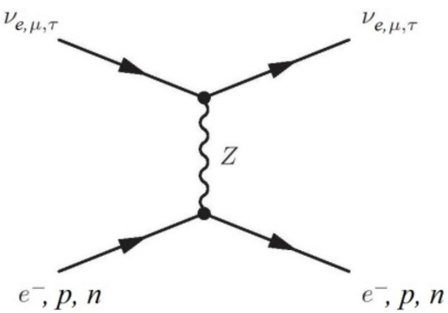
- Standard model neutrinos are massless and maintain their polarization: neutrinos are left-polarized and anti-neutrinos are right-polarized.
- If neutrinos are massive they can change their polarization in external fields (e.g., in a magnetic field).
- The change of the neutrino polarization will result in the effective reduction of the neutrino flux since right-polarized neutrinos interact much weaker with background particles in a detector.
- The observation of neutrino spin oscillations will be the indication of the nonzero neutrino mass and magnetic moment.
- We shall study the evolution of the neutrino polarization within one neutrino eigenstate ($\nu_{\alpha L} \rightarrow \nu_{\alpha R}$) assuming that other channels of neutrino oscillations (e.g., neutrino flavor oscillation $\nu_{\alpha L} \rightarrow \nu_{\beta L}$, where polarization is conserved) are suppressed.

Interaction of neutrinos with background matter

Charged currents interaction



$$L_{eff}^{(CC)} = -\frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e \gamma^\mu (1-\gamma^5)\nu_e\right]\cdot\left[\bar{e}\gamma_\mu(1-\gamma^5)e\right]$$



Neutral currents interaction

$$L_{eff}^{(NC)} = -\frac{G_F}{\sqrt{2}}\left[\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5)\nu_\alpha\right]\cdot\left[\bar{f}\gamma_\mu\left(I_{3L}^{(f)}(1-\gamma^5)-2Q^{(f)}\sin^2\theta_W\right)f\right]$$

After the averaging over the background fermions, we get:

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5)\nu_\alpha \cdot G_\mu,\ G_\mu = \sum_{f=e,p,n}\left[q_f^{(1)} J_\mu^{(f)} + q_f^{(2)}\Lambda_\mu^{(f)}\right],$$

$$J^\mu = n_0 U^\mu,\ \ \Lambda^\mu = n_0\left[(\boldsymbol{\zeta}\mathbf{U}), \boldsymbol{\zeta} + \frac{\mathbf{U}(\boldsymbol{\zeta}\mathbf{U})}{1+U^0}\right],\ U^\mu = (U^0, \mathbf{U})$$

Relativistic spin operator

We shall study a Dirac neutrino with mass ***m*** and energy ***E*** propagating in background matter and interacting with an external electromagnetic field $\boldsymbol{F_{\mu\nu}} = (\mathbf{E},\mathbf{B})$

- Relativistic spin operator

$$\hat{\mathbf{O}} = \gamma^0 \mathbf{\Sigma} + \gamma^5 \frac{\mathbf{p}}{E} - \gamma^0 \frac{\mathbf{p}(\mathbf{\Sigma p})}{E(E+m)}$$

- Dirac equation in presence of external fields

$$i\frac{\partial \nu}{\partial t} = \hat{H}\nu, \;\; \hat{H} = \hat{H}_0 + \hat{H}_{mat} + \hat{H}_{em}, \;\; \hat{H}_0 = (\boldsymbol{\alpha}\hat{\mathbf{p}}) + \gamma^0 m,$$

$$\hat{H}_{mat} = \frac{G_F}{\sqrt{2}}\gamma^0\gamma^\mu(1-\gamma^5)G_\mu, \;\; \hat{H}_{em} = -\frac{\mu}{2}\gamma^0\sigma^{\mu\nu}F_{\mu\nu}$$

- Heisenberg equation for spin operator

$$\frac{d\hat{\mathbf{O}}}{dt} = i\left[\hat{H}, \hat{\mathbf{O}}\right]_-$$

- Elimination of zitterbewegung (Schrödinger 1930)

$$\frac{d\hat{\mathbf{O}}}{dt} \to \frac{1}{2E}\left\{\frac{d\hat{\mathbf{O}}}{dt}, \hat{H}_0\right\}_+$$

- Averaging over the neutrino state

$$\left\langle \hat{\mathbf{O}} \right\rangle = \zeta, \;\; S^\mu = \left[\frac{(\zeta\cdot\langle\mathbf{p}\rangle)}{m}, \zeta + \frac{\langle\mathbf{p}\rangle(\zeta\cdot\langle\mathbf{p}\rangle)}{m(m+E)}\right],$$

Spin dynamics in flat space-time

- Quasi-classical evolution equation for the 3-vector of the neutrino spin in in matter and electromagnetic field

$$\frac{d\zeta}{dt} = 2(\zeta \times \mathbf{\Omega}),$$

$$\mathbf{\Omega} = \frac{\mu}{\gamma}\mathbf{B}_0 + \frac{G_F}{\sqrt{2}\gamma}\mathbf{M}_0$$

- The effective Schrödinger equation can be associated with the dynamics of 3-vector of spin

$$i\frac{\partial \nu}{\partial t} = \hat{H}_{eff}\nu, \ \nu = \begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix},$$

$$\hat{H}_{eff} = -(\sigma \cdot \mathbf{\Omega})$$

- This equation can be rewritten in the covariant form (MD & Studenikin 2002)

$$\frac{dS^\mu}{d\tau} = 2\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F \varepsilon^{\mu\nu\lambda\rho}G_\nu U_\lambda S_\rho$$

Spinning particles in curved space-time

$$\frac{DS^{\mu\nu}}{D\tau} = p^\mu v^\nu - p^\nu v^\mu,$$

$$\frac{Dp^\mu}{D\tau} = -\frac{1}{2} R^\mu{}_{\nu\rho\sigma} v^\nu S^{\rho\sigma},$$

$$S_\rho = \frac{1}{2m}\sqrt{-g}\,\varepsilon_{\mu\nu\lambda\rho} p^\mu S^{\nu\lambda}$$

- Papapetrou (1951) eqns for a spinning body in a gravitational field. Here $\boldsymbol{v^\mu}$ is the unit tangent vector to the center-of-mass world line and $\boldsymbol{R^\mu{}_{\nu\rho\sigma}}$ is Riemann tensor.
- The motion of a spinning body deviates from geodesics. Rietdijk and van Holten (1993) showed that for point like particles this deviation is negligible.

We get the new spin evolution equation for a neutrino interacting with background matter under the influence of electromagnetic and gravitational fields (MD 2013):

$$\frac{DS^\mu}{D\tau} = 2\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F \frac{\varepsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}} G_\nu U_\lambda S_\rho, \quad \frac{DU^\mu}{D\tau} = 0$$

Spin evolution in vierbein frame

- Metric is Minkowskian in the vierbein frame
- All the objects should be transformed to the vierbein frame

$$g_{\mu\nu} = e^a_{\ \mu} e^b_{\ \nu} \eta_{ab}, \ \eta_{ab} = \mathrm{diag}(+1,-1,-1,-1)$$

$$s^a = e^a_{\ \mu} S^\mu, \ u^a = e^a_{\ \mu} U^\mu, \ \ldots$$

Neutrino spin evolution equation in the vierbein frame:

$$\frac{ds^a}{dt} = \frac{1}{\gamma}\left[G^{ab} s_b + 2\mu\left(f^{ab} s_b - u^a u_b f^{bc} s_c \right) + \sqrt{2} G_F \varepsilon^{abcd} g_b u_c s_d \right], \ \frac{du^a}{dt} = \frac{1}{\gamma} G^{ab} u_b$$

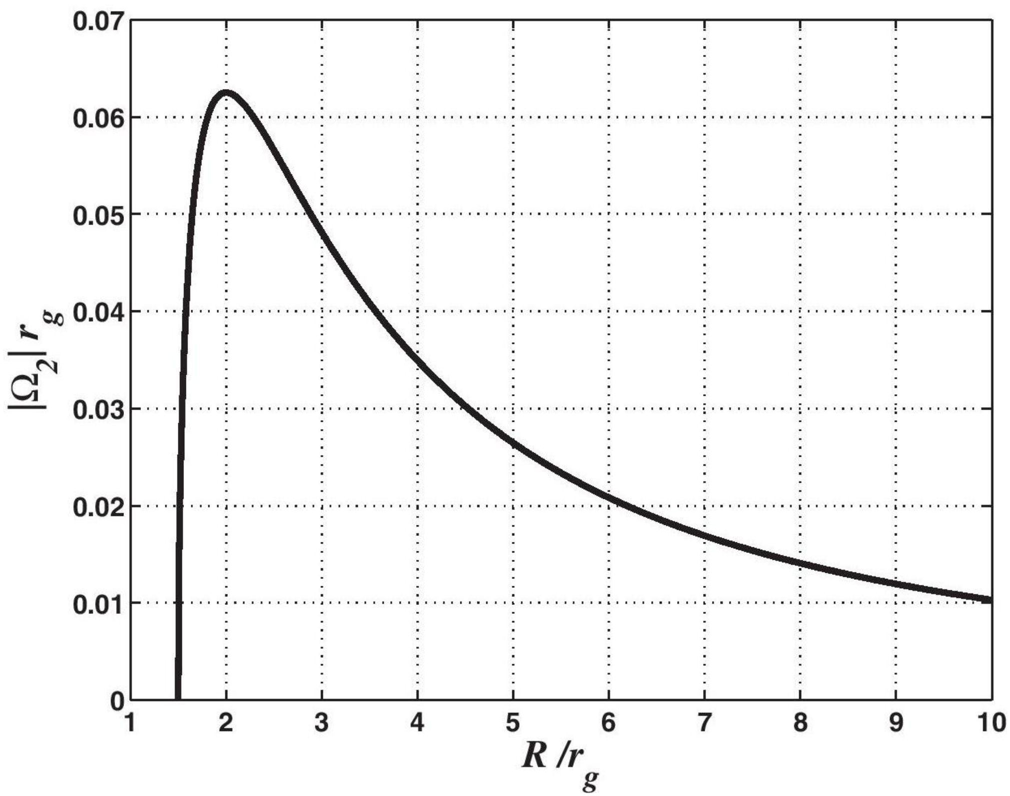
$$G_{ab} = \gamma_{abc} u^c = (\mathbf{e}_g, \mathbf{b}_g), \ \gamma_{abc} = \eta_{ad} e^d_{\ \mu;\nu} e_b^{\ \mu} e_c^{\ \nu}, \ \gamma = U^0, \ f_{ab} = (\mathbf{e}, \mathbf{b})$$

Evolution of the three vector of the neutrino spin:

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \ \boldsymbol{\Omega} = \frac{1}{\gamma}\left\{ \frac{1}{2}\left[\mathbf{b}_g + \frac{1}{1+u^0}(\mathbf{e}_g \times \mathbf{u}) \right] + \frac{G_F}{\sqrt{2}}\left[\mathbf{u}\left(g^0 - \frac{(\mathbf{g}\cdot\mathbf{u})}{1+u^0} \right) - \mathbf{g} \right] + \mu\left[u^0 \mathbf{b} - \frac{\mathbf{u}(\mathbf{u}\cdot\mathbf{b})}{1+u^0} + (\mathbf{e}\times\mathbf{u}) \right] \right\}$$

Schwarzschild metric

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$



We shall study a circular orbit with the radius ***R***. In this case a neutrino moves along the third axis line in the vierbein frame. The spin rotates around the second axis.

$$P_{L\to R}(t) = \sin^2(\Omega_2 t)$$

$$\Omega_2 = -\frac{v_\phi}{2\gamma}, \quad \Omega_1 = \Omega_3 = 0,$$

$$v_\phi = \left(\frac{r_g}{2R^3}\right)^{1/2}, \quad \gamma = \left(1 - \frac{3r_g}{2R}\right)^{-1/2}$$

Kerr metric

$$d\tau^2 = \left(1-\frac{rr_g}{\Sigma}\right)dt^2 + 2\frac{rr_g\sin^2\theta}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - \frac{\Xi}{\Sigma}\sin^2\theta d\phi^2,$$

$$\Delta = r^2 - rr_g + a^2,\ \Sigma = r^2 + a^2\cos^2\theta,\ \Xi = \left(r^2+a^2\right)\Sigma + rr_g a^2\sin^2\theta$$

We shall take that BH is surrounded by an accretion disk. The asymptotic value of the magnetic field, $\boldsymbol{B_0}$, is constant and directed along the rotation axis of BH. Then (Wald 1974):

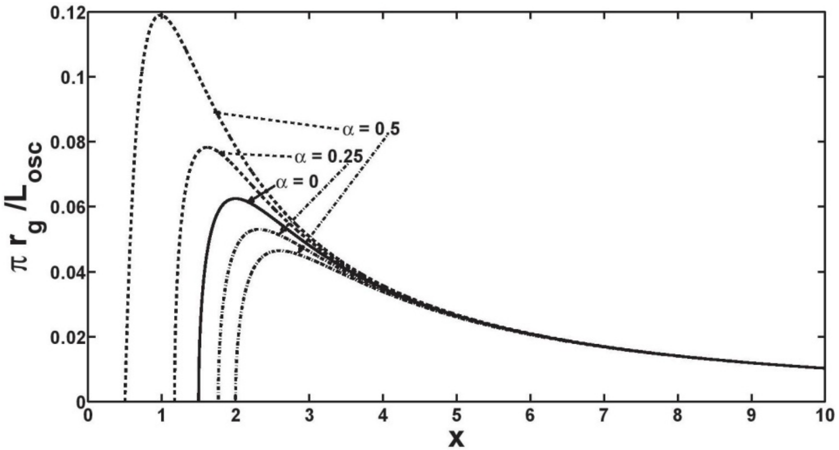
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu,\ A_t = B_0 a\left[1-\frac{rr_g}{2\Sigma}\left(1+\cos^2\theta\right)\right],\ A_\phi = -\frac{B_0}{2}\left[r^2+a^2-\frac{rr_g a^2}{\Sigma}\left(1+\cos^2\theta\right)\right]\sin^2\theta$$

Circular neutrino motion

$$P(t) = \frac{\Omega_2^2}{\Omega_2^2+\Omega_3^2}\sin^2\left(\sqrt{\Omega_2^2+\Omega_3^2}t\right),$$

$$\Omega_2 = \frac{1}{2\gamma r_g}\left\{\mp\frac{1}{\sqrt{2}x^{3/2}} - \mu B_0 r_g\frac{2\sqrt{2}x^2(x-1)\pm\alpha\sqrt{x}(2x-1)+\sqrt{2}\alpha^2}{x^{3/2}\sqrt{2x^3-3x^2\pm2\sqrt{2}\alpha x^{3/2}}}\right\},$$

$$\Omega_3 = \frac{G_F n_{eff}}{\sqrt{2}\gamma}\frac{\pm U_f^0 - r_g U_f^\phi\left(\sqrt{2}x^{3/2}\pm\alpha\right)}{\sqrt{2x^3-3x^2\pm2\sqrt{2}\alpha x^{3/2}}},\ x=\frac{R}{r_g},\ \alpha=\frac{a}{r_g}$$



Radial propagation of UHE neutrinos

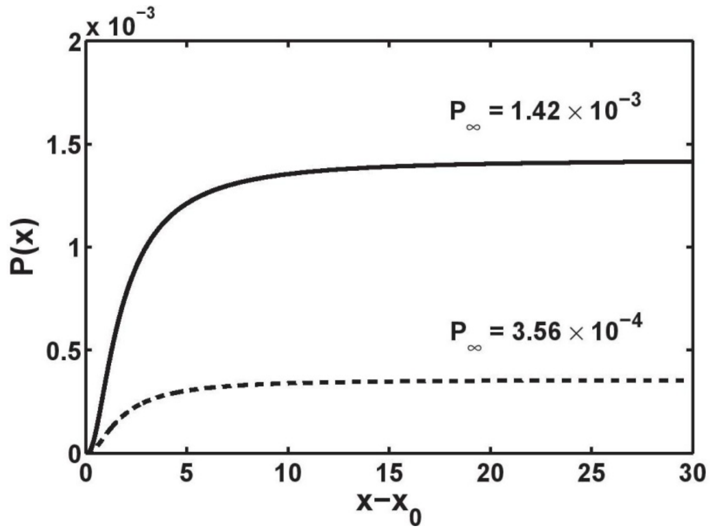
- Because of the symmetry reasons a purely radial motion in Kerr metric is possible only along the rotation axis of BH.
- If $U^0 >> 1$ (UHE neutrinos), the approximate treatment of spin oscillations is possible.
- We shall study the motion in the equatorial plane. A neutrino moves along the first axis in the vierbein frame.

$$i\frac{d\nu}{dx} = -\frac{x}{x-1} r_g (\boldsymbol{\sigma}\cdot\boldsymbol{\Omega})\nu, \quad \nu_{R,L} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix},$$

$$\Omega_1 = \frac{G_F n_{eff}}{\sqrt{2}}\left[U_f^0\left(1-\frac{1}{x}\right) + r_g U_f^\phi \frac{\alpha}{x}\right],$$

$$\Omega_2 = -\mu B_0\left(1-\frac{1}{x}\right) + \frac{\alpha}{4r_g}\frac{2x-3}{x^{7/2}\sqrt{x-1}},$$

$$\Omega_3 = -\frac{G_F n_{eff}}{\sqrt{2}}\frac{\alpha}{x^2}\sqrt{1-\frac{1}{x}}U_f^0,$$



Gravity does not produce a sizable effect on neutrino spin oscillations

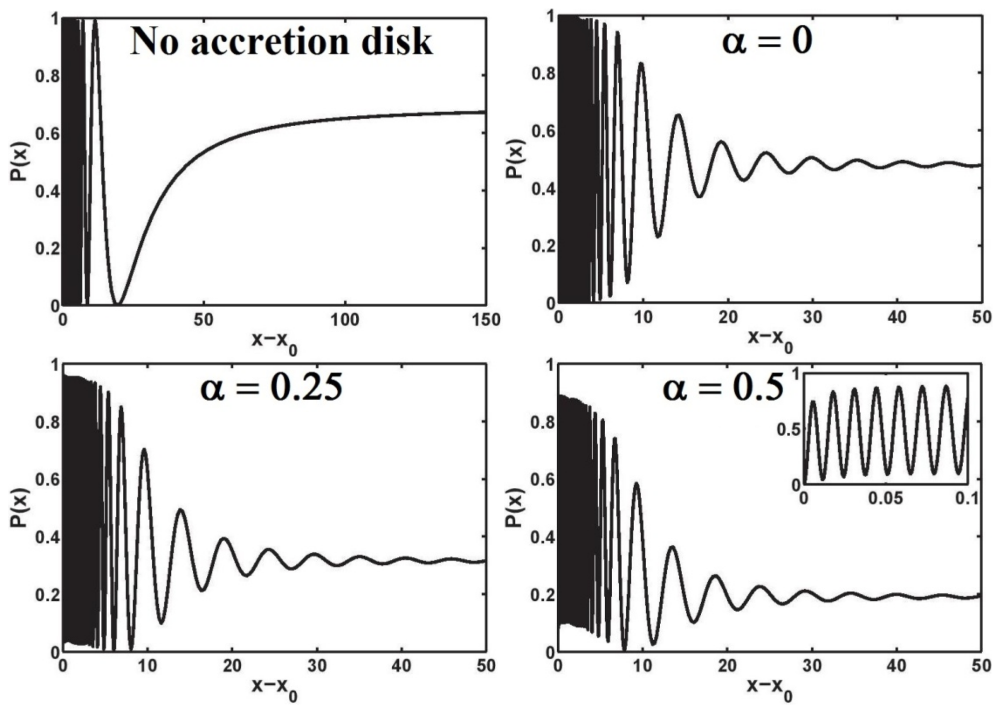
Interaction of UHE neutrinos with a relativistic accretion disk

Motivation

- Deficit of UHE ν-s was reported by IceCube (2012). In 2013 some UHE ν_e-s were observed by IceCube. Still there is a lack of signal for UHE $\nu_{\mu,\tau}$-s.
- Barranco et al. (2012) : neutrino spin oscillations in strong magnetic field ➔ the neutrino flux is reduced

Input data

- UHE ν-s emitted in GRB
- Dipole magnetic field: $\boldsymbol{B_0 = 10^{12}}$ **G /** $\boldsymbol{x^3}$
- Magnetic moment: $\boldsymbol{\mu = 10^{-12} \mu_B}$ (Kuznetsov et al. 2009)
- Accretion disk density: $\boldsymbol{\rho = 10^2}$ **g/cm³** (MacFadyen & Woosley 1999)



Result: Spin oscillations cannot explain the deficit of UHE ν-s. The neutrino interaction the a realistic relativistic accretion disk will suppress the transition probability even if magnetic field is strong.

Summary

- The new covariant equation for neutrino spin evolution in matter and electromagnetic field in Minkowski space was derived.
- For the first time this equation was generalized to include the effects of gravity.
- Neutrino spin oscillations in Schwarzschild and Kerr metrics were described.
- Spin oscillations of UHE neutrinos in dense magnetized relativistic accretion disk were studied.
- It was shown that spin oscillations cannot solve the problem of the UHE neutrinos deficit.

Acknowledgments

- To the organizers of IWARA 2013 for the invitation.
- To FAPESP (Brazil) for the financial support.