Thermodynamical Analysis of the Black Hole with a Global Monopole in a f(R) Theory

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Motivation 1: History…

- Since the introduction of the Kibble Mechanism (T. W. B. Kibble, J. Phys. A 9, 1387 (1976)), many aspects of cosmological topological defects have been studied – Cosmic Strings, Domain Walls and Global Monopoles
- In 1989 Barriola and Vilenkin (Phys. Rev. Lett. 63, 341 (1989)) showed that in the presence of a Global Monopole the Static Spherically Symmetric (SSS) solution to Einstein's Equation can be written as

$$ds^2 = (1 - 8\pi G\eta^2 - \frac{2GM}{r})dt^2 - \frac{dr^2}{1 - 8\pi G\eta^2 - \frac{2GM}{r}}$$
$$- r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- In 1993 J. Jing, H. Yu, Y. Wang (Phys. Lett. A, 178, (1993)) studied the thermodynamics of a Black Hole with a Global Monopole.

Motivations 2: Nowadays

- More recently T. R. P. Caramês, E. R. Bezerra de Mello and M. E. X. Guimarães (arXiv:1106.4033) studied the gravitational effects of a global monopole in a modified theory of gravity (f(R)).

- Soon after H. Cheng and J. Man (arXiv:1301.2739 [hep-th]) derived the thermodynamic quantities of a Black Hole with a Global Monopole in the same model of modified gravity.

- In this presentation we will derive and analyze the thermodynamic quantities for a black hole + global monopole system in a more general f(R) model.

f(R) Theories

- Theories that propose an alteration of the Einstein-Hilbert action introducing a function of the Ricci scalar.

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S_m$$

Giving rise to a new set of field equations...

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = \kappa T_{\mu\nu}$$

With trace

$$\Box F(R) + \frac{1}{3} (F(R)R - 2f(R)) = \frac{1}{3} \kappa T$$

where $F(R) \equiv \frac{df(R)}{dR}$ is the *scalaron (*it carries an scalar degree of freedom) and it is subjected to stability conditions F(R) > 0 and F'(R) > 0.

f(R) Field Equations for the BH+GM system

- Using the general f(R) field equations one can see that the quantity

$$C_\mu = \frac{F(R)R_{\mu\mu} - \nabla_\mu \nabla_\mu F(R) - \kappa T^m_{\mu\mu}}{g_{\mu\mu}}$$

is index independent and can be used to obtain the field equations for the spherically symmetric metric of the BH+GM system. Assuming an SSS metric and $F(R(r)) = \mathcal{F}(r)$, and defining

$$\beta \equiv \frac{B'}{B} + \frac{A'}{A}$$

we obtain the field equations as:

$$\frac{\beta}{r} = \frac{\mathcal{F}''}{\mathcal{F}} - \frac{1}{2}\frac{\mathcal{F}'}{\mathcal{F}}\beta \ ,$$

$$\begin{aligned} &- \quad 4B + 4AB - 4rB\frac{\mathcal{F}'}{\mathcal{F}} + 2r^2B'\frac{\mathcal{F}'}{\mathcal{F}} \\ &+ \quad 2r^2B'' - r^2B'\beta + 2Br\beta - \frac{4AB\kappa\eta^2}{\mathcal{F}} = 0 \ . \end{aligned}$$

Solutions in the Weak Field Regime

- Assuming $B(r) = 1 + b(r)$ and $A(r) = 1 + a(r)$, with $|b(r)|$ and $|a(r)|$ smaller then 1 (where $ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2$)
- Assuming that the modification of gravity is a small perturbation of GR, namely $\mathcal{F}(r) = 1 + \psi(r)$, with $|\psi(r)| << 1.$
- We obtain the linearized field equations;

$$\frac{\beta}{r} = \psi''$$ and

$$4a - 4r\psi' + 2r(a' + b') + 2r^2 b'' - 4(1 + a + b - \psi)\kappa\eta^2 = 0\ .$$

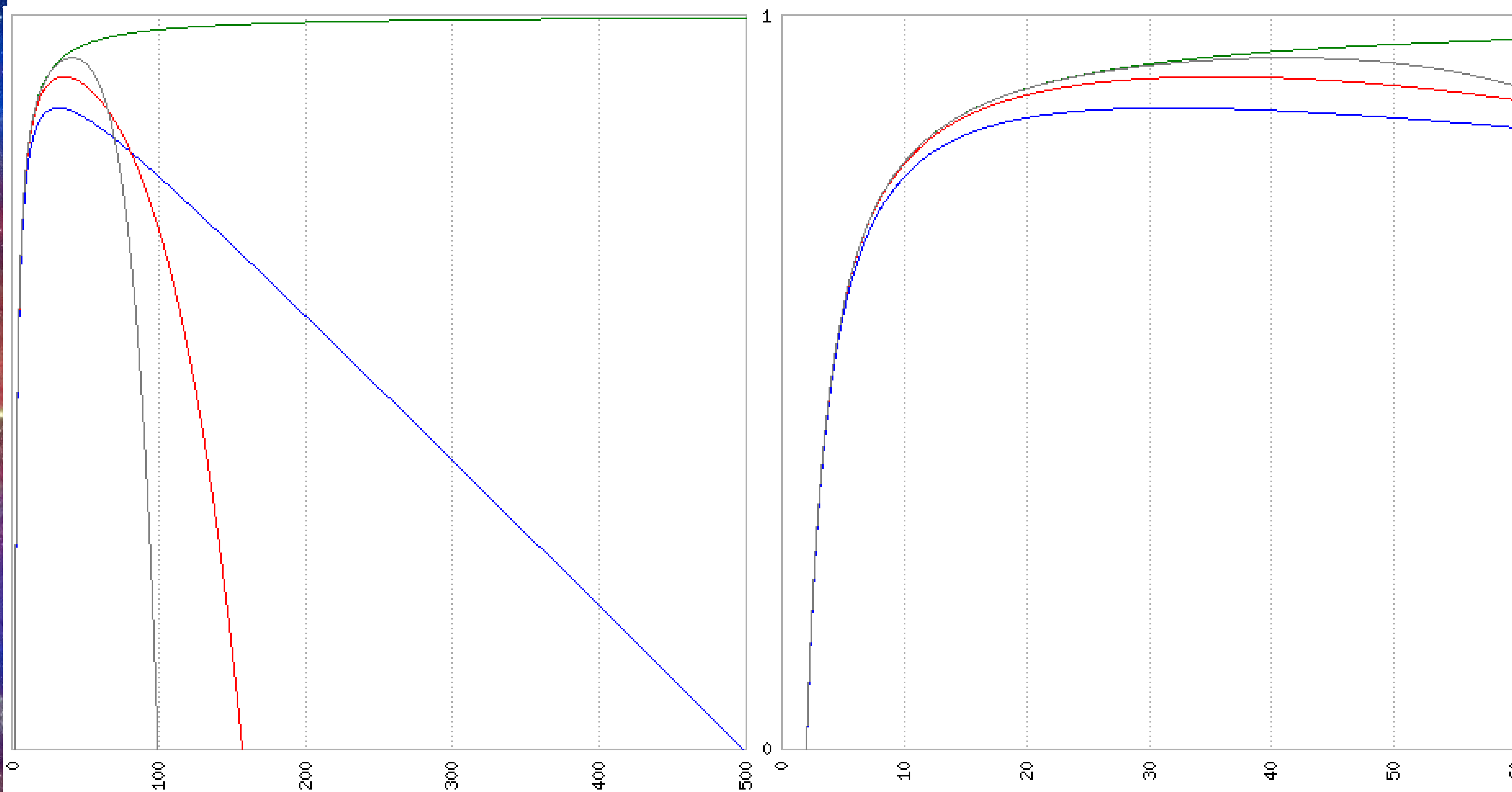
- Different choices for the function $\psi(r)$ give rise to different effects on the metric. Here, we are interested in the cases $\psi(r) = \psi_n r^n$ and the effects of the variation of the degree n in the solution.
- The Solutions for a(r) and b(r) are given by

$$a(r) = \frac{2GM}{r} + 8\pi G\eta^2 + (n - \frac{8\pi G\eta^2}{n+1})\psi_n r^n$$

$$b(r) = \frac{e^{(n-1)\psi_n r^n}}{1+a(r)}$$

Black Hole Solutions?

- To show that this are black hole solutions for a wide variety of n's we analyze the graphs for $B(r) \approx e^{(n-1)\psi_n r^n}(1-a(r))$, since it is not possible to find real roots for the equation B(r) = 0 for n>3;



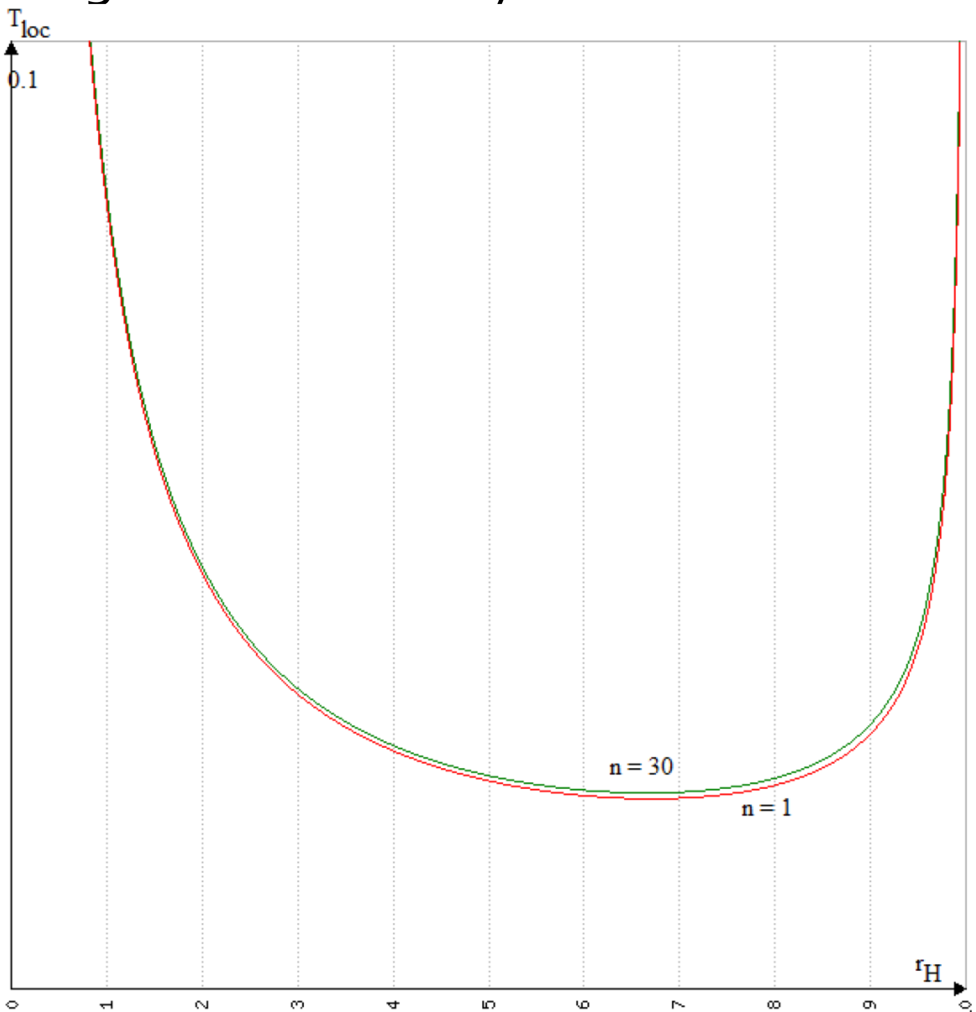
Green: RG solution; Blue: n =1; Red: n = 2; Grey: n = 5 The apparent outer horizon is in a region where $\psi(r) \sim 1$, so we have only one horizon.

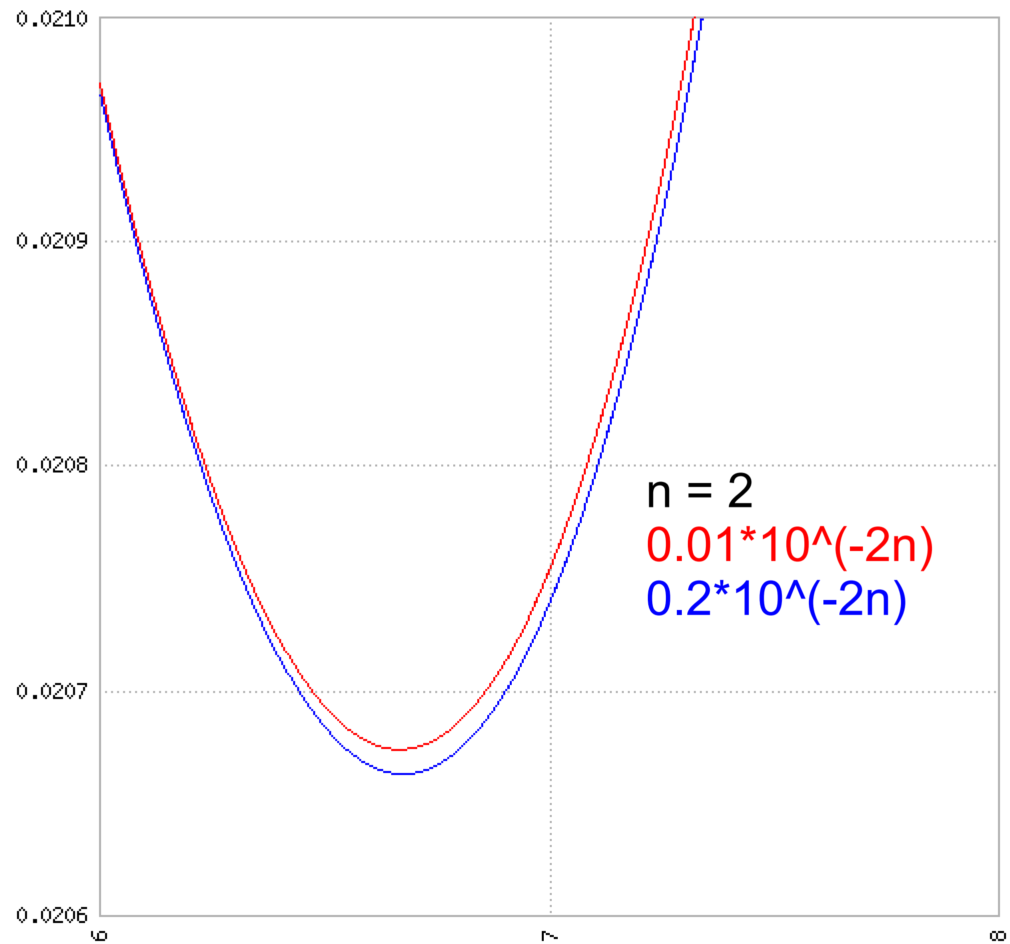
Thermodynamic quantities

- Calculating the Hawking Temperature from the superficial gravity we obtained

$$T_H = \frac{1}{4\pi}[\sqrt{-g^{tt}g^{rr}}g'_{tt}]|_{r_H} \approx \frac{1}{4\pi}b'(r_H) = \frac{1}{4\pi r_H}[1 - 8\pi G\eta^2 - (n+1)\psi_n r_H^n]$$

- Using Tolman's argument to calculate a local temperature (our results are restrict to the regions where $r^n << \frac{1}{\psi_n}$), we obtained the function $T_{loc} = \frac{b'(r_H)}{4\pi\sqrt{B(r)}}$ for generic n an analyzed the effects of varying n and ψ_n





Thermodynamic quantities

Using Bekenstein's relation of the black hole entropy with the event horizon area together with the first law of thermodynamics we can obtain a *local heat capacity*, as follows:

$$dE_{loc} = T_{loc} dS,$$

$$E_{loc} = E_0 + \int_{S_0}^{S} T_{loc} 2\pi r_H \, dr_H,$$

$$C = \left(\frac{\partial E_{loc}}{\partial T_{loc}}\right)_r$$
$$= 2\pi r_H^2 (1-\kappa\eta^2-(n+1)\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r_H^n)(r(1-\kappa\eta^2-\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r^n)-r_H(1-\kappa\eta^2-\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r_H^n)] \times [\tfrac{r_H}{2}(1-\kappa\eta^2-(n+1)\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r_H^n)^2-(1-\kappa\eta^2+(n^2-1)\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r_H^n)(r(1-\kappa\eta^2-\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r^n)-r_H(1-\kappa\eta^2-\psi_n(n-\tfrac{2\kappa\eta^2}{n+1})r_H^n)]^{-1}$$

This complicated expression gives almost the same curves for a wide variety of n's.

Local phase transitions

- Plot of the heat capacity for a position r = 10. As the horizon grows to 10 the heat capacitiy goes trough a transition before reaching it

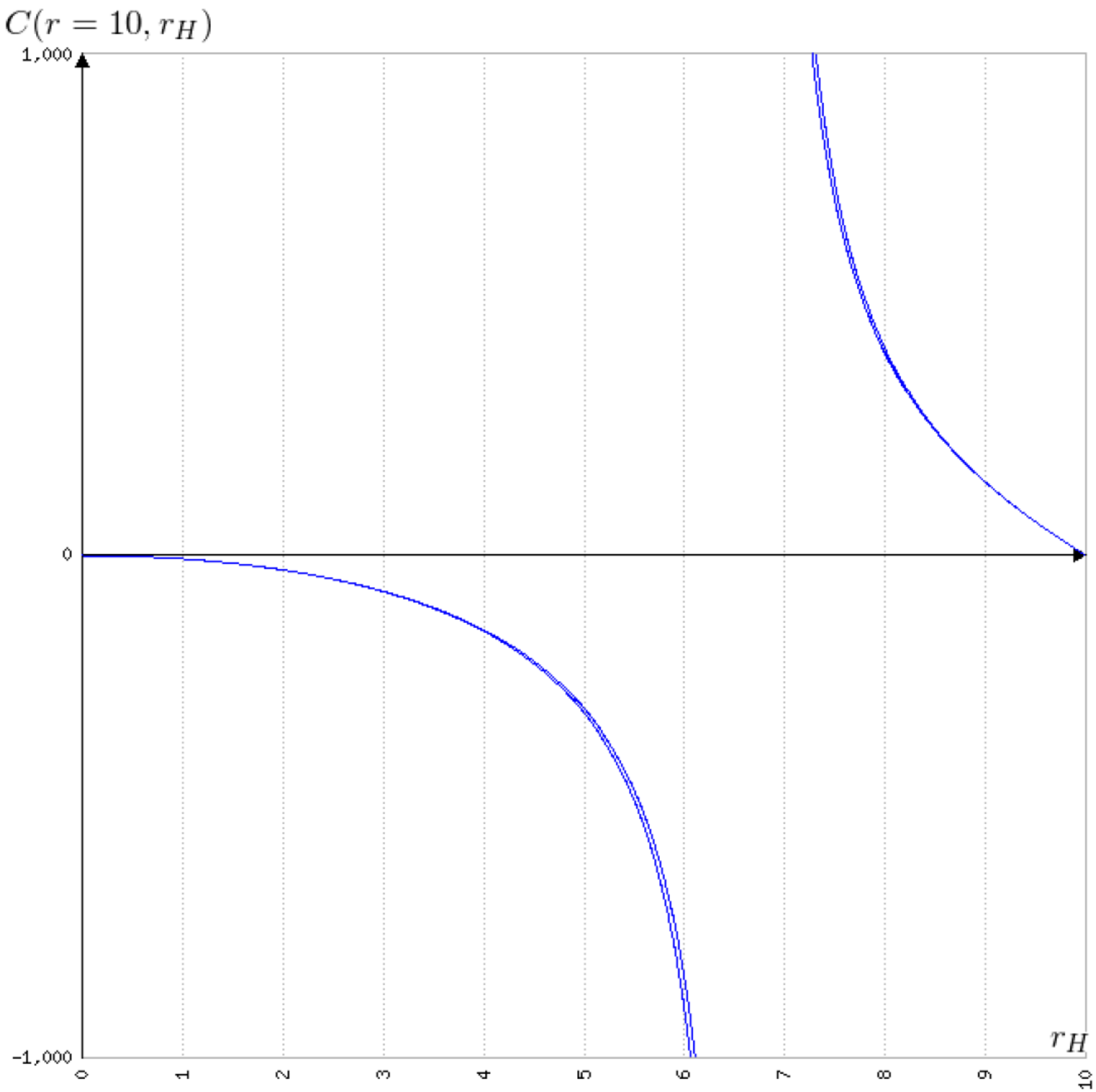


Figure 4: Plot of the heat capacity vs the event horizon for n varying from 1 to 40, at r = 10 and with $\psi_n = 0.2 \times 10^{-2n}$ *and* $\kappa\eta^2 = 10^{-5}$.

Local phase transitions

Plot of the heat capacity for a fixed horizon radius 2. As the observer goes away from the horizon the heat capacity assumes a negative const. Value, as expected

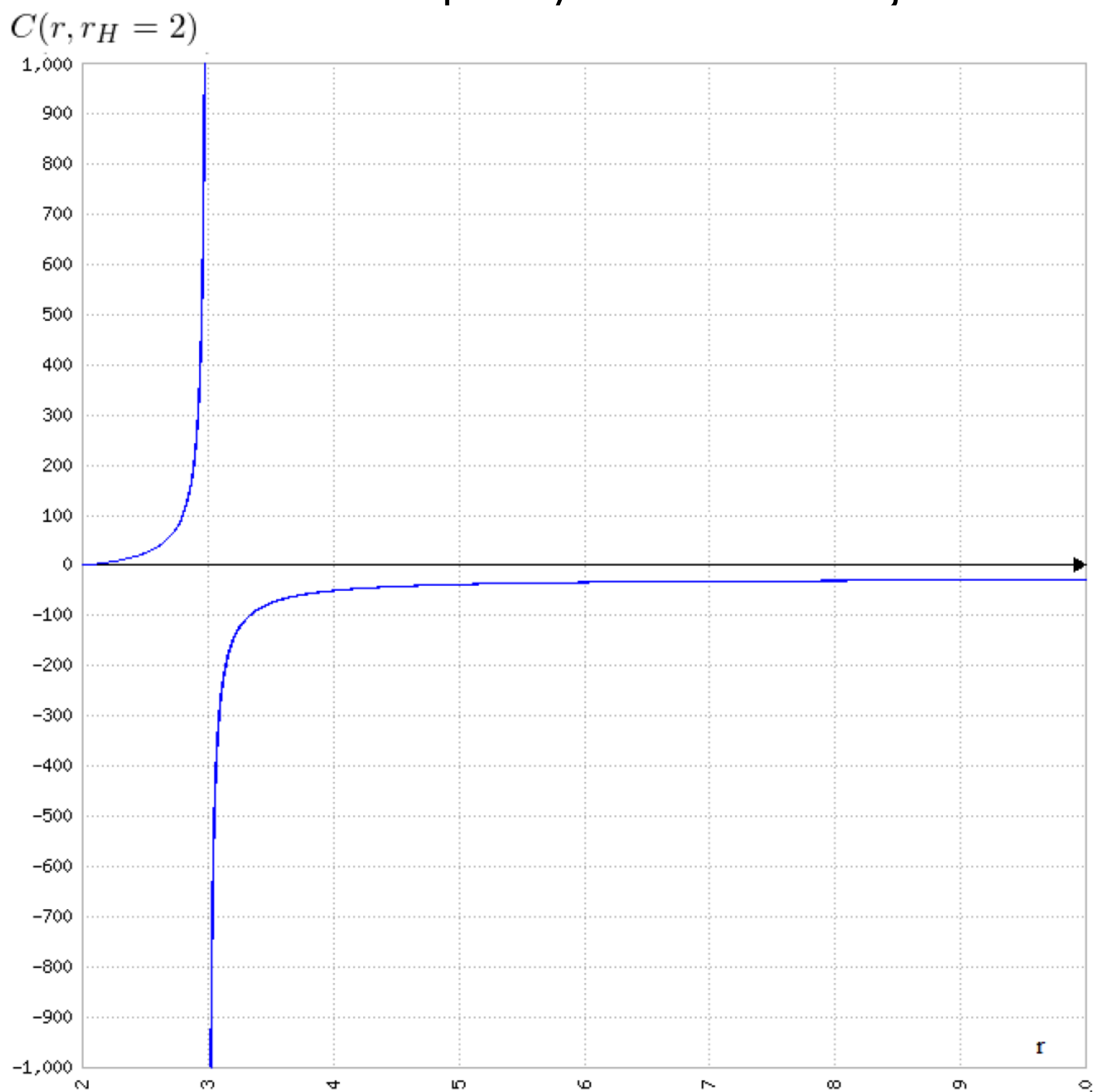


Figure 5: Plot of the heat capacity vs the position r, for a black hole with event horizon of size 2 for n varying from 1 to 40 with $\psi_n = 0.2 \times 10^{-2n}$ *and* $\kappa\eta^2 = 10^{-5}$.

Local phase transitions

For higher values of the horizon radius the phase transition becomes more drastic

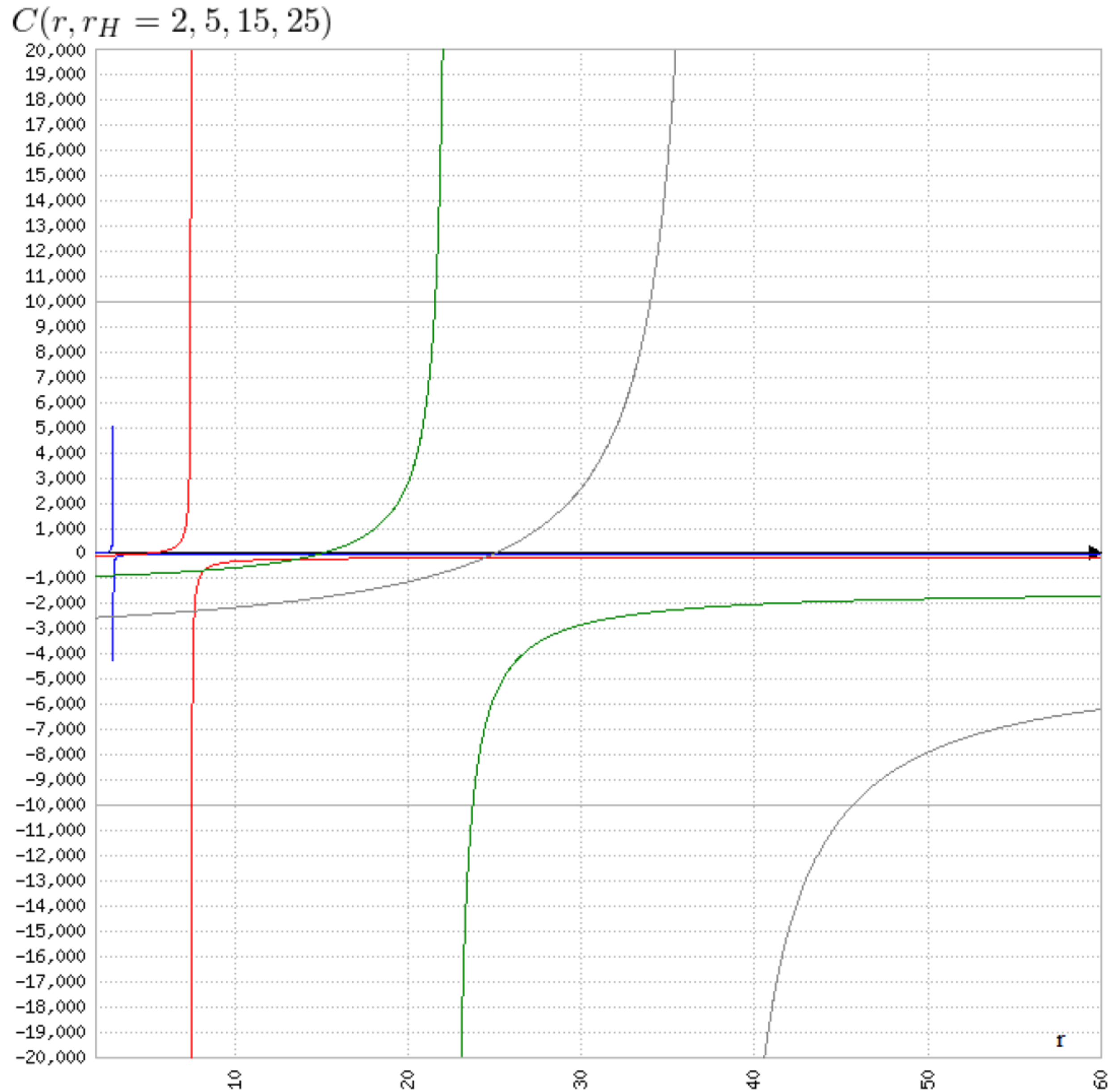


Figure 6: Plot of the heat capacity vs the radial position with n = 10 for $r_H = 2, 5, 15, 25$ *, with* $\psi_n = 0.2 \times 10^{-2n}$ *and* $\kappa\eta^2 = 10^{-5}$.

Conclusions and open questions

- We first conclude that considering F(R) a radial function of the form 1 + $\psi_n r^n$ leads to a BH + GM solution for the metric and that the thermodynamics of this BH is *almost* independent of the degree n.
- The *almost* comes from the fact that, as we showed in the graph for the local temperature, if n is very high may have a little difference in the minimum of local temperature.
- All values of n (and even without the modification) have a heat capacity that indicates a thermodynamical phase transition at the surrounding of the BH.
- The phase transitions are instabilities of the configuration?
- The Unruh Temperature of this configuration is compatible with the Hawking Temperature?

Thank you!

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