Stellar entropy and stellar evolution: a brief review

Marcio G. B. de Avellar

Jorge E. Horvath

Instituto de Astronomia, Geofísica e Ciências Atmosféricas (Universidade de São Paulo)

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Context

I am going to present you part of our research about:

- the relation between the thermodynamic and Shannon entropies along the stellar evolution and
- how we can use information theoretic methods to address the composition of neutron stars.

In this presentation my main focus will be the second problem.

Stellar sequence and adopted hypotheses

We assumed the schematic path of evolution of a single object:

- MC $\rightarrow$ MSS $\rightarrow$ WDS $\rightarrow$ PNS $\rightarrow$ NS $\rightarrow$ BH.

Conserved quantity:

- Baryon number, $N_b = 1.6 \times 10^{57} \Rightarrow 1.35 M_\odot$.

Hypotheses:

- the total energy is given by $E_{tot} = E_{int} + E_{kin} + E_{pot}$;
- the virial condition $E_{pot} = -2 \times E_{kin}$ is satisfied;
- the components have equilibrium particle distributions, for example, equipartition of energy holds for ideal gases, $E_{kin} \sim kT$.

Calculation

Internal conditions were properly chosen and were based on astrophysical observations and computer simulations:

- MC: agglutinating blobs with $T \sim 20K$;
- MSS: four different ages, 0, 1 Gyr, 3 Gyr and 4 Gyr with internal structure simulated by computer;
- WDS: hot and cold phases, taking into account melting and crystallization;
- PNS: $T \sim 10^{11}K$, from explosion mechanisms;
- NS: hot and cold phases, observations and theory;
- BH: theory.

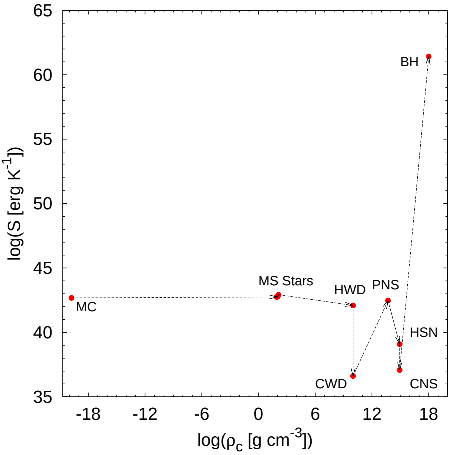
Results

Each component of matter had its entropy calculated appropriately:

	Radiation	Ideal baryons	Ideal electrons	Degenerate e^-	Crystal	Degenerate n	Area
MC	–	$\sim 10^{42}$	–	–	–	–	–
Star0	$\sim 10^{40}$	$\sim 10^{42}$	$\sim 10^{42}$	–	–	–	–
Star1	$\sim 10^{40}$	$\sim 10^{42}$	$\sim 10^{42}$	–	–	–	–
Star3	$\sim 10^{41}$	$\sim 10^{42}$	$\sim 10^{42}$	–	–	–	–
Star4	$\sim 10^{42}$	$\sim 10^{42}$	$\sim 10^{42}$	–	–	–	–
HWD	$\sim 10^{37}$	$\sim 10^{42}$	–	$\sim 10^{40}$	–	–	–
CWD	$\sim 10^{26}$	–	–	$\sim 10^{36}$	$\sim 10^{33}$	–	–
PNS	$\sim 10^{42}$	$\sim 10^{42}$	–	$\sim 10^{42}$	–	–	–
HNS	$\sim 10^{32}$	–	–	–	–	$\sim 10^{39}$	–
CNS	$\sim 10^{26}$	–	–	–	–	$\sim 10^{37}$	–
BH	$\sim 10^{-17}$	–	–	–	–	–	$\sim 10^{61}$

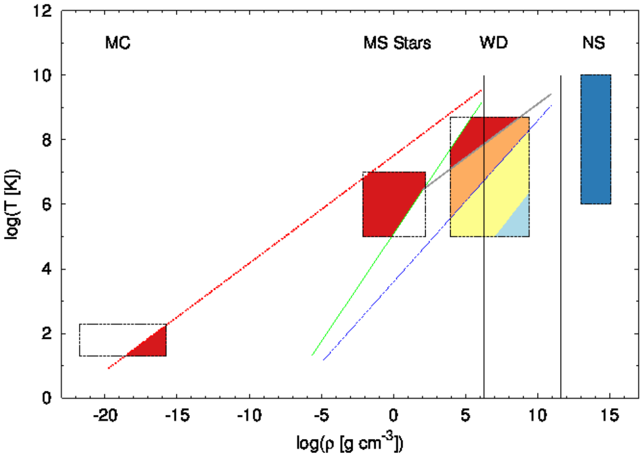
Results

Evolution of thermodynamic entropy:



Results

The entropy regimes do not follow the pressure regimes (see WD):



Complexity and Information

First, concepts and definitions:

- Information: what we can get from observing the occurrence of an event (how surprising, or unexpected or what else).

With a certain reductionism:

- definition of information in terms of the probability of an event to occur.

Information

From some desired mathematical properties of information we can derive:

$$I(p) = -log_b(p) \tag{1}$$

for some probability p and basis b (that gives the unit). $b = 2$ give us *bits*.

- flipping a fair coin once give you $-log_2(1/2) = 1$ bit of information.

Information

If a source provides n symbols $\{a_i\}$ with probability $\{p_i\}$, then the average amount of information in the stream of symbols is:

$$\frac{I}{N} = -\sum_{i=0}^{N} p_i log_b(p_i) \equiv H(P). \qquad (2)$$

This quantity is defined as the *entropy* of the probability distribution $P = \{p_i\}$

Property:

- The maximum of this quantity is achieved at equiprobability $p_i = 1/n$.

Information

The generalization we seek should be valid for continuous systems admitting a probability distribution.

$$H = -K \int p(x) log_b \big(p(x)\big) d\vec{x} \quad (3)$$

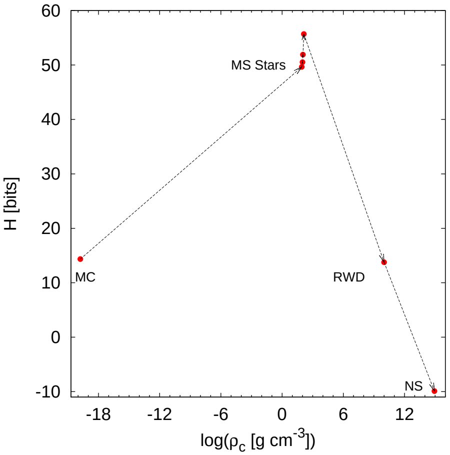
In order to apply this concept to physical systems we have to define properly what quantity to use as a probability distribution.

Information

- In condensed matter, the momentum and position distributions in the phase space or even the atomic number have been used.
- Here we assumed that the probability distribution is proportional to the energy density (or the mass density in the non-relativistic case) profile of the star.
- For each object we obtained the energy density profile by solving the appropriate set of structure equations and calculated the quantity H.

Information

Evolution of information entropy:



Complexity in physical systems: ideal cases

- Complexity: what does not match the requirements of being simple (tautology?). In physics, we always begin with ideal systems as the simplest systems possible.

Let us allow complexity to encode order and disorder (or the self-organization of a system): two ideal systems, extremes in all aspects and opposites as well:

- Perfect crystal: zero complexity by definition; strict symmetry rules $\Rightarrow$ probability density centered around the prevailing state of perfect symmetry $\Rightarrow$ minimal information. Completely ordered.
- Ideal gas: zero complexity by definition; accessible states are equiprobable $\Rightarrow$ maximal information. Totally disordered.

Key concept: Disequilibrium

- Here we show how to use the information we have just calculated to discriminate among the myriad of equations of state.

However, the information alone is not enough to define complexity. We define then the *disequilibrium* as the distance to the equiprobability.

Now we mathematically define complexity as:

$$C \equiv H \times D \tag{4}$$

Getting some intuition first

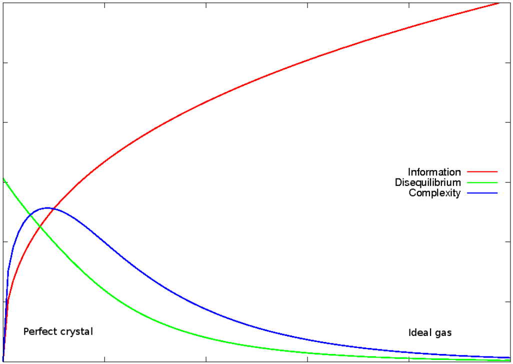


Figure: Intuitive definition of complexity

Disequilibrium

As an expression to disequilibrium the proposal is

$$D = \sum_{i=1}^{N} \left(p_i - \frac{1}{N} \right)^2 \rightarrow \int p^2(x) dx \tag{5}$$

Compact stars

Application to neutron stars with two different compositions:

- Hadronic composition with SLy4 equation of state;
- Quark composition with three flavours in equal amounts or strange quark matter.

How does the composition affect the measures of these quantities?

Compact stars

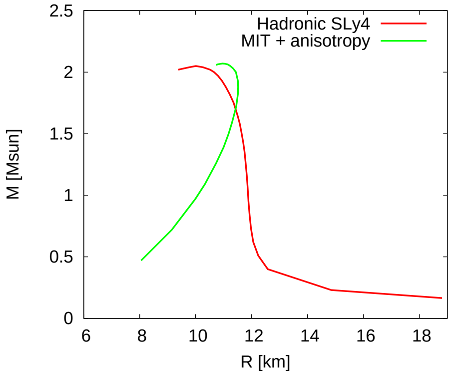


Figure: Mass-radius relation for two different EoSs

Results

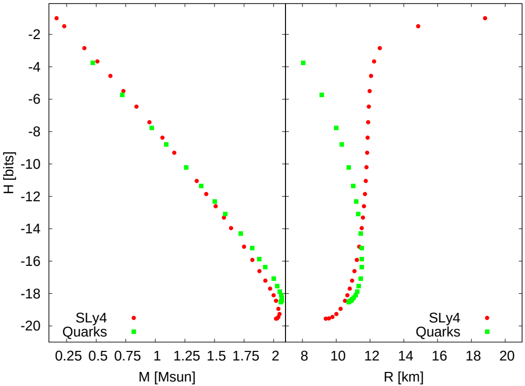


Figure: S *vs* M *vs* R

Results

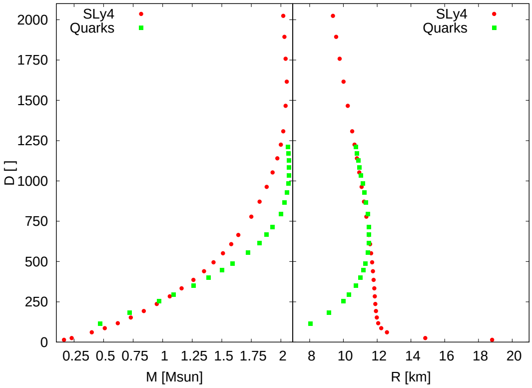


Figure: D *vs* M *vs* R

Results

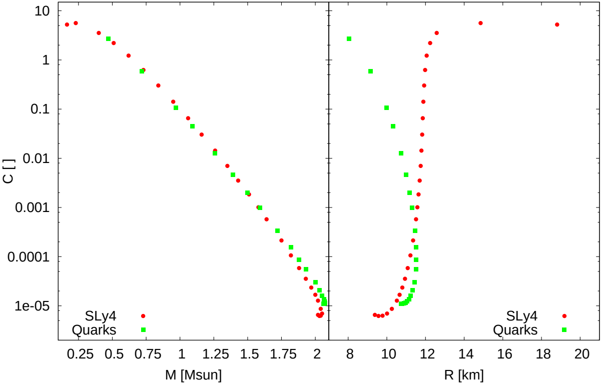


Figure: C *vs* M *vs* R

Summary

Summary of the results comparing the two equations of state:

H: $M \geq 1.3\,M_\odot$ ⇒ higher I for quarks ⇒ quarks more “gaseous”;
$M \leq 1.3\,M_\odot$ ⇒ lower I for quarks ⇒ quarks more “crystalline”;
D: $M \geq 1.1\,M_\odot$ ⇒ lower D for quarks ⇒ quarks less ordered ⇒ quarks more “gaseous”;
$M \leq 1.1\,M_\odot$ ⇒ higher D quarks ⇒ quarks more ordered ⇒ quarks more “crystalline”;
C: $M \geq 1.6\,M_\odot$ ⇒ quarks more complex;
$M \leq 1.6\,M_\odot$ ⇒ equivalent complexity for both compositions.

The white dwarf case: complexity *grows* with increasing mass, reaching a maximum finite value at the Chandrasekhar mass. Resemblance to atomic systems.

Conclusions

Conclusions:

- If higher D implies higher distance to the equiprobability and a trend to be more crystalline (and ordered), and additionally we assume that order has a cost, then for each EoS separately Nature prefers to form less massive objects, albeit more complex ones.
- Therefore, comparing the two EoSs presented here it seems that for masses $\leq 1.1 - 1.3\,M_{\odot}$ the quarks tend to be more crystalline than the hadrons, i.e., Nature would prefer to make hadronic "neutron" stars. For masses $\geq 1.1 - 1.3\,M_{\odot}$ the quarks tend to be more "gaseous", and quark "neutron" stars are preferred by Nature.

Conclusions

Remark:

- Note that this is roughly in line with naive expectations about the presence of quarks in massive stars only. The exact nature of the objects formed in Nature still depends on the actual astrophysical mechanism(s) allowing the birth of a compact star. For example, in stellar collapses the expected iron cores would never form compact stars with masses below $\geq 1\,M_{\odot}$, therefore, essentially all the observed "'neutron stars" should be quark stars.
- The mechanism of formation is not being tested; this method tests the complexity and information entropy for different compositions.

Other EoSs and the effects of interactions

The following Figures show:

- how different values of the bag constant affect these statistical measures;
- the effects of the strong coupling constant;
- the effects of the anisotropy of pressure (exact solution versus MIT original);
- other EoSs;
- ...

Other EoSs and the effects of interactions

EoSs we are studying now: can we quantify these differences?

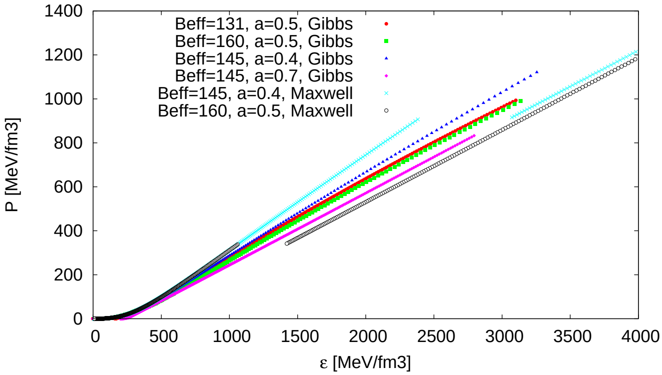


Figure: Hybrid stars: Bag Model + relativistic mean field TM1; For details, see: S. Weissenborn et al. Astrophysical Journal, 740, L14 (2011)

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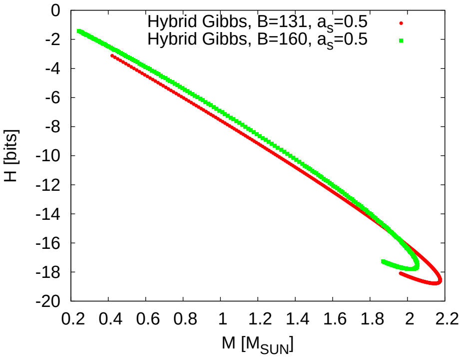

Figure: $B_{eff} = 131 MeV/fm^3$ and $B_{eff} = 160 MeV/fm^3$ for $\alpha = 0.5$

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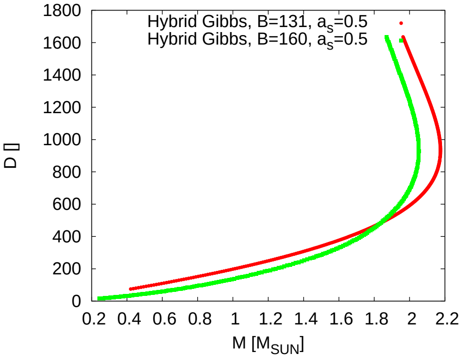

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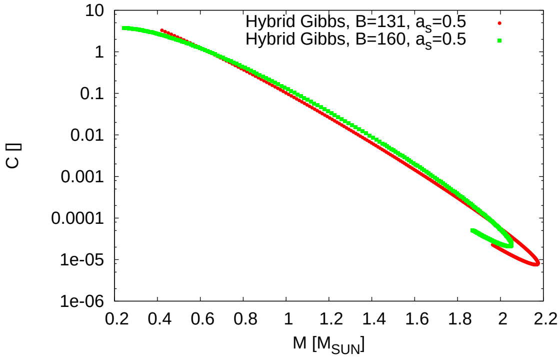

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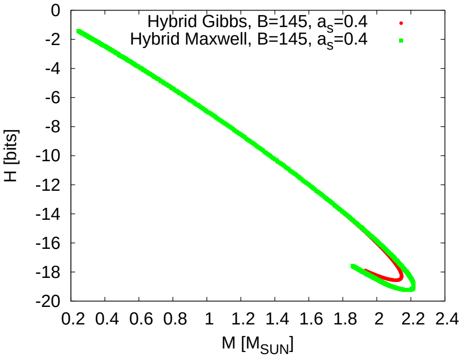

Figure: $B_{eff} = 145 MeV/fm^3$ and $\alpha = 0.4$ for Gibbs and Maxwell transitions

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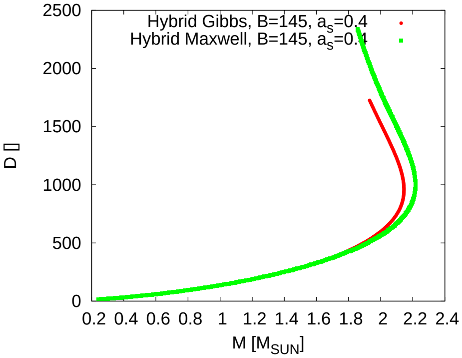

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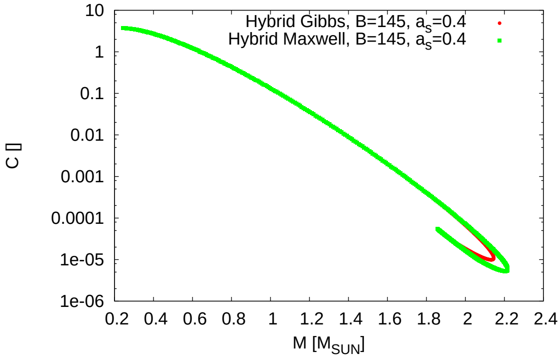

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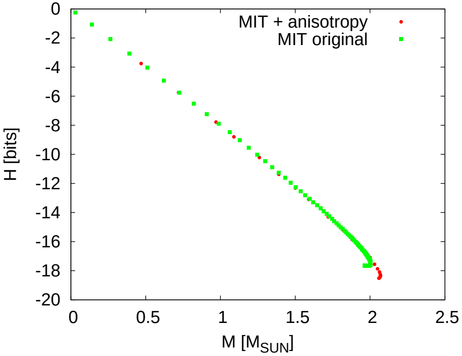

Figure: Original Bag Model *vs* Bag Model with anisotropic pressure (exact solution)

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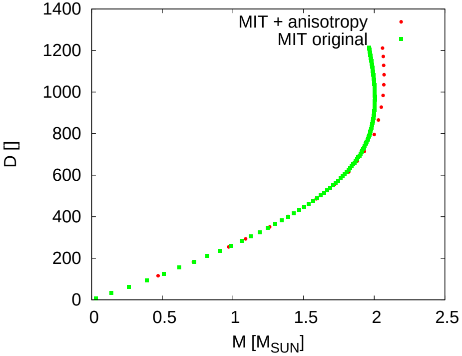

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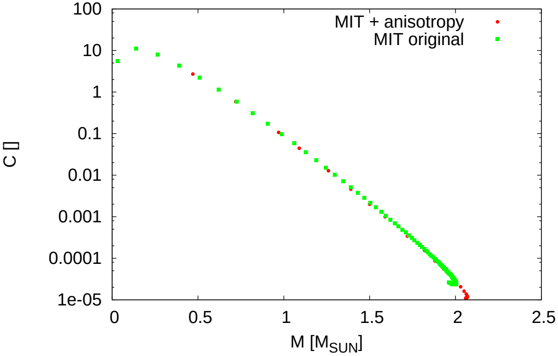

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References

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