Quark matter and the high-density frontier

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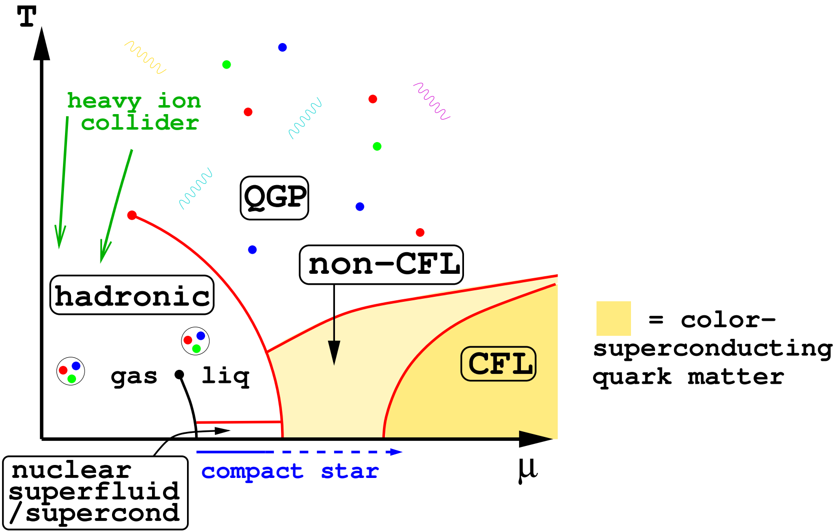
Outline

- **I** Quarks at high density
  Confined, quark-gluon plasma, color superconducting
- **II** Color superconducting phases
  Color-flavor locking (CFL), and beyond
- **III** Quark matter in the real world
  Battle between color superconductivity and the strange quark

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, `arXiv:0709.4635` (RMP)
A. Schmitt, `arXiv:1001.3294` (Springer Lecture Notes)

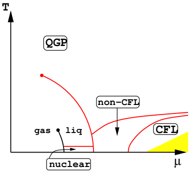
I. Quarks at high density



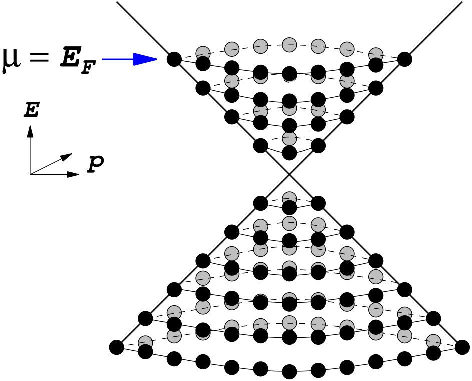
heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Color superconductivity



At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

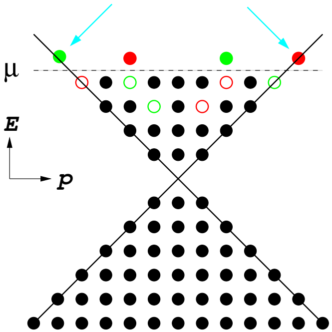
$$\frac{dF}{dN} = 0$$

But quarks have attractive QCD interactions.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.

BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

What is a condensate of Cooper pairs?



$$|\phi_0\rangle = \prod_{\mathbf{p}} \Big(\cos(\theta_{A\mathbf{p}}) + \sin(\theta_{A\mathbf{p}})\, a^\dagger(\mathbf{p}) a^\dagger(-\mathbf{p})\Big)$$
$$\Big(\cos(\theta_{B\mathbf{p}}) + \sin(\theta_{B\mathbf{p}})\, b^\dagger(\mathbf{p}) b^\dagger(-\mathbf{p})\Big) \quad \times \quad |\text{Fermi sea}\rangle$$

$|\phi_0\rangle$, not $|\text{Fermi sea}\rangle$, is the ground state.

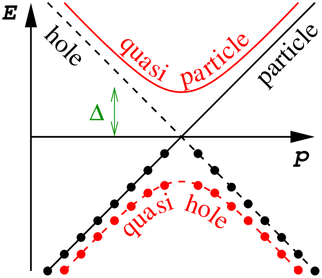
Physical consequences of Cooper pairing

Changes low energy excitations, affecting *transport properties*.

- Spontaneous breaking of global symmetries ⇒ Goldstone bosons, massless degrees of freedom that dominate low energy behavior. E.g.: Superfluidity
- Spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (Meissner effect). E.g.: Superconductivity
- Gap in fermion spectrum.

  Adding a fermion near the Fermi surface now costs energy because it disrupts the condensate.

$$a_p^\dagger(\cos\theta + \sin\theta\, a_p^\dagger a_{-p}^\dagger) = \cos\theta\, a_p^\dagger$$

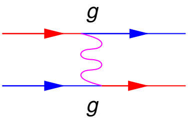


Interactions between Quarks

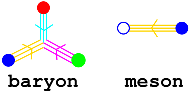
Dominant interaction between quarks is the strong interaction, mediated by exchange of gluons that couple to "color" charge (QCD).

Properties of QCD

- Short distances, $r \ll 1$ fm, asymptotically free: gauge coupling $g \ll 1$, single gluon exchange dominates, the theory is analytically tractable.



- Long distances $r > 1$ fm, QCD confines: color electric fields form flux tubes, only color-neutral states, baryons and mesons, exist.



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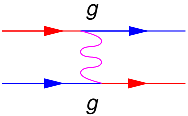

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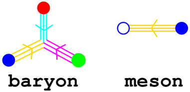
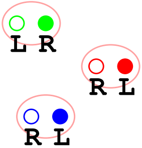


- At low temperature ($T \lesssim 170$ MeV), Chiral (left-right) symmetry is broken: color force can't turn a LH quark to RH, but our vacuum is full of $\bar{q}_L q_R$ pairs



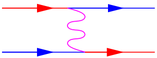
Handling QCD at high density

Lattice: “Sign problem”—negative probabilities

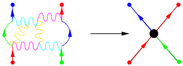
SUSY: Statistics crucial to quark Fermi surface

large N: Quarkyonic phase?

pert: Applicable far beyond nuclear density.
Neglects confinement and instantons.



NJL: Model, applicable at low density.
Follows from instanton liquid model.



EFT: Effective field theory for lightest degrees of freedom.
“Parameterization of our ignorance”: assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.

II. Color superconducting phases

Attractive QCD interaction ⇒ Cooper pairing of quarks.

We expect pairing between *different flavors*.

Quark Cooper pair: $\langle q^\alpha_{ia} q^\beta_{jb} \rangle$

color $\alpha, \beta = r, g, b$
flavor $i, j = u, d, s$
spin $a, b = \uparrow, \downarrow$

Each possible BCS pairing pattern P is an 18×18 color-flavor-spin matrix

$$\langle q^\alpha_{ia} q^\beta_{jb} \rangle_{1PI} = \Delta_P \, P^{\alpha\beta}_{ij\,ab}$$

The attractive channel is:

	space symmetric	[*s*-wave pairing]
	color antisymmetric	[most attractive]
	spin antisymmetric	[isotropic]
⇒	flavor antisymmetric	

Initially we will assume the most symmetric case, where all three flavors are massless.

High-density QCD calculations

- Guess a color-flavor-spin pairing pattern P
- to obtain gap Δ_P, minimize free energy Ω with respect to Δ_P (imposing color and electric neutrality)

$$\frac{\partial\Omega}{\partial\Delta_P} = 0 \qquad \frac{\partial\Omega}{\partial\mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

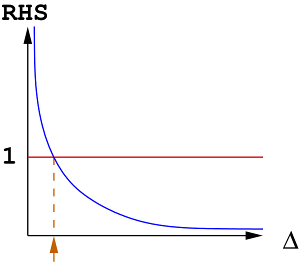
1. **Weak-coupling** methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.
2. **Nambu–Jona-Lasinio models**, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10-100$ MeV at $\mu \sim 400$ MeV.

Gap equation in a simple NJL model

Minimize free energy wrt Δ:

$$1 = \frac{8K}{\pi^2}\int_0^{\Lambda} p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p-\mu)^2}} \right\}$$



Note BCS divergence as $\Delta \to 0$: there is *always* a solution, for any interaction strength K and chemical potential μ.
Roughly,

$$1 \sim K\mu^2 \ln(\Lambda/\Delta)$$
$$\Rightarrow \Delta \sim \Lambda \exp\left(-\frac{1}{K\mu^2}\right)$$

Superconducting gap is non-perturbative.

Color supercond. in 3 flavor quark matter

Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, `hep-ph/9804403`)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta n} \epsilon_{ijn}$$

color α, β	This is invariant under equal and opposite
flavor i, j	rotations of color and (vector) flavor

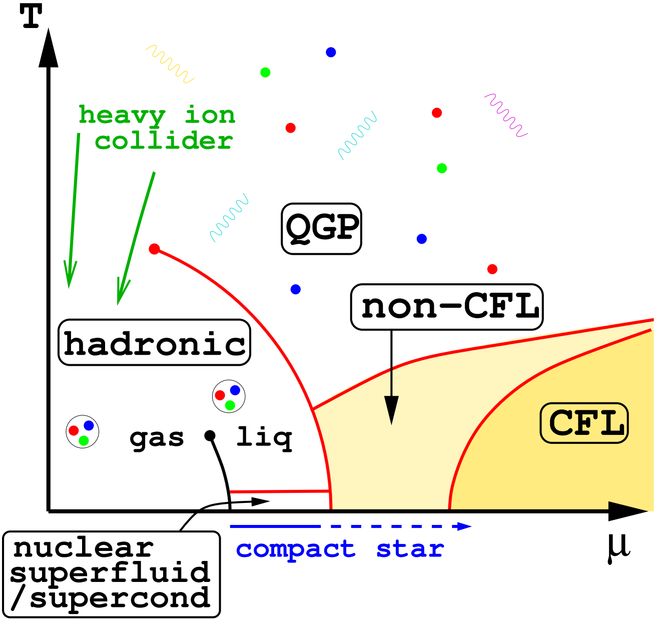
$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset\, U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset\, U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor-locked (“CFL”) quark pairing

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

Conjectured QCD phase diagram



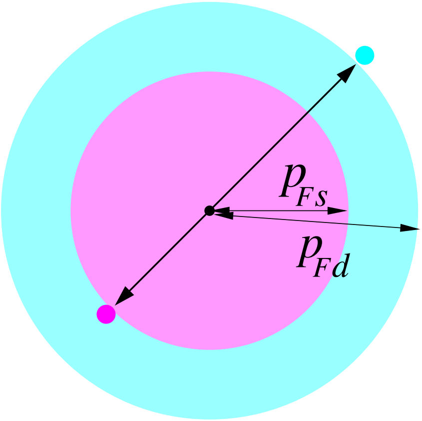
III. Quark matter in the real world

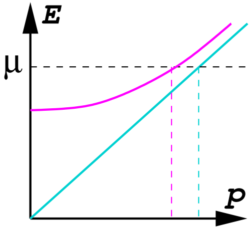
In the real world there are three factors that combine to oppose pairing between different flavors.

1. **Strange quark mass** is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. **Neutrality requirement.** Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. **Weak interaction equilibration.** In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

These factors favor different Fermi momenta for different flavors which *obstructs* pairing between different flavors.

Mismatched Fermi surfaces vs. Cooper pairing

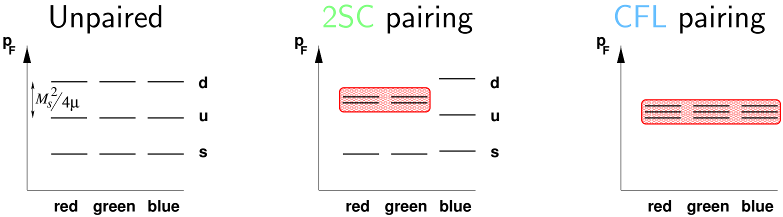




s and d quarks near their Fermi surfaces cannot have equal and opposite momenta.

The strange quark mass is the cause of the mismatch.

Cooper pairing vs. the strange quark mass



CFL: Color-flavor-locked phase, favored at the highest densities.

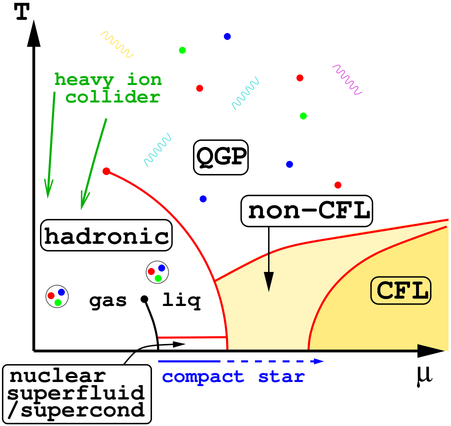
$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN}$$

2SC: Two-flavor pairing phase. May occur at intermediate densities.

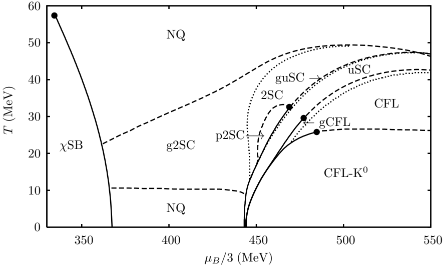
$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: CFL with kaon condensation (CFL-K^0),
crystalline phase (LOFF), p-wave "meson" condensates,
single-flavor pairing (color-spin locking, ∼liq ^{3}He-B).

Phases of quark matter, again



NJL model, uniform phases only

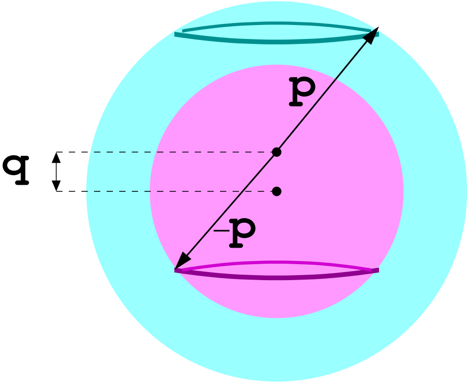


Warringa, `hep-ph/0606063`

But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, `hep-ph/0008208`)

Crystalline (LOFF) superconductivity

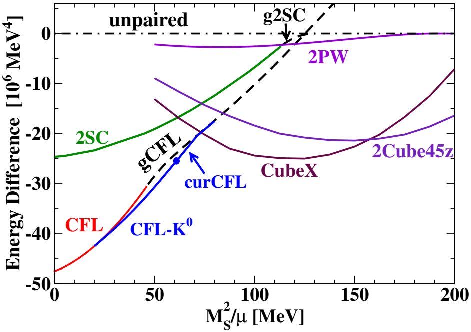
When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.



Every quark pair in the condensate has the same nonzero total momentum 2**q** (single plane wave LOFF).

Free energy comparison of phases

Assuming $\Delta_{\mathrm{CFL}} = 25$ MeV.



CFL-K^0	K^0 condensate
curCFL	K^0 cond current
2PW	LOFF, 2-plane-wave
CubeX	LOFF crystal, G-L approx
2Cube45z	LOFF crystal, G-L approx

(Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`)

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

IV. Looking to the future

- Observability of quark matter in neutron stars:
  - mass-radius relation
  - spindown
  - glitches
  - cooling
- instability of gapless phases; better treatment of LOFF
- role of large magnetic fields
- better weak-coupling calculations
- better models of quark matter: Functional RG, Schwinger-Dyson
- solve the sign problem and do lattice QCD at high density.