III. Teoria Quântica do Paramagnetismo; Ferromagnetismo

III.1 Paramagnetismo - Função de Brillouin

$$\mu = -g\mu_B J \tag{1}$$

$$E = -\mu \cdot B = -\mu_z B = g\mu_B M_J B \tag{2}$$

M_J projeções de J na direção de B: $M_J = J, J-1, J-2, \ldots -J+1, -J$.

Para $J = 1/2$, $M_J = +1/2; -1/2$; com $g = 2$, projeções $\mu_z = \pm\mu_B$, e a energia fica:

$$E = \pm\mu_B B \tag{3}$$

$$N_1 = Ae^{-E_1/kT} = Ae^{\mu_B B/kT} \tag{4}$$

$$N_2 = Ae^{-E_2/kT} = Ae^{-\mu_B B/kT} \tag{5}$$

$N = N_1 + N_2$. Donde,

$$\frac{N_1}{N} = \frac{e^{\mu_B B/kT}}{e^{\mu_B B/kT} + e^{-\mu_B B/kT}} \tag{6}$$

$$\frac{N_2}{N} = \frac{e^{-\mu_B B/kT}}{e^{\mu_B B/kT} + e^{-\mu_B B/kT}} \tag{7}$$

$$M = \mu_B(N_1 - N_2) = \mu_B N \frac{e^{\mu_B B/kT} - e^{-\mu_B B/kT}}{e^{\mu_B B/kT} + e^{-\mu_B B/kT}}$$

$$= \mu_B N \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad (8)$$

onde $x = \mu_B B/kT$.

$$M = \mu_B N \,\mathrm{tgh} x = N \mu_B \,\mathrm{tgh}\left(\frac{\mu_B B}{kT}\right) \qquad (9)$$

Para J qualquer,

$$M = g \mu_B J B_J(x) \qquad (10)$$

$$x = g \mu_B J B/kT \qquad (11)$$

Função de Brillouin:

$$B_J(x) = (1 + \frac{1}{2J})[\mathrm{cotgh}(1 + \frac{1}{2J})x] - \frac{1}{2J}\mathrm{cotgh}(\frac{x}{2J}) \qquad (12)$$

Para x pequeno:

$$\mathrm{cotgh}(x) = \frac{1}{x} + \frac{x}{3} + \cdots \qquad (13)$$

$$B_J(x) \approx \left(1+\frac{1}{2J}\right)\left[\frac{1}{(1+1/2J)x} + \frac{(1+1/2J)}{3}x\right] - \frac{1}{2J}\left[\frac{2J}{x} + \frac{x}{6J}\right] = \frac{J+1}{3J}x \qquad (14)$$

$$M = n < \mu_J^z >_T = n g \mu_B J B_J(x) \approx$$

$$\approx n g \mu_B J \frac{g \mu_B J B}{kT}\frac{(J+1)}{3J} = \frac{n g^2 \mu_B^2 J(J+1) B}{3kT} \qquad (15)$$

A suscetibilidade:

$$\chi = \frac{M}{H} = \mu_0 \frac{M}{B} \qquad (16)$$

ou

$$\chi = \frac{\mu_0 n g^2 \mu_B^2 J(J+1)}{3kT} = \frac{C}{T} \qquad (17)$$

Constante de Curie:

$$C = \frac{\mu_0 n g^2 \mu_B^2 J(J+1)}{3k} \qquad (18)$$

III.2 Ferromagnetismo - Modelo de Campo Médio (Weiss)

$$M = n < \mu_J^z >_T \qquad (19)$$

$$B_m = \lambda_m M = \lambda_m n < \mu_J^z >_T \; k \qquad (20)$$

$$< \mu_J^z >_T = g \mu_B J B_J(x') \qquad (21)$$

com

$$x' = g \mu_B J \frac{\lambda_m n < \mu_J^z >_T}{kT} \qquad (22)$$

Portanto,

$$< \mu_J^z >_T = g \mu_B J B_J \left(g \mu_B J \frac{\lambda_m n < \mu_J^z >_T}{kT} \right) \qquad (23)$$

$$< \mu_J^z >_T = \frac{x'}{g \mu_B J \lambda_m n / kT} \qquad (24a)$$

$$< \mu_J^z >_T = g \mu_B J B_J(x') \qquad (24b)$$

$$\frac{< \mu^z >_T}{< \mu^z >_0} = B_J(x') = \frac{< \mu_J^z >_T}{g \mu_B J} = \frac{x' kT}{(g \mu_B J)^2 n \lambda_m} \qquad (25)$$

$$B_J(x') \approx \frac{J+1}{3J}x' \tag{26}$$

$$\frac{J+1}{3J}x' = \frac{x'kT}{(g\mu_B J)^2 n_{\lambda m}} \tag{27}$$

Temperatura de Curie:

$$T_C = \frac{g^2\mu_B^2 n_{\lambda m} J(J+1)}{3k} \tag{28}$$

III.3 Comportamento Magnético Acima de T_C

$$\langle \mu_J^z \rangle_T = g\mu_B J B_J(x') \approx \frac{1}{3} g\mu_B (J+1) x' \quad (29)$$

$$x' = g\mu_B J \frac{B + \lambda_m n \langle \mu_J^z \rangle_T}{kT} \quad (30)$$

$$\langle \mu_J^z \rangle_T \approx g^2 \mu_B^2 J(J+1) \frac{B + \lambda_m n \langle \mu_J^z \rangle_T}{3kT} \quad (31)$$

Com $C = \mu_0 g^2 \mu_B^2 n J(J+1)/3k$, fica

$$n \langle \mu_J^z \rangle_T T = \frac{C}{\mu_0}(B + \lambda_m n \langle \mu_J^z \rangle_T) \quad (32)$$

$$M = n \langle \mu_J^z \rangle_T = \frac{CB/\mu_0}{T - C\lambda_m/\mu_0} \quad (33)$$

A suscetibilidade é dada por $\chi = M/H$:

$$\chi = \frac{n \langle \mu_J^z \rangle_T}{H} = \frac{C}{T - C\lambda_m/\mu_0} = \frac{C}{T - \theta_p} \quad (34)$$

com

$$\theta_p = C\lambda_m/\mu_0 = \frac{g^2 \mu_B^2 n \lambda_m J(J+1)}{3k} \quad (35)$$