Introdução à Correspondência AdS/CFT

Parte 4

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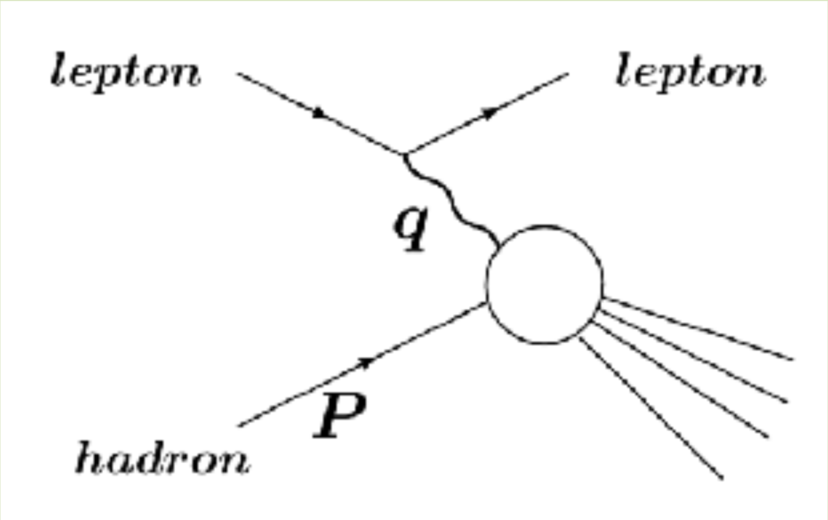
Summary of the talk:

- Deep inelastic scattering in AdS/QCD.
- Hadronic form factors from D4-D8 brane model.
- Other Results

Deep inelastic scattering in AdS/QCD.



Inclusive cross section characterized by the hadronic tensor

$$W^{\mu\nu} = i \int d^4y \, e^{iq\cdot y} \langle P, \mathcal{Q}| \, [J^\mu(y), J^\nu(0)] \, |P, \mathcal{Q}\rangle$$

Dynamical variables: $q^2 \,, \quad x = -\dfrac{q^2}{2P\cdot q}$ **Bjorken variable**

Hadronic tensor (spin independent case)

$$W^{\mu\nu} = F_1(x, q^2)\,(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2} F_2(x, q^2)\,(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x})$$

The Structure functions $F_{1,2}(x, q^2)$ contain informations about the distribution of constituents inside the hadron.

Polschinski and Strassler (2003) found prescriptions for calculating the structure functions, using the hard wall model (depending on the kinematical regime; here we just review the case when supergravity approximation holds).

The matrix element of the hadronic current is given by a 10-dimensional supergravity interaction action. For scalars:

$$\eta_\mu \langle P_X | \tilde{J}^\mu(q) | P \rangle \;=\; i \int d^{10}x \sqrt{-g}\, A^m (\Phi_i \partial_m \Phi_X^* \;-\; \Phi_X^* \partial_m \Phi_i)$$

$\updownarrow$ $\updownarrow$

Gauge theory $\leftrightarrow$ Supergravity (~ low energy String theory)

(Just one hadron in the final state)

Result for fermions (τ is the twist = *d* - *s*):

$$F_2(x, q^2) \;=\; 2F_1(x, q^2) \;=\; C\, Q^2 \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1}\, (1-x)^{\tau-2}$$

(For spin ½ in the hard wall, Polchinski & Strassler)

Hadronic structure functions at small x.

C.A.Ballon Bayona, H.B.-F. and N.Braga, 2008

Center of mass energy $\sqrt{s}$ is larger at small x:

$$\boxed{s \sim q^2/x}$$

So, we expect more hadrons in the final state.

Δ = scaling (conformal) dimension of hadronic operator.

↔ minimum number of constituents of the state.

Bulk/Boundary relation: Δ is "regulated" by

the 5-dimensional mass of the dual bulk field

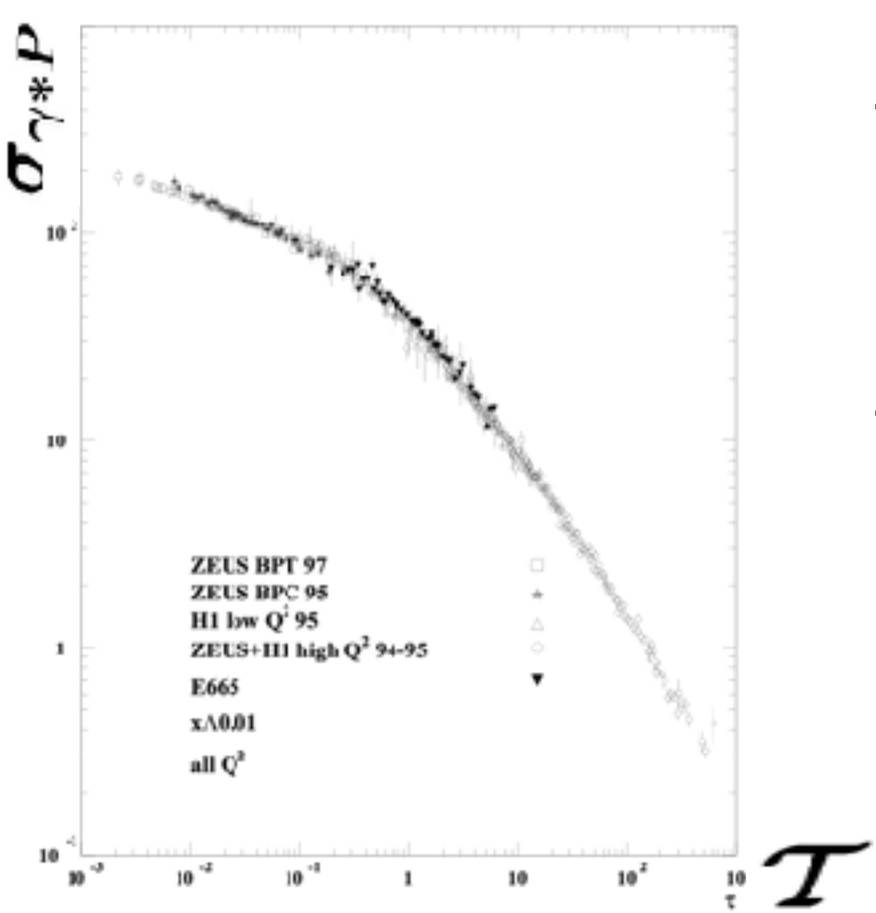
$$m_5^2 = \frac{\Delta(\Delta - 4)}{R^2}$$

So, we summed over final hadronic states with all allowed values of Δ (in Polchinski & Strassler article $\Delta_{initial} = \Delta_{final}$), finding a behaviour similar to GEOMETRIC SCALING :

$$F_2(x, q^2) \sim \left(\frac{q^2}{\Lambda^2}\right)^{1/2} x^{-1/2}$$

Geometric Scaling:

Stásto, Golec-Biernat, Kwiecinski; PRL 2001



For Bjorken parameter x < 0.01 the observed total cross section:

$$\sigma(q^2, x) = 4\pi^2 \alpha_{EM} \frac{F_2(x, q^2)}{q^2}.$$

depends on x and q^2 only through the combination:

$$\tau = q^2 x^\lambda$$

with: $0.3 < \lambda < 0.4$

Our AdS/QCD result imply a similar scaling with $\lambda = 1$

D4-D8 brane model for hadrons
Sakai and Sugimoto (2005)

- D8 (probe) branes embedded in D4 brane space.
- Holographic model for (large N_C , strongly coupled) QCD.

D4 brane background:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left[\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right] + \left(\frac{R}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right]$$

With: $f(U) = 1 - (U_{KK}/U)^3$, τ is a compact dimension.
Period of τ is related to minimum value of U → mass scale.

In this model mesons correspond to fluctuations of the D8 brane solutions in the D4 background.

Vector and axial vector mesons are described by U(N_F) gauge field fluctuations.

4-dim effective action (after field redefinitions, ...)

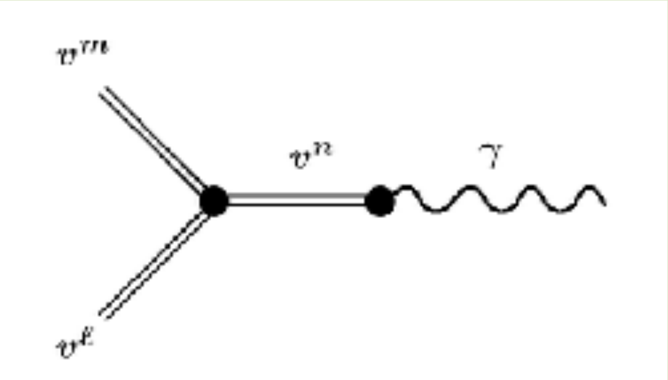
$$\mathcal{L}^{4d}_{eff} = \frac{1}{2}\mathrm{tr}\left(\partial_\mu \tilde{v}^n_\nu - \partial_\nu \tilde{v}^n_\mu\right)^2 + \frac{1}{2}\mathrm{tr}\left(\partial_\mu \tilde{a}^n_\nu - \partial_\nu \tilde{a}^n_\mu\right)^2 + \mathrm{tr}\left(i\partial_\mu \Pi + f_\pi \mathcal{A}_\mu\right)^2$$
$$+ M^2_{v^n}\,\mathrm{tr}\left(\tilde{v}^n_\mu - \frac{g_{v^n}}{M^2_{v^n}}\mathcal{V}_\mu\right)^2 + M^2_{a^n}\,\mathrm{tr}\left(\tilde{a}^n_\mu - \frac{g_{a^n}}{M^2_{a^n}}\mathcal{A}_\mu\right)^2 + \sum_{j\geq 3}\mathcal{L}_j$$

Vector and axial-vector mesons: $\tilde{v}^n_\mu\,,\quad \tilde{a}^n_\mu$

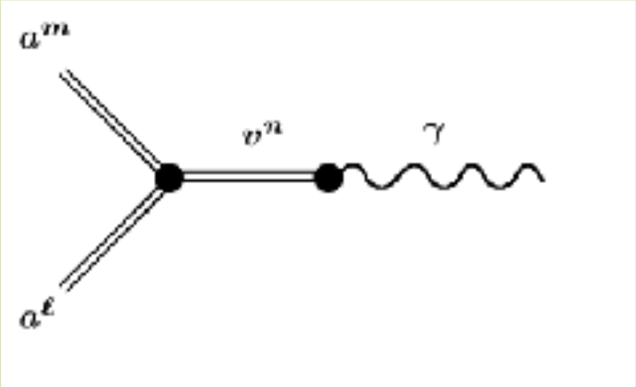
→ The effective actions show up with a set of prescritions for calculating masses and couplings. (everything is solved numerically)

- **Important: D4-D8 model realizes vector meson dominance.**

Form factors for vector and axial-vector mesons in the D4-D8 model:
C.A.Ballon Bayona, H.B.-F., N.Braga, M.A.C.Torres, JHEP 2010



VMD (Vector meson dominance)
Interaction with a photon mediated by the exchange of vector mesons



Generalized form factors for vector mesons:

$$
\langle v^{m\,a}(p), \epsilon | J^{\mu c}(0) | v^{\ell\,b}(p'), \epsilon' \rangle
$$
$$
= \epsilon^{\nu} \epsilon'^{\rho} f^{abc} \left[\eta_{\nu\rho}(2p+q)_{\sigma} + 2(\eta_{\sigma\nu} q_{\rho} - \eta_{\rho\sigma} q_{\nu}) \right] \left(\eta^{\mu\sigma} - \frac{q^{\mu} q^{\sigma}}{q^2} \right) F_{v^m v^\ell}(q^2)
$$

Where, in the model:

$$
F_{v^m v^\ell}(q^2) = \sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n v^m v^\ell}}{q^2 + M_{v^n}^2}
$$

g_{v^n} is the coupling between the photon and the vector meson,

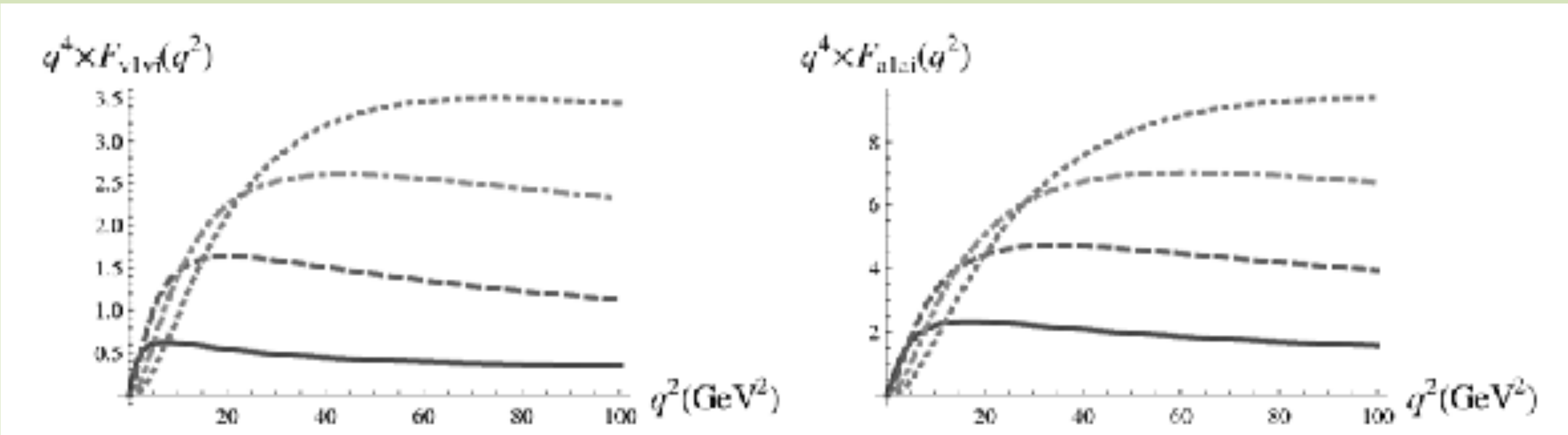
$g_{v^n v^\ell v^m}$ Is the 3 vertex on vector mesons, ...

$$
g_{v^n} = \kappa M_{v^n}^2 \int d\tilde{z}\, K(\tilde{z})^{-1/3} \psi_{2n-1}(\tilde{z}), \qquad K(\tilde{z}) \equiv 1 + \tilde{z}^2
$$

$$
g_{v^n v^\ell v^m} = \kappa \int d\tilde{z}\, K(\tilde{z})^{-1/3} \psi_{2n-1}(\tilde{z}) \psi_{2\ell-1}(\tilde{z}) \psi_{2m-1}(\tilde{z}),
$$

... and similar expressions for the axial-vector mesons.

Results: appropriate decrease with q^{-4} **for large q**.



q^4 times the form factors. Left panel: $F_{v^1v^i}$, right panel: $F_{a^1a^i}$, for i =1 (solid line), 2 (dashed line), 3 (dot-dashed line), 4 (dotted line).

Interesting quantities in the elastic case:

• Form factors for vector mesons with transversal and longitudinal polarizations

$$F_{TT}(q^2) = \frac{\langle p, \epsilon_T | J_0(0) | p', \epsilon'_T \rangle}{2E} \quad , \quad F_{LT}(q^2) = \frac{\langle p, \epsilon_T | J_x(0) | p', \epsilon'_L \rangle}{2E}$$

$$F_{LL}(q^2) = \frac{\langle p, \epsilon_L | J_0(0) | p', \epsilon'_L \rangle}{2E} .$$

In the D4-D8 model we found:

$$F_{TT}^{(v^m)} = F_{v^m} \quad , \quad F_{LT}^{(v^m)} = \frac{q}{M_{v^m}} F_{v^m} \quad , \quad F_{LL}^{(v^m)} = \left(1 - \frac{q^2}{2M_{v^m}^2}\right) F_{v^m}$$

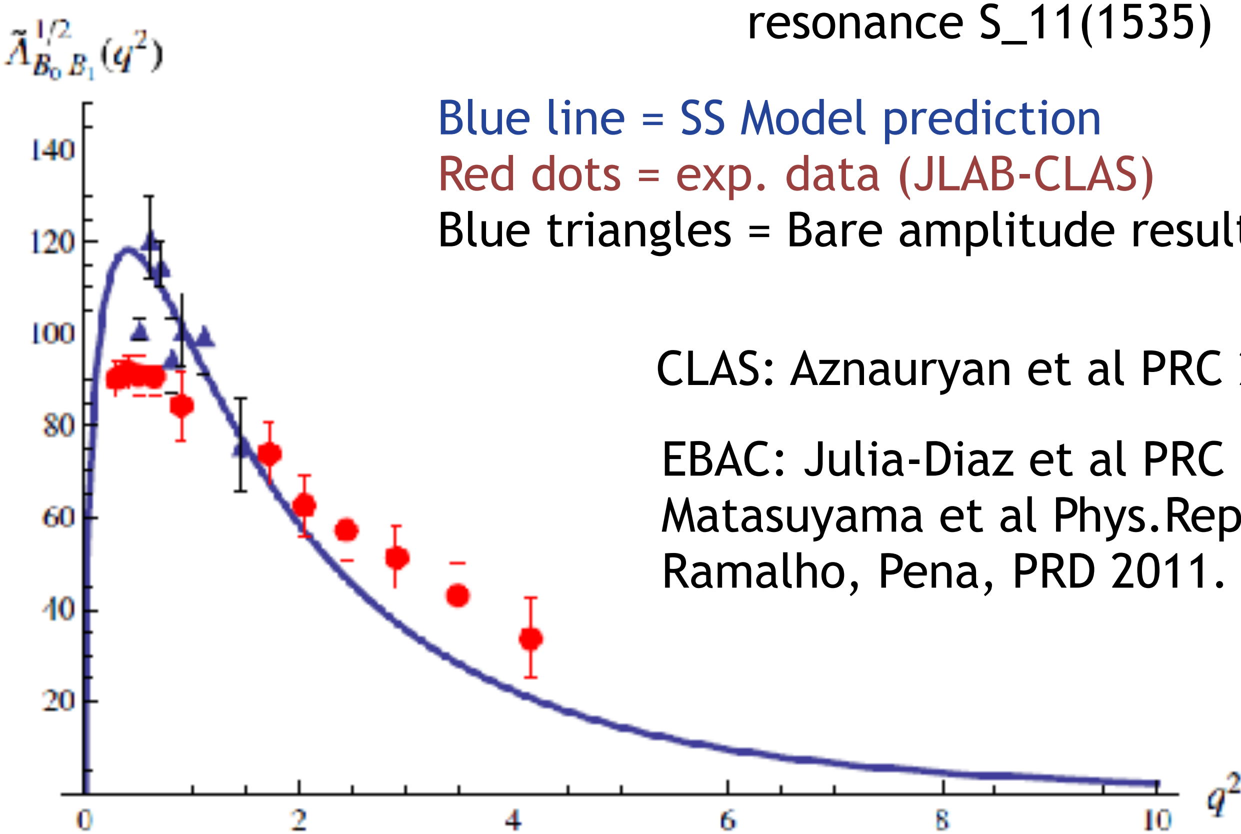
That imply the large q^2 behaviour expected from QCD:

$$F_{TT}^{(v^m)} \sim q^{-4} \, , \, F_{LT}^{(v^m)} \sim q^{-3} \, , \, F_{LL}^{(v^m)} \sim q^{-2}$$

Baryons Form Factors and Proton Structure in the Holographic Sakai-Sugimoto D4-D8 Model

Ballón-Bayona, HBF, Braga, Ihl, Torres, PRD 2012; NPB 2013

Helicity Amplitude $[10^{-3}(\text{GeV})^{-1/2}]$ for the observed negative parity resonance S_11(1535)



(GeV)^2

Blue line = SS Model prediction
Red dots = exp. data (JLAB-CLAS)
Blue triangles = Bare amplitude results (EBAC)

CLAS: Aznauryan et al PRC 2009.

EBAC: Julia-Diaz et al PRC 2009;
Matasuyama et al Phys.Rep. 2007;
Ramalho, Pena, PRD 2011.

Pion (and vector meson) Form Factors in the Kuperstein-Sonnenschein Holographic model

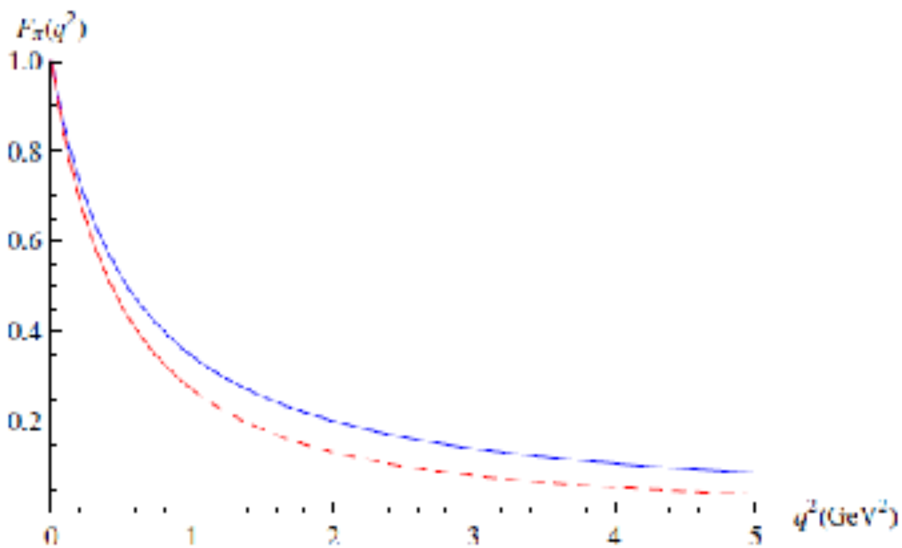
Ballón-Bayona, HBF, Ihl, Torres, JHEP (2010)

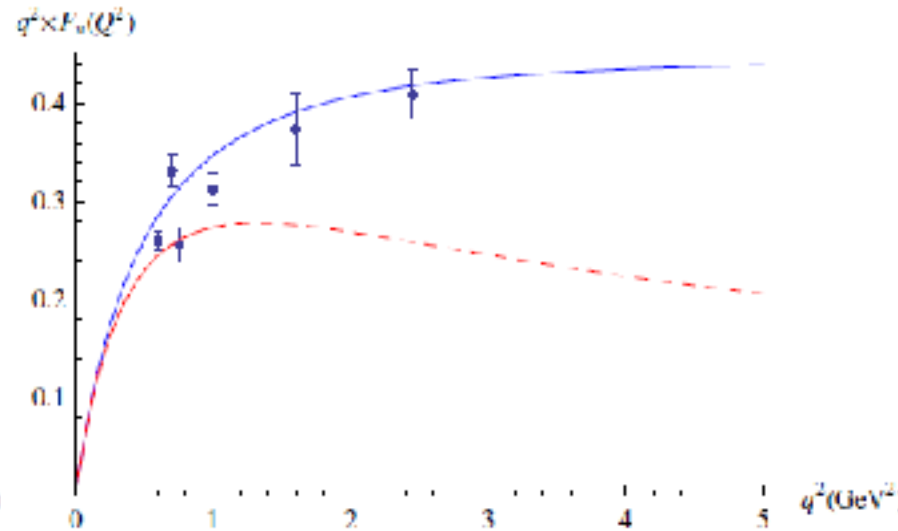
D3-brane background

D7-brane profiles

The KS model is based on the $D3$-brane background with a conical singularity in type IIB superstring theory first studied by Klebanov and Witten

Stable, non-supersymmetric, but similar to D4-D8 with VMD





Red = SS model; Blue = KS model

Dots = experimental data (PDG)