A Systematic Study of the Singular Isothermal Elliptical Lens Models in the Strong Regime

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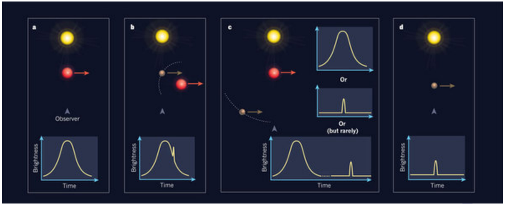
Content

What is Gravitational Lensing?

. . . a phenomenon where foreground mass distributions deflect the light rays from background sources.

On smaller angular scale, galaxies are made of stars. The Einstein radius of a solar mass star at a cosmological distance is of the order of microarcseconds, hence the name of microlensing.

- Stars within our galaxy.
- Resolution of current instruments is insufficient to detect microlensing.
- Relative motion of stars with respect to the background sources ⇒ Magnification pattern is modified over time-scales of just few years.



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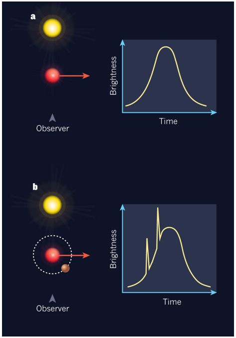
Exo-planet detection



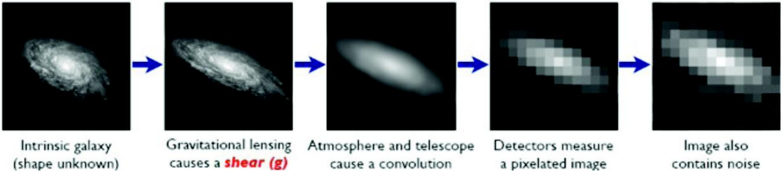
- Stars within our galaxy.
- Resolution of current instruments is insufficient to detect microlensing.
- Relative motion of stars with respect to the background sources $\Rightarrow$ Magnification pattern is modified over time-scales of just few years.

Stars in other galaxies: Typical Einstein radii are of the order of milliarcsecond, hence the name of millilensing.

. . . to Macro: Weak Lensing

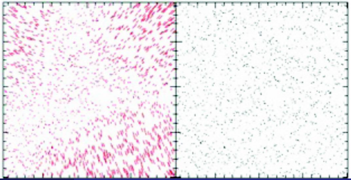
Weak Lensing effect causes small distortions and variations of the magnifications of sources.

Galaxies have intrinsic ellipticities $\Rightarrow$ Lensing signal induces an ellipticity



Sarah Bridle

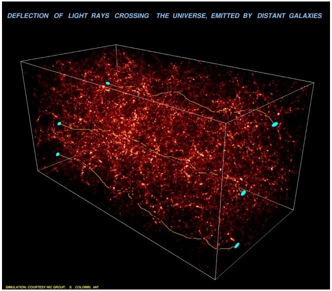
For each galaxy, the size and orientation are not known $\Rightarrow$ Statistical sample to detect in average the weak lensing signal.

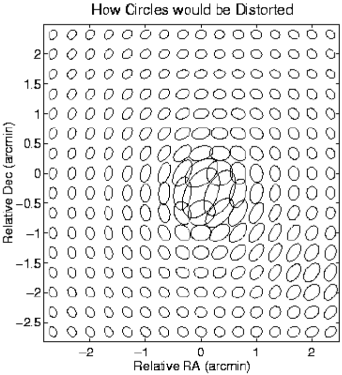


CFHTLS Deep 1
Gavazzi, Soucail 2006

Weak lensing is a powerful method for measuring the growth of structure over time.

- Cluster with small mass
- Outer part of clusters
- Large Scale Structure

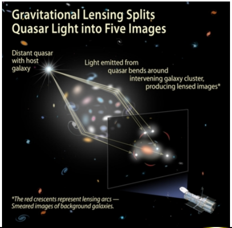


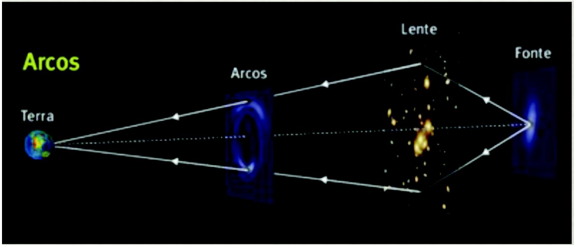


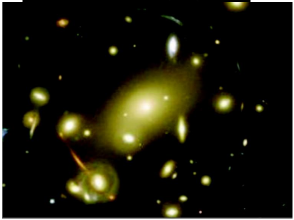
. . . to Macro: Strong Lensing

On large angular scale, when the lenses are galaxies or cluster of galaxies, it is possible to observe the image split. Multiple images, high magnifications and strong deformations (gravitational arcs and Einstein rings)

- Unique probes of the inner structure from galaxy to clusters ($\rho \propto r^{-\gamma}$, Ω_m, Ω_b, substructures, to study formation and evolution of these objects along the cosmic history.).
- Complementary probe to other cosmological observables (dependence of N_{arcs} on Ω_Λ,time delay between multiple images, primordial spectrum).
- Natural telescopes for high-z background galaxies $\Longrightarrow$ Galaxy evolution..







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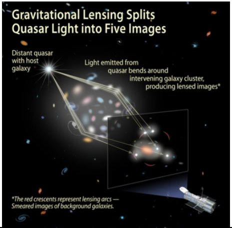
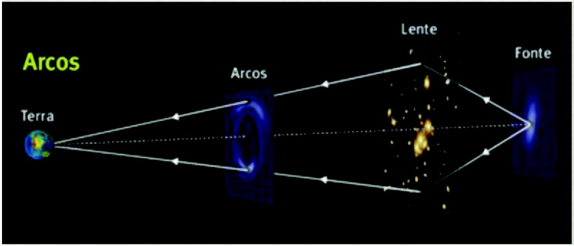
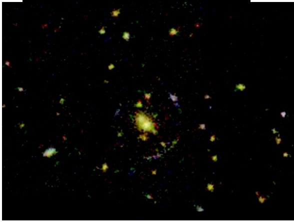






Content

A pinch of mathematics

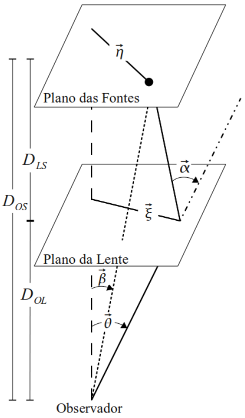


Figure: Schematic of Gravitational Lensing Geometry

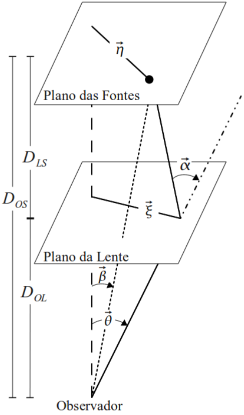
- Object emitting light (source): star, planet, galaxy, etc.
- Matter concentration (lens): stellar system, individual galaxies, cluster of galaxies, black holes, etc.
- Telescope/Detector (observer): localized at certain distance from the lens and source.
- Space-time: defined by the cosmological model and the matter distribution.

Lens Equation:

$$\vec{\beta} = \vec{\theta} - \vec{\hat{\alpha}}(\vec{\theta}) \left(\frac{D_{LS}}{D_{OS}} \right);$$

where $\vec{\xi} = D_{OL}\vec{\theta}$ and $\vec{\eta} = D_{OS}\vec{\beta}$

A pinch of mathematics



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Using length-scales:

$\vec{x} = \vec{\xi}/\xi_0$ and $\vec{y} = \vec{\eta}/\eta_0$ $(\eta_0 = D_{OS}/D_{OL}\xi_0)$

Dimensionless Lens Equation:

$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x}), \quad \vec{\alpha}(\vec{x}) = \nabla_x \varphi(\vec{x})$$

Lensing mapping

image $\rightarrow$ source mapping (relating areas)

$$J_{ij} = \delta_{ij} - \partial_{ij}\varphi(\vec{x})$$

$$\mathbf{J}_{ij} = \begin{pmatrix} 1-\kappa(\vec{x})+\gamma_1(\vec{x}) & -\gamma_2(\vec{x}) \\ -\gamma_2(\vec{x}) & 1-\kappa(\vec{x})-\gamma_1(\vec{x}) \end{pmatrix}$$

Lensing functions (single plane)

$$\begin{aligned} \kappa(\vec{x}) &= \frac{1}{2}\nabla_x^2\varphi(\vec{x}) \\ \gamma_1(\vec{x}) &= \frac{1}{2}\left(\partial_{11}\varphi(\vec{x}) - \partial_{22}\varphi(\vec{x})\right) \\ \gamma_2(\vec{x}) &= \partial_{12}\varphi(\vec{x}) \\ \gamma(\vec{x}) &= \sqrt{\gamma_1^2(\vec{x}) + \gamma_2^2(\vec{x})} \end{aligned}$$

Eigenvalues

$$\begin{aligned} \mu_r^{-1} = \lambda_r &= 1-\kappa(\vec{x})+\gamma(\vec{x}) \\ \mu_t^{-1} = \lambda_t &= 1-\kappa(\vec{x})-\gamma(\vec{x}) \end{aligned}$$

Point where the eigenvalues vanish $\rightarrow$ Critical Curves (lens plane) and Caustics (source plane).

Local magnification and axial ratio:

$$\begin{aligned} \mu(\vec{x}) &= \mu_r(\vec{x})\mu_t(\vec{x}) \\ R_\lambda(\vec{x}) &= \frac{\lambda_r(\vec{x})}{\lambda_t(\vec{x})} \end{aligned}$$

Content

Motivation for simple toy models

- Lensing properties of combined model of dark matter with non-isothermal stellar density profiles, are very close to the isothermal models (van den Ven et al. 2009, Mandelbaum et al. 2009)
- Fit well most of the galaxy-galaxy strong lensing systems (Ratnatunga et al. 1999; Bolton et al. 2006; Koopmans et al. 2006; Faure et al. 2008; Jackson 2008).

And are consistent with

- Spiral galaxy rotation curves (e.g., Rubin, Ford & Thonnard 1978, 1980)
- Stellar dynamics of elliptical galaxies (e.g., Rix et al. 1997)
- X-ray halos of elliptical galaxies (Fabbiano 1989)

Summary of SI models: Axial Symmetry

- Density profile

$$\rho(\vec{\xi}, z) = \frac{\sigma_v^2}{2\pi G(\xi^2 + z^2)}, \tag{1}$$

σ_v: velocity dispersion
$\xi = |\vec{\xi}|$: perpendicular coordinate to the line-of-sight
z: coordinate on the lens-observer direction

- Einstein Ring as a length-scale $\left(\vec{x} = \vec{\xi}/\xi_0\right)$

$$\xi_0 = \frac{\sigma_v^2}{G\Sigma_{\text{crit}}}, \quad \Sigma_{\text{crit}} = \frac{c^2}{4\pi G}\frac{D_{OS}}{D_{OL}D_{LS}} \tag{2}$$

- Dimensionless lensing potential and convergence

$$\varphi(\vec{x}) = x, \quad \kappa(\vec{x}) = \frac{1}{2x} \tag{3}$$

Summary of SI models: Elliptical Symmetry

"Introducing the ellipticity either on the lensing potential (pseudo-elliptical model, Dúmet-Montoya et al 2012) or on the mass distribution (elliptical model, Caminha et al 2013)"

It is done by

$$x \to \frac{x}{ab}\sqrt{\Delta}, \quad \Delta = a^2 \sin^2\phi + b^2 \cos\phi, \tag{4}$$

on $\varphi(x)$ or on $\kappa(x)$ with $a > b$ and defining the ellipticity

$$\varepsilon = 1 - \frac{b}{a} \tag{5}$$

To fix notation: subscript φ and Σ refer to Singular Isothermal with Elliptic Potential (SIEP) and Singular Isothermal Ellipsoid (SIE) models, respectively.

Summary of SI models: Elliptical Symmetry

(a) Deflection Angles:

$$\begin{aligned}
\alpha(\vec{x}) &= (\cos\phi, \sin\phi), \\
\alpha_\varphi(\vec{x}) &= \frac{1}{\sqrt{\Delta}}\left(\frac{b_\varphi\cos\phi}{a_\varphi}, \frac{a_\varphi\sin\phi}{b_\varphi}\right), \\
\vec{\alpha}_\Sigma(\vec{x}) &= f(\varepsilon_\Sigma)(\arctan(\zeta(\phi)\cos\phi), \operatorname{arctanh}(\zeta(\phi)\sin\phi)),
\end{aligned}$$

where

$$f(\varepsilon_\Sigma) = \frac{a_\Sigma b_\Sigma}{\sqrt{a_\Sigma^2 - b_\Sigma^2}} \quad \text{and} \quad \zeta(\phi) = \left(\frac{a_\Sigma^2 - b_\Sigma^2}{\Delta}\right)^{1/2},$$

(b) Convergences:

$$\kappa(\vec{x}) = \frac{x_t(\phi)}{2x}, \quad x_t(\phi) = \begin{cases} 1 & \text{(SIS)}, \\ a_\varphi b_\varphi \Delta^{-3/2} & \text{(SIEP)}, \\ a_\Sigma b_\Sigma \Delta^{-1/2} & \text{(SIE)}. \end{cases}$$

(c) Components of the Shear:

$$\gamma_1(\vec{x}) = -\cos 2\phi\kappa(\vec{x}), \quad \gamma_2(\vec{x}) = -\sin 2\phi\kappa(\vec{x}),$$

next: SI + external fields

Content

SI models $+\kappa_{\rm ext} + \gamma_{\rm ext}$

- Lensing galaxies often are found in group or cluster of galaxies $\rightarrow$ mimic the interaction with neighbour galaxies.
- Improving the fits of observed systems.

Adding small perturbations to the angular structure of the lensing potential:

Deflection Angle:

$$\vec{\alpha}_{\rm ext}(\vec{x}) = \begin{pmatrix} \kappa_{\rm ext} + \gamma_{\rm ext}\cos 2\phi_\gamma & \gamma_{\rm ext}\sin 2\phi_\gamma \\ \gamma_{\rm ext}\sin 2\phi_\gamma & \kappa_{\rm ext} - \gamma_{\rm ext}\cos 2\phi_\gamma \end{pmatrix} \vec{x} + \vec{\alpha}(\vec{x}).$$

where $\alpha(\vec{x})$ is the deflection angle of the ISI model.

Convergence: $K(\vec{x}) = \kappa(\vec{x}) + \kappa_{\rm ext}$

Strength of the Shear: $\Gamma(\vec{x}) = \left[\kappa^2(\vec{x}) + \gamma_{\rm ext}^2 - 2\kappa(\vec{x})\gamma_{\rm ext}\cos 2(\phi - \phi_\gamma)\right]^{1/2}.$

Solutions for constant magnification and constant distortion curves

Constant Magnification Curves: Typical forming-region of images with magnification above $\mu_{\rm th}$. (solution of $\mu_t(\vec{x})\mu_r(\vec{x}) = \pm\mu_{\rm th}$)

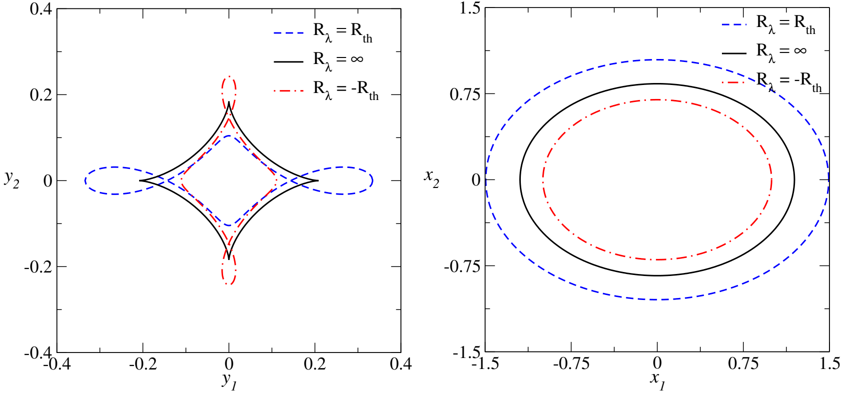
$$x_\mu(\phi, \kappa_{\rm ext}, \gamma_{\rm ext}) = x_t(\phi)\left[\frac{1-\kappa_{\rm ext}-\gamma_{\rm ext}\cos 2(\phi-\phi_\gamma)}{(1-\kappa_{\rm ext})^2-\gamma_{\rm ext}^2-\mu_{\rm th}^{-1}}\right],$$

Constant Distortion Curves: Typical forming-region of arc-like images with $L/W > R_{\rm th}$ (solution of $R_\lambda(\vec{x}) = \lambda_r(\vec{x})/\lambda_t(\vec{x}) = \pm R_{\rm th}$)

$$x_\lambda(\phi, \kappa_{\rm ext}, \gamma_{\rm ext}) = \frac{x_t(\phi)}{2C_\lambda}\left[B_\lambda + \sqrt{B_\lambda^2 - A_\lambda C_\lambda}\right].$$

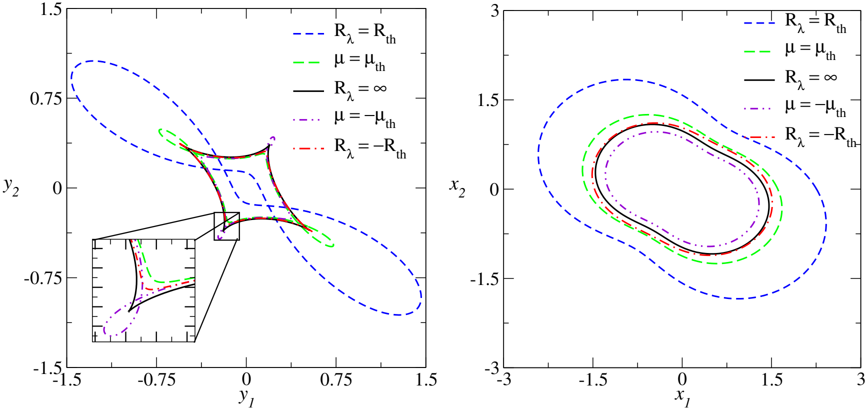
$$\begin{aligned} A_\lambda &= 1 - Q_\lambda^2; \quad Q_\lambda = \frac{R_\lambda+1}{R_\lambda-1} \\ B_\lambda &= 1-\kappa_{\rm ext} - Q_\lambda^2\gamma_{\rm ext}\cos 2(\phi-\phi_\gamma), \\ C_\lambda &= (1-\kappa_{\rm ext})^2 - Q_\lambda^2\gamma_{\rm ext}^2. \end{aligned}$$

Solutions for constant magnification and constant distortion curves



Constant magnification $\mu = \pm\mu_{\rm th}$ and distortion $R_\lambda = \pm R_{\rm th}$ curves for $\mu_{\rm th} = R_{\rm th} = 10$ and the tangential critical curve $(R_\lambda = \infty)$ for the SIE model with $\varepsilon_\Sigma = 0.3$,

Solutions for constant magnification and constant distortion curves



Constant magnification $\mu = \pm\mu_{\rm th}$ and distortion $R_\lambda = \pm R_{\rm th}$ curves for $\mu_{\rm th} = R_{\rm th} = 10$ and the tangential critical curve ($R_\lambda = \infty$) for the SIE model with $\varepsilon_\Sigma = 0.3$, $\kappa_{\rm ext} = 0.1, \gamma_{\rm ext} = 0.2$ and $\phi_\gamma = 45^o$. (next: applications)

- A big sample of lenses and sources (doesn't need detailed information on arcs).
- Ideal for a wide field surveys having a big number of clusters or massive galaxies.
- Usually approached with simulations
- Boost with observational project as *Dark Energy Survey* and *Large Synoptic Survey Telescope*: $\sim 10^3 - 10^4$ arcs.

Limits on the matter distribution of lenses and cosmological parameters

Content

A motivation: Lens inversion

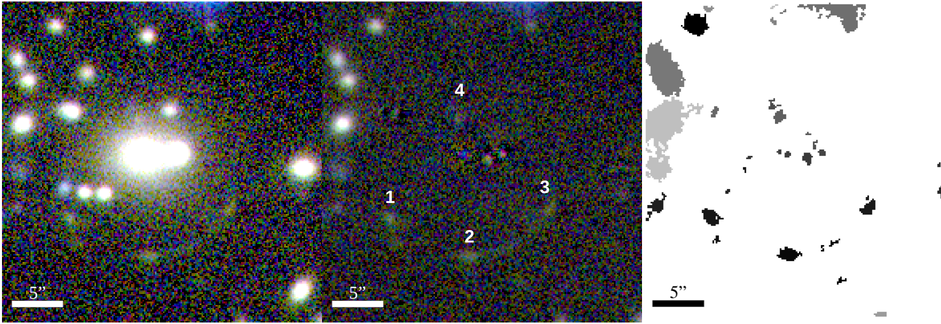


Figure: Images from SOGRASSL094053.70+074425.56. (by G. Caminha)

...Knowing the images, which is the fitting model?

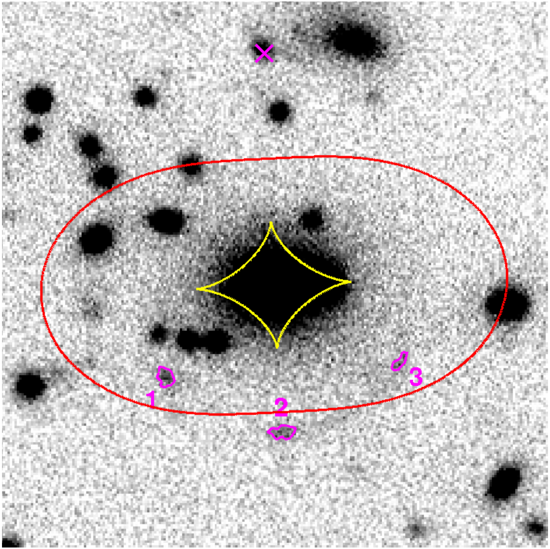


Figure: Lens inversion with a SIEP model (by G. Caminha)

Solution of the lens equation

- The boundary of an elliptical source can be parametrized as

$$
\begin{aligned}
y_1 &= u_0 + R_s(a_s \cos\vartheta)\cos\phi_s - R_s(b_s \sin\vartheta)\sin\phi_s, \\
y_2 &= v_0 + R_s(a_s \cos\vartheta)\sin\phi_s + R_s(b_s \sin\vartheta)\cos\phi_s,
\end{aligned}
$$

  where $0 \leq \vartheta \leq 2\pi$ and a_s and b_s $(a_s > b_s)$ define the source ellipticity.

- Using adequately the lens equation, we obtain a vectorial equation

$$
\vec{R}_s = \frac{1}{a_s}(x\bar{P}_1 - \bar{s}_1)\hat{x}_1 + \frac{1}{b_s}(x\bar{P}_2 - \bar{s}_2)\hat{x}_2,
$$

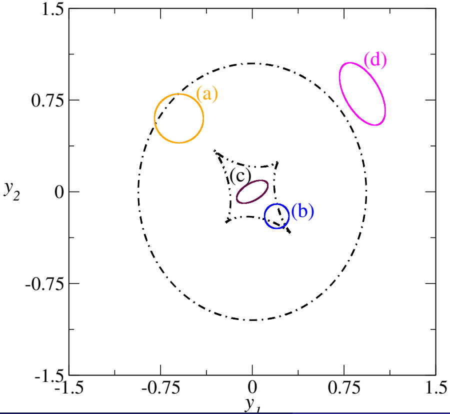
$$
\begin{aligned}
\bar{P}_1 &= (1-\kappa_{\text{ext}})\cos(\phi-\phi_s) - \gamma_{\text{ext}}\cos(\phi+\phi_s-2\phi_\gamma), \\
\bar{P}_2 &= (1-\kappa_{\text{ext}})\sin(\phi-\phi_s) + \gamma_{\text{ext}}\sin(\phi+\phi_s-2\phi_\gamma), \\
\bar{s}_1 &= s_1\cos\phi_s + s_2\sin\phi_s, \quad (s_1 = u_2 + \alpha_1, \quad s_2 = v_s + \alpha_2) \\
\bar{s}_2 &= -s_1\sin\phi_s + s_2\cos\phi_s.
\end{aligned}
$$

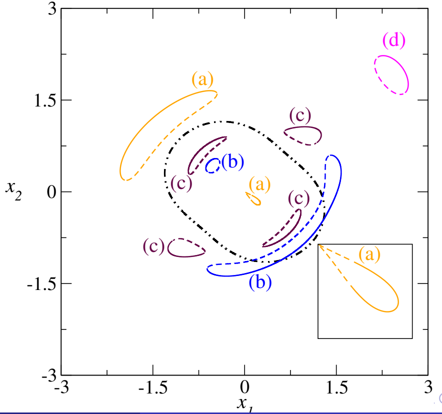
Solution of the lens equation

- Using $|\vec{R}_s|^2 = R_s^2$ and solving for x, we obtain

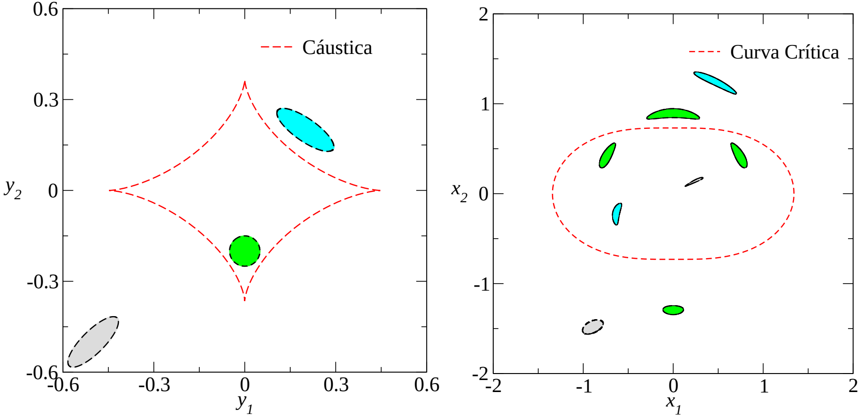
$$x_{\pm}(\phi) = \frac{1}{\bar{S}}\left[b_s^2 \bar{s}_1 \bar{P}_1 + a_s^2 \bar{s}_2 \bar{P}_2 \pm a_s b_s \sqrt{\bar{S} R_s^2 - \left(-\bar{s}_1 \bar{P}_2 + \bar{s}_2 \bar{P}_1\right)^2}\right]$$

where $\bar{S} = b_s^2 \bar{P}_1^2 + a_s^2 \bar{P}_2^2$.



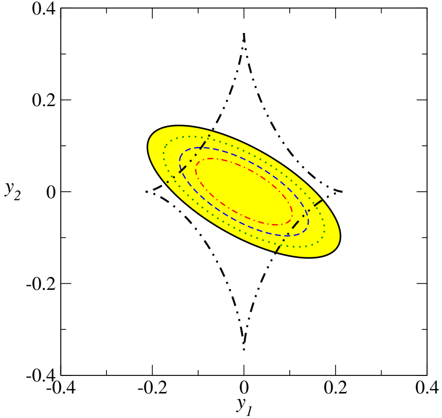


An example: Isolated SIEP



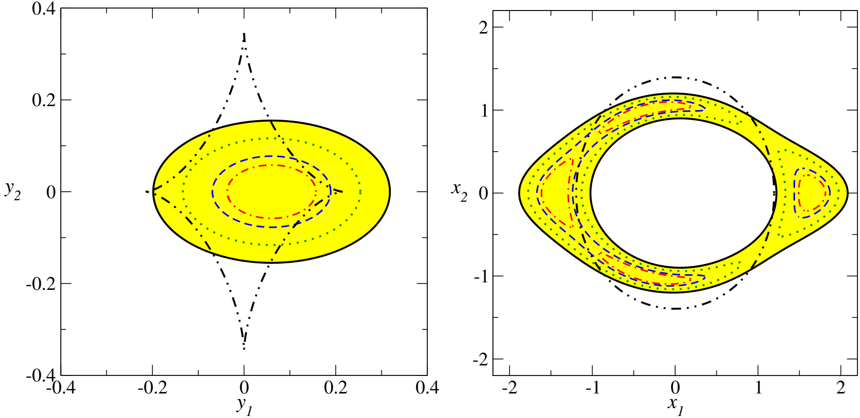
Solution for the SIEP model with $\varepsilon_\varphi = 0.18$.

Fold-arcs, Cusp-arcs and Fold-Cusps-Arcs



Solution for the SIE model with $\varepsilon_\Sigma = 0.3$, $\kappa_{\rm ext} = \gamma_{\rm ext} = 0.2$ and $\phi_\gamma = 0$, and sources with different radii.

Fold-arcs, Cusp-arcs and Fold-Cusps-Arcs



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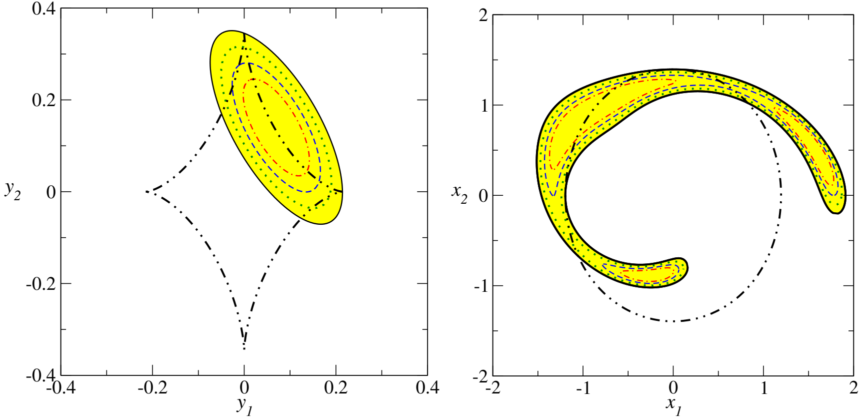

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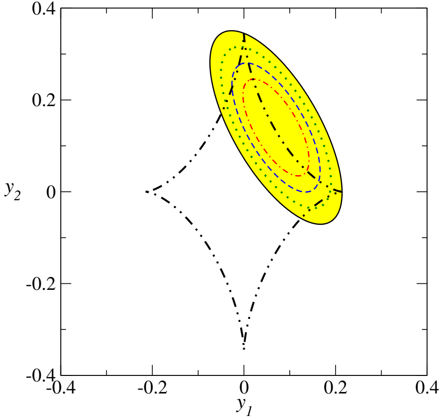

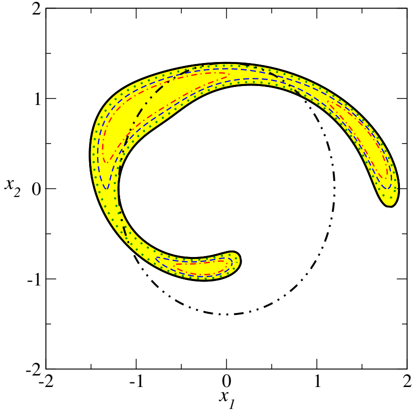

Solution for the SIE model with $\varepsilon_\Sigma = 0.3$, $\kappa_{\rm ext} = \gamma_{\rm ext} = 0.2$ and $\phi_\gamma = 0$, and sources with different radii.

Could give information about baryonic cooling process!

Content

Concluding Remarks

- We derived common analytic expressions for the convergence and shear for Isothermal models in presence of external fields.
- We derived analytic solutions for constant magnification and constant distortion curves.
- We developed an analytic method to invert the lens equation for elliptical sources with arbitrary orientation.

Including external fields

- Expands or diminishes the size of the critical curve and caustic.
- Break the equivalence between constant magnification and constant distortion curves.
- Changes the morphology of the images of lensed sources.

The results presented here belong to a set of papers to be submitted soon to Astronomy & Astrophysics.