Introdução à correspondência AdS/CFT
Parte 5

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Soft-wall AdS/QCD Model

Soft cut off (Karch, Katz, Son, Stephanov PRD 2006)

$$\int d^5x\sqrt{-g}\,\mathcal{L} \quad \Longrightarrow \quad \int d^5x\sqrt{-g}e^{-\Phi}\mathcal{L}\,. \quad ; \quad \Phi(z) = cz^2$$

spectrum of vector mesons $\quad m^2_{V_n} = 4c(n+1)\,,$

Glueballs in the soft-wall

[Colangelo, De Fazio, Jugeau, Nicotri PLB(2007)]

The corresponding glueball spectrum is

$$m^2_{G_n} = 4c(n+2)\,.$$

Softwall Model

1

Colangelo et al 2007 (scalar, vector and tensor glueballs)
Capossoli, HBF 2016 (higher spin glueballs)

$$S = \int d^5x \sqrt{-g} e^{-\Phi(z)} \left[g^{mn} \partial_m \mathcal{G} \partial_n \mathcal{G} + M_5^2 \mathcal{G}^2 \right],$$

$$ds^2 = g_{mn} dx^m dx^n = \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dy^\mu dy^\nu),$$

$$\Phi(z) = kz^2$$

$$\partial_m [\sqrt{-g}\, e^{-\Phi(z)} g^{mn} \partial_n \mathcal{G}] - \sqrt{-g} e^{-\Phi(z)} M_5^2 \mathcal{G} = 0,$$

Softwall Model

2

$$\mathcal{G}(z, x^\mu) = v(z) \exp i q_\mu x^\mu ,$$

$$v(z) = \psi(z)(z/R)^{3/2} \exp \tfrac{1}{2}(kz^2),$$

"Schrödinger-like" equation

$$-\psi''(z) + \left[k^2 z^2 + \frac{15}{4z^2} + 2k + \left(\frac{R}{z}\right)^2 M_5^2 \right] \psi(z) = -q^2 \psi(z)$$

which has a well known solution:

$$\psi_n(z) = \mathcal{N}_n \, z^{t(M_5)+\frac{1}{2}} \, {}_1F_1(-n; t(M_5)+1, kz^2) \exp\{-kz^2/2\}$$

where $t(M_5) = \sqrt{4 + R^2 M_5^2},$

Softwall Model

The corresponding "eigenenergies" $-q^2 = -q_\mu q^\mu$ are identified with the 4-d glueball squared masses

$$m_n^2 = \left[4n + 4 + 2\sqrt{4 + M_5^2 R^2}\right] k; \quad (n = 0, 1, 2, \cdots).$$

$$\Delta = 2 + \sqrt{4 + R^2 M_5^2}$$

scalar glueball state 0^{++} $\mathcal{O}_4 = Tr(F^2) = Tr(F^{\mu\nu} F_{\mu\nu})$

$(M_5^2 = 0)$ $\Delta = 4.$

$$m_n^2 = [4n + 8] k; \quad (n = 0, 1, 2, \cdots).$$

Higher spin glueballs in the softwall model [1]

Analoglous to the higher spin Glueballs in the Hardwall

$$\Delta = 2 + \sqrt{4 + R^2 M_5^2} \qquad 0^{++},\ 2^{++},\ 4^{++},\ \text{etc.}$$

$$\mathcal{O}_{4+J} = F D_{\{\mu 1 \ldots} D_{\mu J\}} F\,, \qquad \Delta = 4 + J$$

$$M_5^2 R^2 = J(J+4)\,; \qquad (\text{even } J)\,.$$

$$m_n^2 = \left[4n + 4 + 2\sqrt{4 + J(J+4)}\right] k; \quad (n = 0, 1, 2, \cdots, \text{even } J)\,,$$

Higher spin glueballs in the softwall model 2

Analogous to the higher spin Glueballs in the Hardwall

$$\Delta = 2 + \sqrt{4 + R^2 M_5^2} \qquad 1^{--},\ 3^{--},\ 5^{--},\ \text{etc.}$$

$$\mathcal{O}_{6+J} = SymTr\left(\tilde{F}_{\mu\nu} F D_{\{\mu 1 \ldots} D_{\mu J\}} F\right),$$

$$M_5^2 R^2 = (J+6)(J+2)\,; \qquad (\text{odd } J),$$

$$m_n^2 = \left[4 + 2\sqrt{4 + (J+6)(J+2)}\right] k; \quad (\text{odd } J). \qquad (n=0)$$

Not good when compared with the literature!!!

A Dynamical Softwall Model 1

Li & Huang JHEP 2013 (scalar glueballs)
Capossoli, Li, HBF 2016 (higher spins)

The 5D action for the graviton–dilaton coupling in the string frame is given by:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_s}\, e^{-2\Phi(z)} \left(R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi)\right)$$

The metric tensor has the following form:

$$ds^2 = g_{mn}^s dx^m dx^n = b_s^2(z)(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu);$$

$$b_s(z) \equiv e^{A_s(z)}$$

A Dynamical Softwall Model 2

Einstein frame and equations of motion

$$g_{mn}^{E} = g_{mn}^{s} e^{-\frac{2}{3}\Phi}, \qquad V_G^E = e^{\frac{4}{3}\Phi} V_G^s ,$$

$$b_E(z) = b_s(z) e^{-\frac{2}{3}\Phi(z)} = e^{A_E(z)}, \qquad A_E(z) = A_s(z) - \frac{2}{3}\Phi(z) .$$

$$-A_E'' + A_E'^2 - \frac{4}{9}\Phi'^2 = 0 ,$$

and

$$\Phi'' + 3A_E'\Phi' - \frac{3}{8} e^{2A_E} \partial_\Phi V_G^E(\Phi) = 0 .$$

A Dynamical Softwall Model 3

For a quadratic dilaton

$$\Phi(z) = kz^2$$

one finds the solutions

$$A_E(z) = \log\left(\frac{R}{z}\right) - \log\left({}_0F_1(5/4, \frac{\Phi^2}{9})\right)$$

and

$$V_G^E(\Phi) = -\frac{12\,{}_0F_1(1/4, \frac{\Phi^2}{9})^2}{R^2} + \frac{16\,{}_0F_1(5/4, \frac{\Phi^2}{9})^2\Phi^2}{3R^2}$$

A Dynamical Softwall Model 4

Going back to the String frame

$$A_s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) - \log\left[{}_0F_1\left(\frac{5}{4}, \frac{\Phi^2}{9}\right)\right],$$

which means a deformed AdS space

$$ds^2 = g^s_{mn} dx^m dx^n = b_s^2(z)(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu); \quad b_s(z) \equiv e^{A_s(z)}$$

and a potential

$$V_G^s(\Phi) = \exp\{-\frac{4}{3}\Phi\}\left[-\frac{12\, {}_0F_1(1/4, \frac{\Phi^2}{9})^2}{R^2} + \frac{16\, {}_0F_1(5/4, \frac{\Phi^2}{9})^2 \Phi^2}{3R^2}\right]$$

A Dynamical Softwall Model 5

5D action for Scalar Glueballs in String frame

$$S = \int d^5x \sqrt{-g_s} \frac{1}{2} e^{-\Phi(z)} [\partial_M \mathcal{G} \partial^M \mathcal{G} + M_5^2 \mathcal{G}^2],$$

which implies the equations of motion

$$\partial_M [\sqrt{-g_s}\, e^{-\Phi(z)} g^{MN} \partial_N \mathcal{G}] - \sqrt{-g_s} e^{-\Phi(z)} M_5^2 \mathcal{G} = 0\,.$$

as before

$$\mathcal{G}(z, x^\mu) = v(z) e^{iq_\mu x^\mu}\,, \qquad B(z) = \Phi(z) - 3A_s(z)\,, \qquad v(z) = \psi(z) e^{B(z)/2}$$

so that one gets a Schrödinger-like equation:

$$-\psi''(z) + \left[\frac{B'^2(z)}{4} - \frac{B''(z)}{2} + M_5^2 \left(\frac{R}{z}\right)^2 e^{4kz^2/3} \mathcal{A}^{-2} \right] \psi(z)$$

$$= -q^2 \psi(z)\,, \qquad \text{where } \mathcal{A} = {}_0F_1(5/4, \Phi^2/9).$$

A Dynamical Softwall Model 6

Higher spins from AdS/CFT

$$M_5^2 R^2 = \Delta(\Delta - 4) - J \qquad (J = 0, 1, 2, 3, \cdots)$$

which implies an effective potential of the form

$$V_J(z) = k^2 z^2 + \frac{15}{4z^2} - 2k + \frac{\Delta(\Delta-4) - J}{z^2} e^{4kz^2/3} \mathcal{A}^{-2} .$$

A Dynamical Softwall Model 6

Higher spins from AdS/CFT

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Even spins and the pomeron

twist 2 trajectory $\Delta = J + 2$

Complex masses for 0++ and 2++ → No Regge trajectory

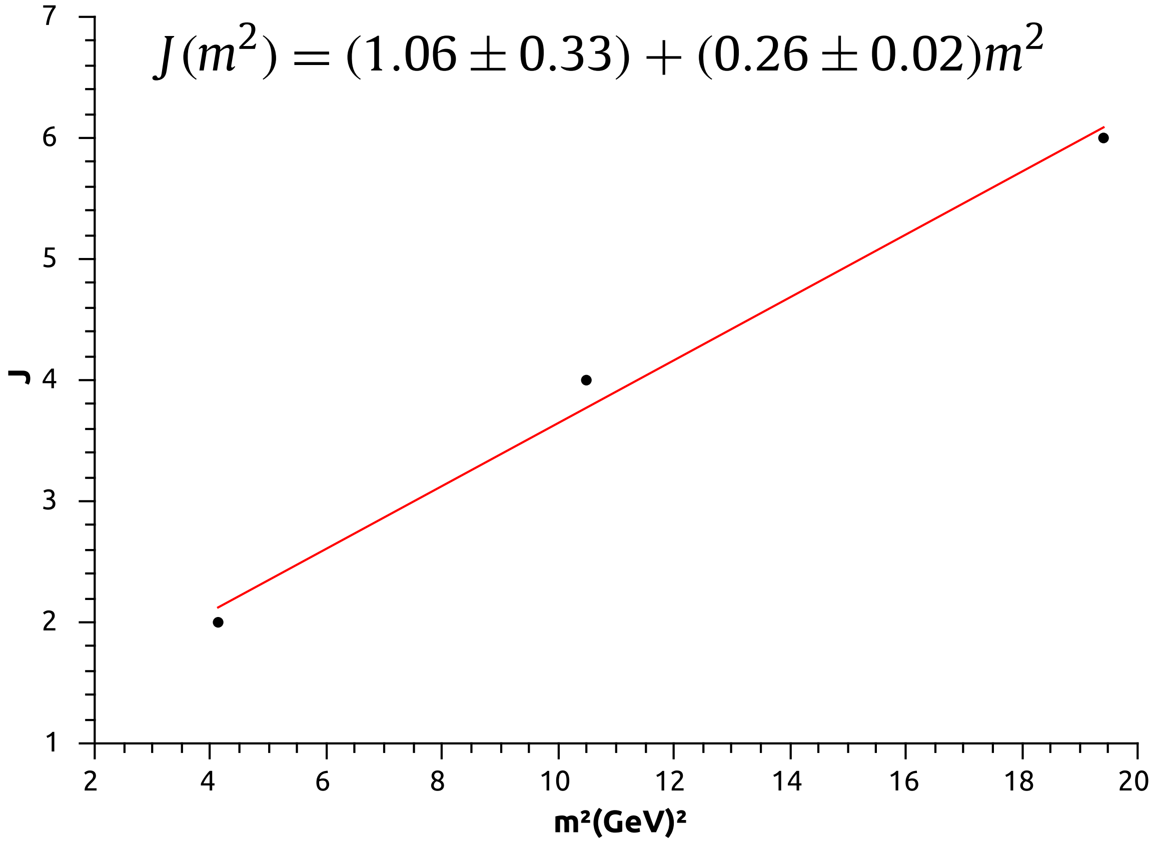
twist 4 trajectory $\Delta = J + 4$

Real masses for 0++ and 2++, etc → Good Regge trajectories

Table 1
Masses m_n expressed in GeV for the glueball states J^{PC} with even J as the eigenstates of Eq. (9) with the potential (12) for $k = 0.10$ GeV^2.

	Glueball states J^{PC}						k
	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}	
m_n	0.51	2.03	3.23	4.40	5.56	6.71	0.10

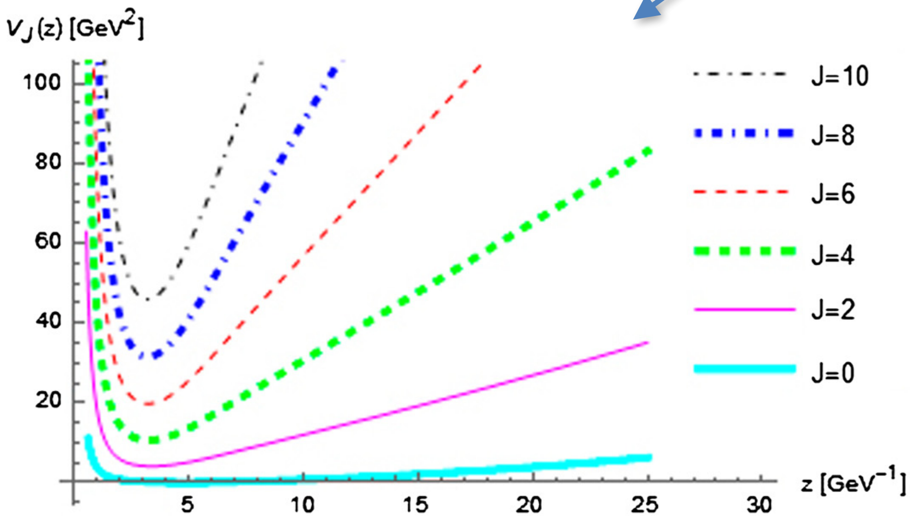
Regge trajectory: glueball states 2^{++}, 4^{++} and 6^{++}

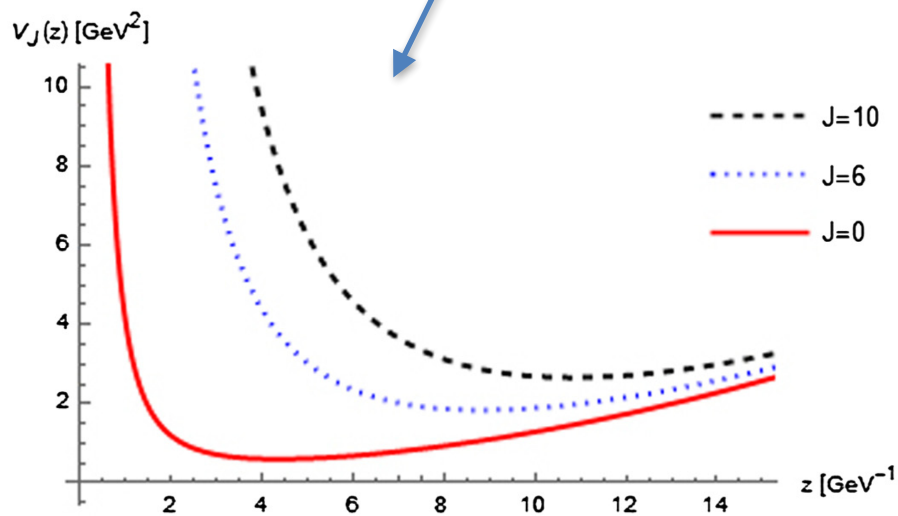


Very good compared to experimental pomeron:

$$J(m^2) \approx 1.08 + 0.25\, \mathrm{m}^2,$$

Effective potentials in the Dynamical Softwall X usual Softwall





Odd spins and the odderon

$$\mathcal{O}_{6+J} = SymTr\left(\tilde{F}_{\mu\nu} F D_{\{\mu 1 \ldots} D_{\mu J\}} F\right),$$

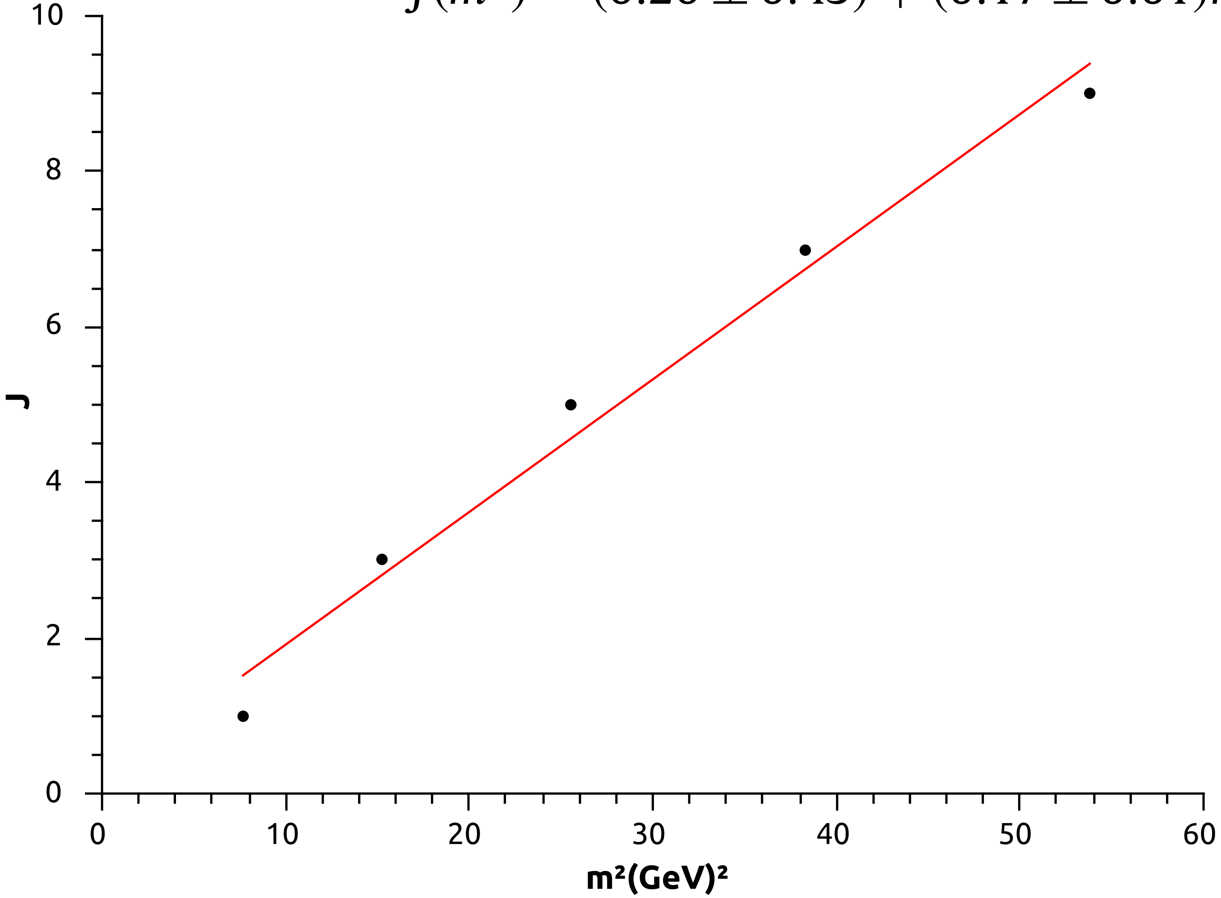
$$\Delta = 6 + J \qquad \text{spin } 1 + J.$$

Table 2
Masses m_n expressed in GeV for the glueball states J^{PC} with odd J solving Eq. (9) with the potential (12) for $k = 0.10$ GeV^2.

	Glueball states J^{PC}						k
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}	
m_n	2.77	3.91	5.05	6.19	7.33	8.47	0.10

Regge trajectories for the odderon

$$J(m^2) \approx (0.20 \pm 0.43) + (0.17 \pm 0.01) m^2$$

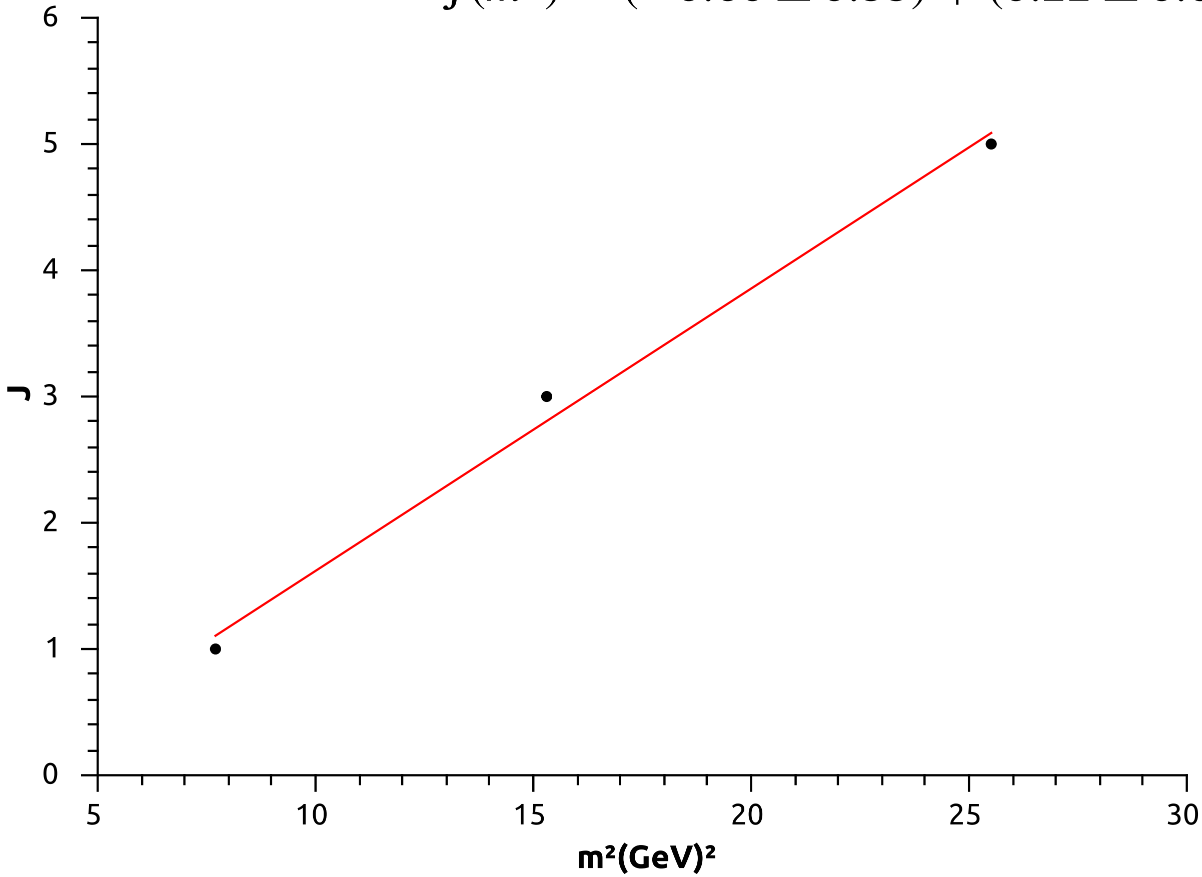


Very good agreement with the non-relativistic model for the odderon:

$$J(m^2) \approx 0.25 + 0.18\,\mathrm{m}^2,$$

Regge trajectories for the odderon

$$J(m^2) = (-0.60 \pm 0.33) + (0.22 \pm 0.01)m^2$$



Very good agreement with the relativistic model for the odderon:

$$J(m^2) \approx -0.88 + 0.23\,\mathrm{m}^2,$$

Finite Temperature AdS/CFT and AdS/QCD

Witten's proposal (1998)

Policastro, Son, Starinets, PRL 2001 (Shear viscosity…)

Finite temperature Yang-Mills theory in 4d dual to a modified AdS(5)xS(5) set up with a Black Hole (Schwarzschild AdS (5) x S (5))

The temperature of the Yang-Mills theory is identified with the Hawking temperature of the Black Hole

Soft-wall model at Finite Temperature

AdS black-hole spacetime

$$ds^2 = e^{2A(z)}\left[-f(z)dt^2 + \sum_{i=1}^{3}(dx^i)^2 + f(z)^{-1}dz^2\right],$$

$$A(z) = -\ln(z/L) \qquad f(z) = 1 - (z/z_h)^4.$$

$$z_h = 1/\pi T.$$

Herzog PRL 2007;

Kajantie, Tahkokallio, Yee, JHEP 2007;

Ballon-Bayona, HBF, Braga, Pando Zayas, PRD 2008.

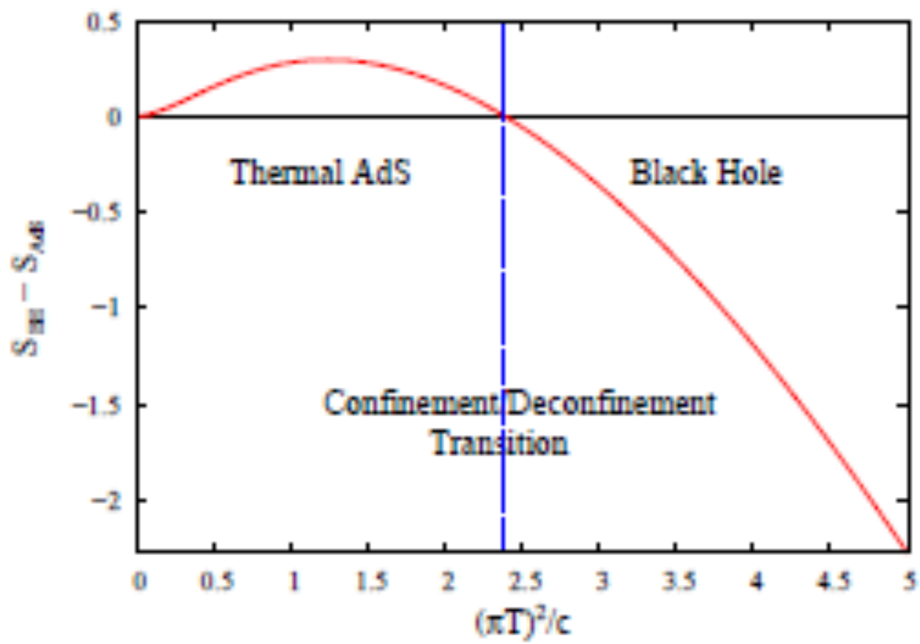
Hard-wall and Soft-wall at Finite Temperature: Confining/deconfining phase transition

Herzog, PRL 2007

Thermal AdS space (low temperature) (confined phase)

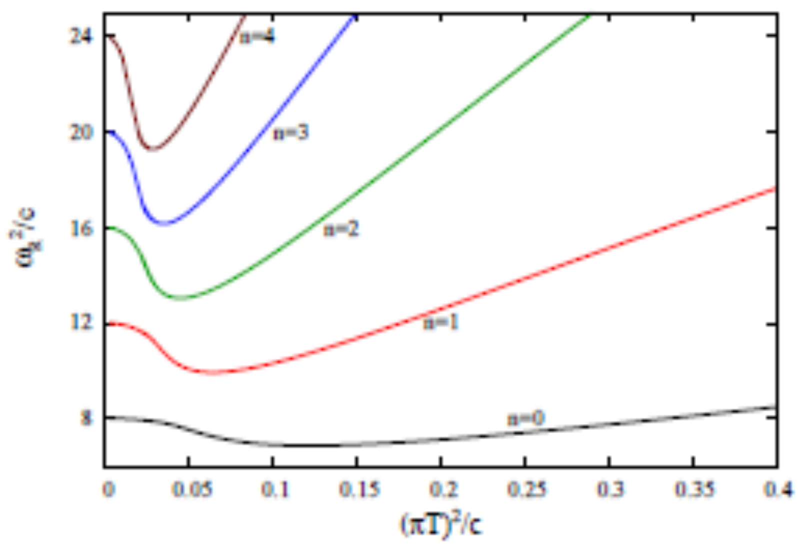
Hawking-Page phase transition

AdS Black hole (high temperature) (deconfined phase)



Quasinormal modes and scalar Glueballs in the Soft-wall at Finite Temperature

Quasinormal modes are formed when a particle/field falls onto a black hole horizon



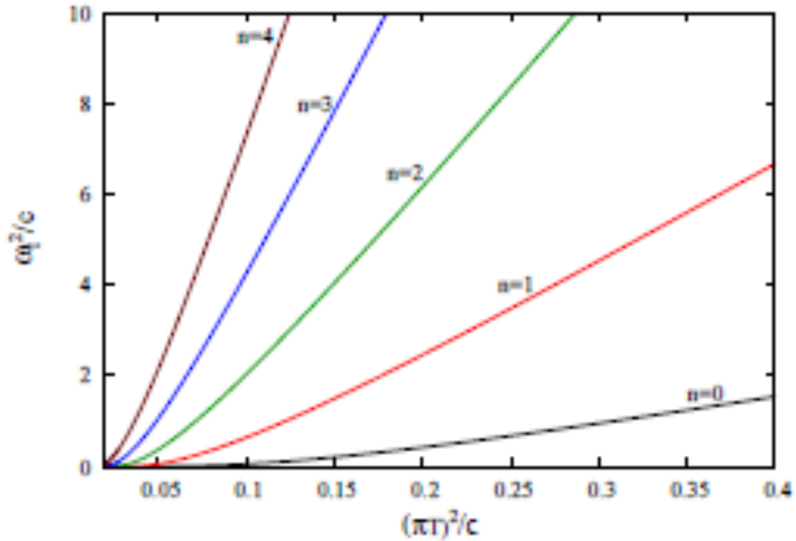


Figure 6. Numerical results for the square of the real and imaginary parts of the QN frequencies, ω_R^2/c and ω_I^2/c, for the first five quasinormal modes $n = 0, 1, \ldots, 4$, with $q = 0$. (zero momentum)

Miranda,Ballon-Bayona, HBF, Braga, JHEP 2009

Quasinormal modes and Vector Mesons in the Soft-wall at Finite Temperature

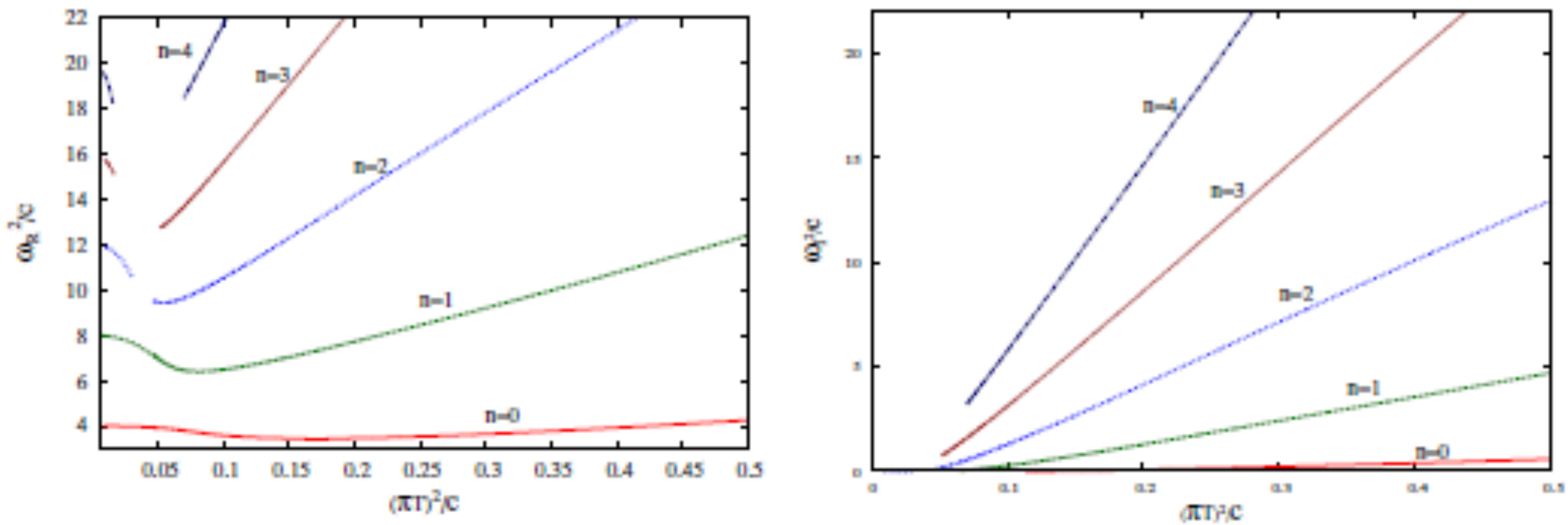
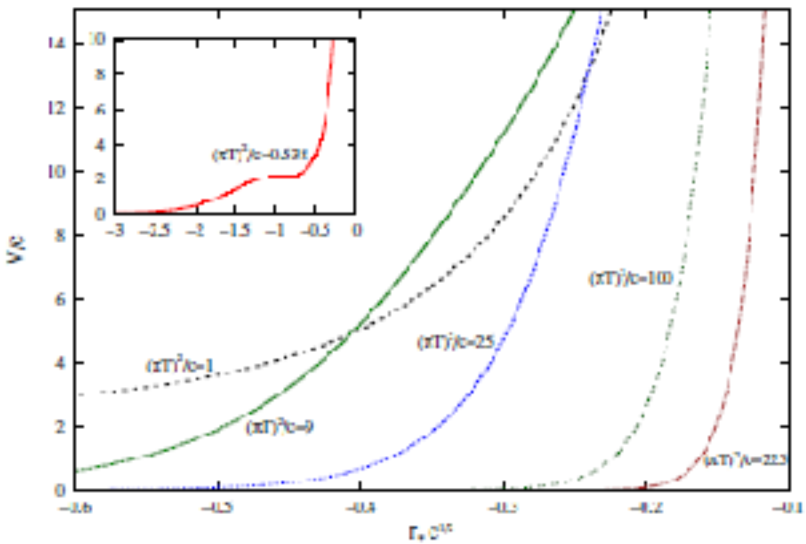


Figure 5. Numerical results for the quasinormal frequencies. On the left panel we show the real part, associated with mass of the vector mesons. On the right panel we show the imaginary part associated with the decay time of the quasiparticle states.

Mamani, Miranda, HBF, Braga, JHEP 2014

Vector Mesons at Finite T in the Soft-wall model

Figure 1. Potential at zero wave number for high temperatures.



the critical value $\widetilde{T}_c^2 = 0.538$ in the detail.

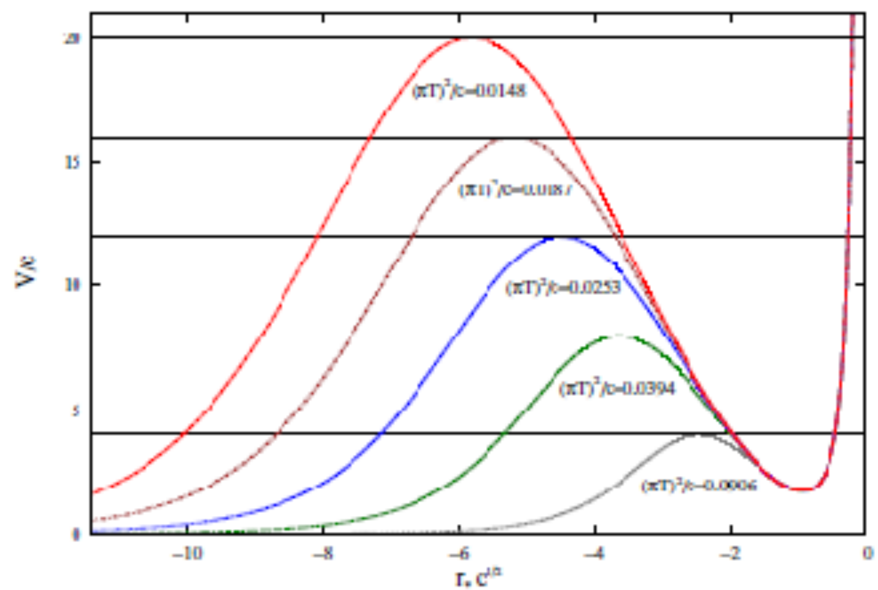


Figure 2. Potential at zero wave number for low temperatures.

The Odderon

F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. 96, 081601 (2006).

Relativistic many-body model (RMB)

$$J_{\rm RMB}(m^2) = -0.88 + 0.23m^2,$$

Non-relativistic constituent model (NRCM)

$$J_{\rm NRCM}(m^2) = 0.25 + 0.18m^2.$$

The Odderon

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$$J_{\mathrm{RMB}}(m^2) = -0.88 + 0.23m^2,$$

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$$J_{\mathrm{NRCM}}(m^2) = 0.25 + 0.18m^2.$$

The Odderon is related to Glueball states $, 3^{--}, 5^{--}, 7^{--}$, ...

and may be $1^{--},$

Experimental signs of the Odderon

The best experimental evidence for the odderon occurred in 1985 at ISR CERN. A difference between differential cross sections for pp and $p\bar{p}$ in the dip-shoulder region $1.1 < |t| < 1.5$ GeV^2 at $\sqrt{s} = 52.8$ GeV was measured, but these results were not confirmed [14].

There are two more evidences related to the nonperturbative odderon, that is, the change of shape in the polarization in $\pi^- p \to \pi^0 n$ from $p_L = 5$ GeV$/c$ [16,17] to $p_L = 40$ GeV$/c$ [18] and a strange structure seen in the $UA4/2$ dN/dt data for pp scattering at $\sqrt{s} = 541$ GeV, namely a bump centered at $|t| = 2 \times 10^{-3}$ GeV^2 [19].

[14] R. Avila, P. Gauronm, and B. Nicolescu, Eur. Phys. J. C **49**, 581 (2007).

[15] Z.-H. Hu, L.-J. Zhou, and W.-X. Ma, Commun. Theor. Phys. **49**, 729 (2008).

[16] D. Hill *et al.*, Phys. Rev. Lett. **30**, 239 (1973).

[17] P. Bonamy *et al.*, Nucl. Phys. **B52**, 392 (1973).

[18] V. D. Apokin *et al.*, AIP Conf. Proc. **95**, 118 (2008).

[19] C. Augier *et al.* (UA4/2 Collaboration), Phys. Lett. B **316**, 448 (1993).

Experimental signs of the Odderon

LCH new results?

Some groups are looking for the Odderon…

Odd spin (*P*=*C*=-*1*) Glueballs and the Odderon

Eduardo Capossoli and H. Boschi PRD 2013

Massive scalar fields in AdS_5

$$\left[z^3\partial_z\frac{1}{z^3}\partial_z + \eta^{\alpha\beta}\partial_\alpha\partial_\beta - \frac{m_5^2R^2}{z^2}\right]\phi(x,z) = 0,$$

$$m_5^2R^2 = (\Delta - p)(\Delta + p - 4).$$

(*p*=*0*)

$$\phi(x,z) = A_{\nu,k}\exp^{-ip.x}z^2J_\nu(u_{\nu,k}z),$$

$$\nu = \sqrt{4 + m_5^2R^2},$$

Boundary operator

$$\mathcal{O}_{6+\ell} = \mathrm{Sym\,Tr}(\tilde{F}_{\mu\nu}FD_{\{\mu_1}\ldots D_{\mu_\ell\}}F)$$

conformal dimension $\Delta = 6 + \ell$

spin $\ell = J \geq 1$

$$\nu = 4 + \ell$$

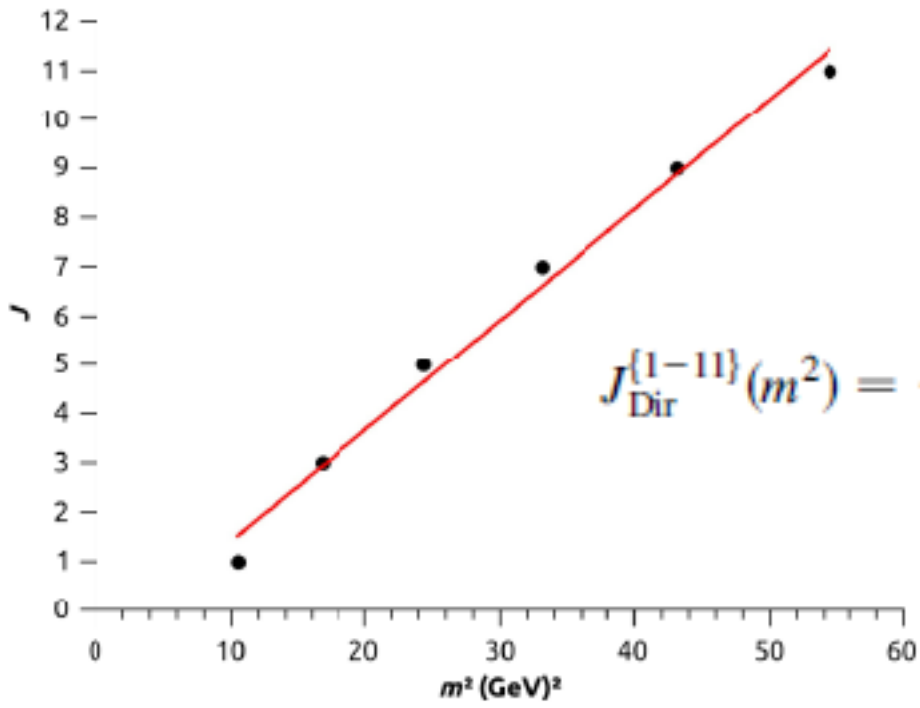
glueball states $1^{--}, 3^{--}, 5^{--}$, etc.

TABLE I. Glueball masses for states J^{PC} expressed in GeV, with odd J estimated using the hardwall model with Dirichlet and Neumann boundary conditions. The mass of 1^{--} is used as an input from the isotropic lattice [36,37]. We also show other results from the literature for comparison.

	Glueball states J^{PC}					
Models used	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
Hardwall with Dirichlet b.c.	3.24	4.09	4.93	5.75	6.57	7.38
Hardwall with Neumann b.c.	3.24	4.21	5.17	6.13	7.09	8.04
Relativistic many body [1]	3.95	4.15	5.05	5.90		
Nonrelativistic constituent [1]	3.49	3.92	5.15	6.14		
Wilson loop [38]	3.49	4.03				
Vacuum correlator [39]	3.02	3.49	4.18	4.96		
Vacuum correlator [39]	3.32	3.83	4.59	5.25		
Semirelativistic potential [40]	3.99	4.16	5.26			
Anisotropic lattice [41]	3.83	4.20				
Isotropic lattice [36,37]	3.24	4.33				

[36] H. B. Meyer and M. J. Teper, Phys. Lett. B **605**, 344 (2005).
[37] H. B. Meyer, arXiv:hep-lat/0508002.

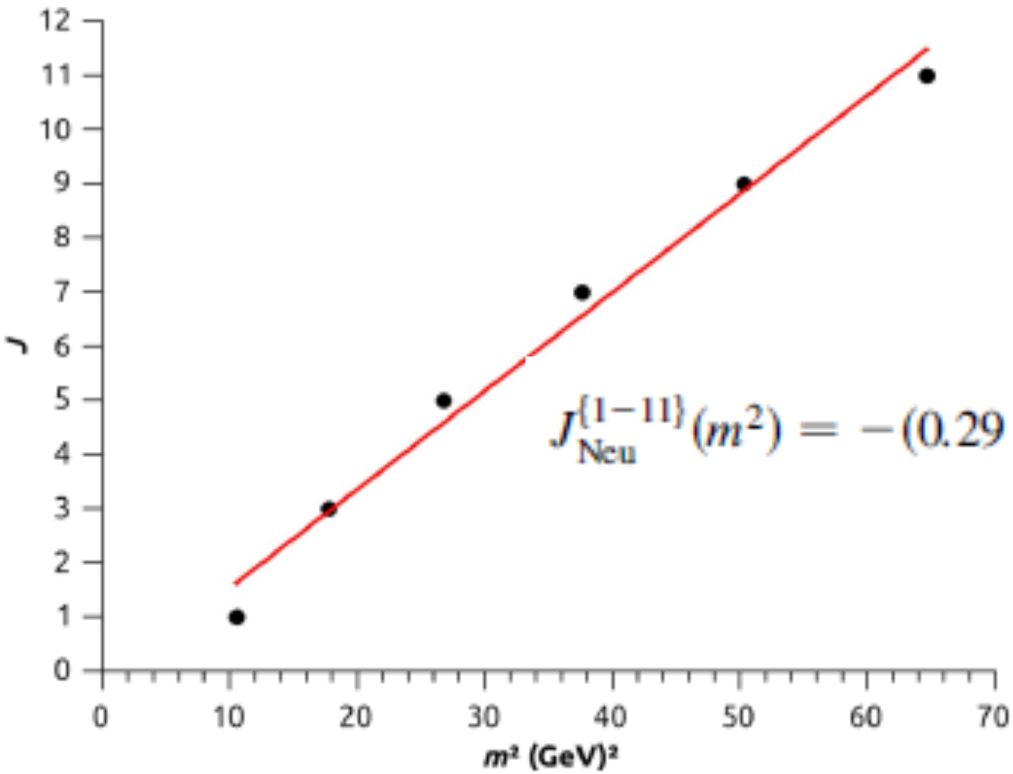
Odd Glueball states in the Hard-wall with Dirichlet Boundary condition



$$J_{\rm Dir}^{\{1-11\}}(m^2) = -(0.83 \pm 0.40) + (0.22 \pm 0.01)m^2.$$

Good agreement with the Relativistic Many-body Model (RMB)

Odd Glueball states in the Hard-wall with Neumann Boundary condition



$$J_{\text{Neu}}^{\{1-11\}}(m^2) = -(0.29 \pm 0.42) + (0.18 \pm 0.01)m^2.$$

Good agreement with the Relativistic Many-body Model (RMB)

Open questions for the Odderon

Experimental confirmation?

The authors

F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. 96, 081601 (2006).

suggest that the state 1^{--} does NOT belong to the Odderon trajectory

Our analysis with the Hard-wall is not conclusive in this regard

Avaliação

Responda às perguntas:

1a. Descreva, com suas palavras, o que é a Correspondência AdS/CFT.

2a. Descreva, com suas palavras, uma aplicação da Correspondência AdS/CFT à Física de Partículas.

Obs.: Consulta livre.

Enviar as respostas com seu nome completo num arquivo em formato livre (pdf, doc, etc) para o email:

hboschi@gmail.com

até o dia 04 de agosto de 2017.

Obrigado!