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TÍTULO DO PROJETO:	Multicriticality in 3D models: from Weyl to Hopf topological insulator

Introduction

The multicriticality study of the Topological Insulators (TI) has been show to be a fruitful branch since the establishment of the table of topological insulators and superconductors classification [1-3]. Furthermore, some topological phases are beyond this standard classification. For instance, the gapless topological phase and the Intrinsic topological (IT) phases [4]. In this present work, we explore the multicriticality along different directions of surface states for the Weyl model [5], a topological insulator AIII and a Hopf topological insulator model [6], as a example of gapless TI, usual TI and IT material, respectively.

Model: Weyl semimetal

The Weyl semimetal with time-reversal symmetry broken can be described by the Dirac equation as:

$$H(k) = t_x \sin k_x \tau_x + t_y \sin k_y \tau_y + t_z (\gamma + 2 - \cos k_x - \cos k_y - \cos k_z) \tau_z \quad (1)$$

where τ are the Pauli matrix in orbital space. A low-energy description reveals:

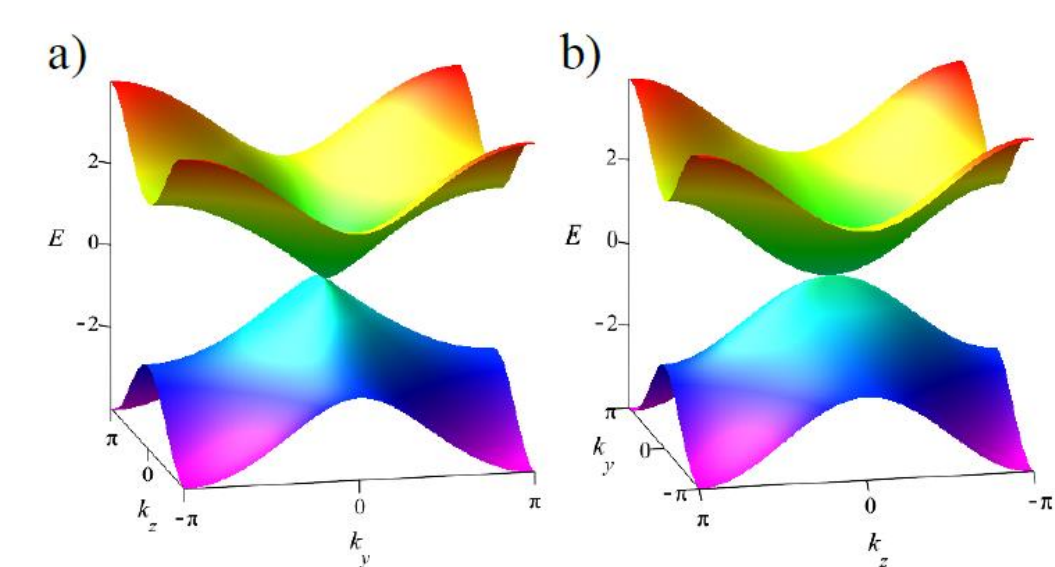


Fig1. Anisotropic bands dispersion at critical point for $k_x = 0$. With linear and quadratic band-crossing behavior along k_y (a) and k_z (b).

To study the surface states, we need preserve the quadratic terms in low energy description. So, the Hamiltonian is now described by the Dirac modified equation:

$$H(k) = t_x k_x \tau_x + t_y k_y \tau_y + [M - B(k_x^2 + k_y^2 + k_z^2)] \tau_z \quad (2)$$

for $M = t_z(\gamma - 1)$ and $B = -\frac{t_z}{2}$.

We investigate the penetration depth of the surface states that only decay ξ_{-} along direction x and y , since there is no wave function along z direction in this case. So, we consider open boundary conditions along each direction studied.

In Fig.2 we present the results about the conditions to obtain a real penetration depth value, since this decay into the bulk.

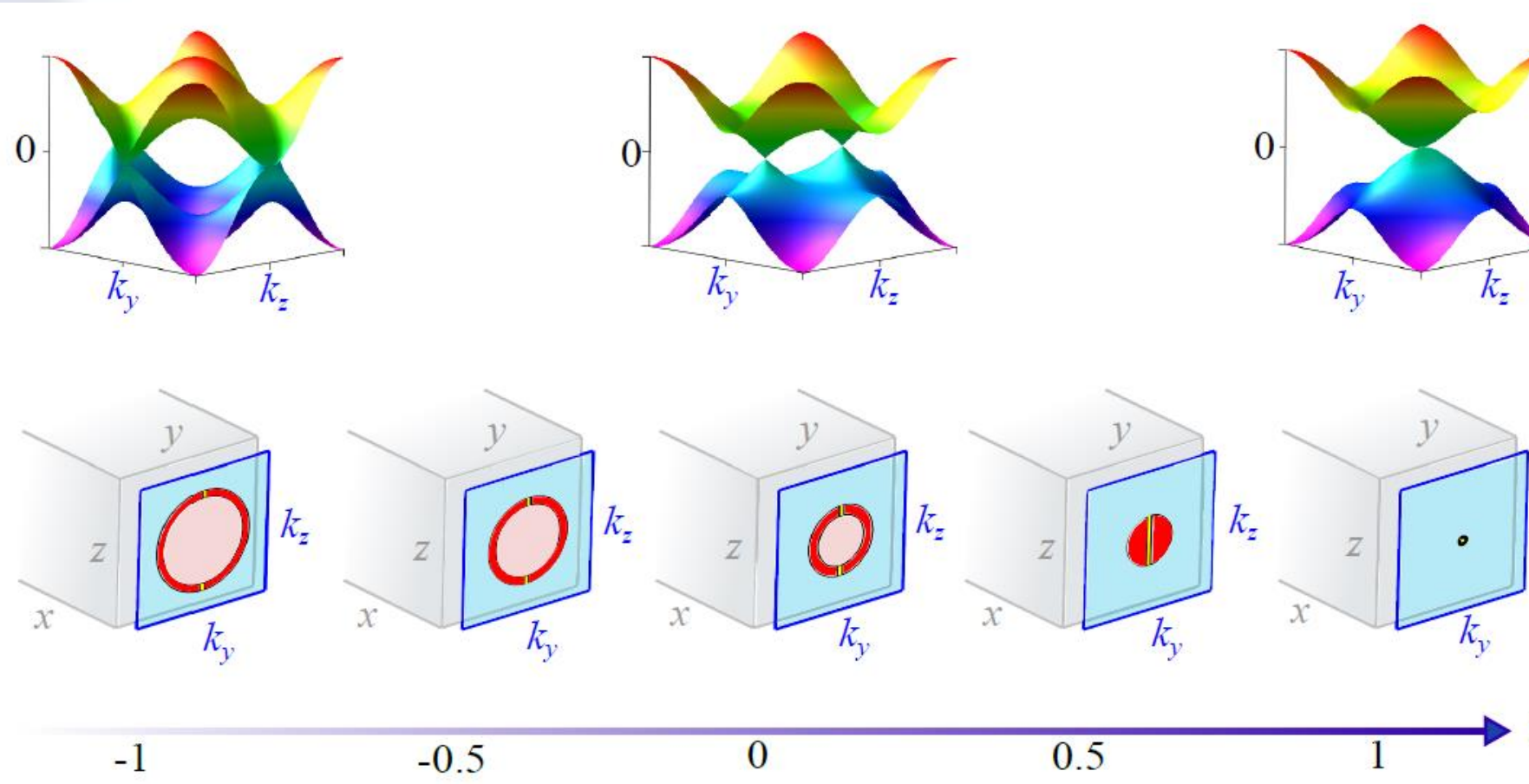
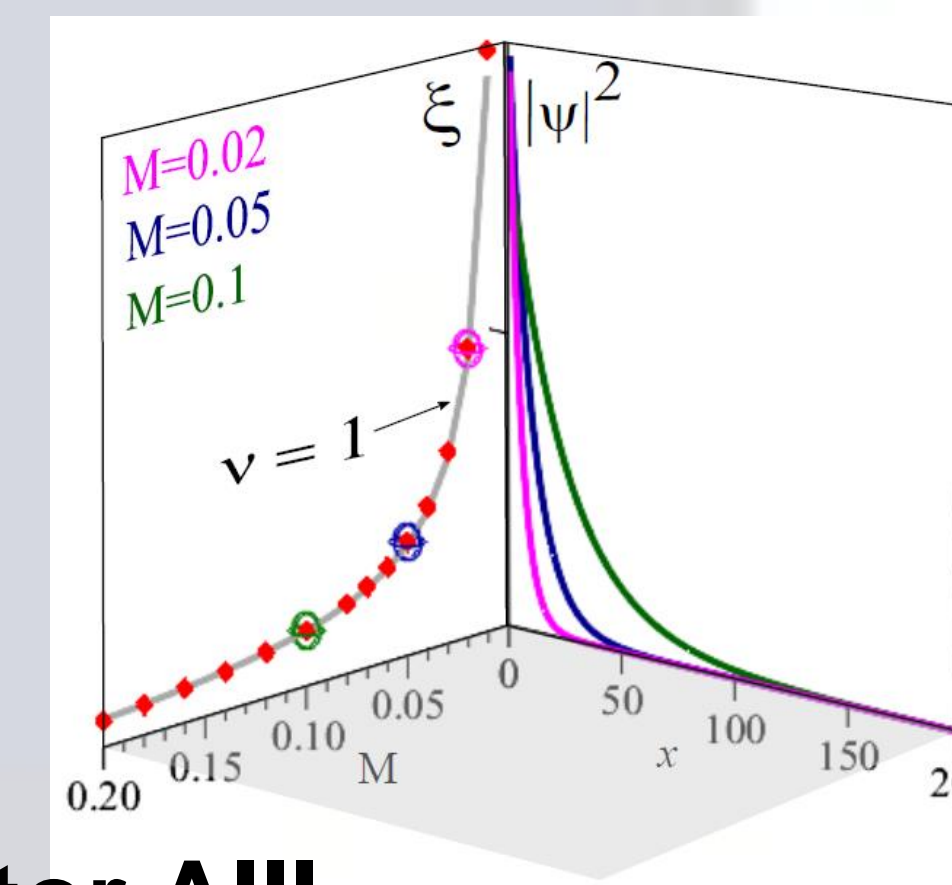


Fig2. The surface state conditions for a real value requires $1 - 2\gamma < k_y^2 + k_z^2 < 2(1 - \gamma)$. This region is highlighted in red (non-zero energy) and yellow (zero-energy) colors in momentum space $k_y - k_z$ (blue plan). The gray shape indicates the x axis with open boundary conditions.

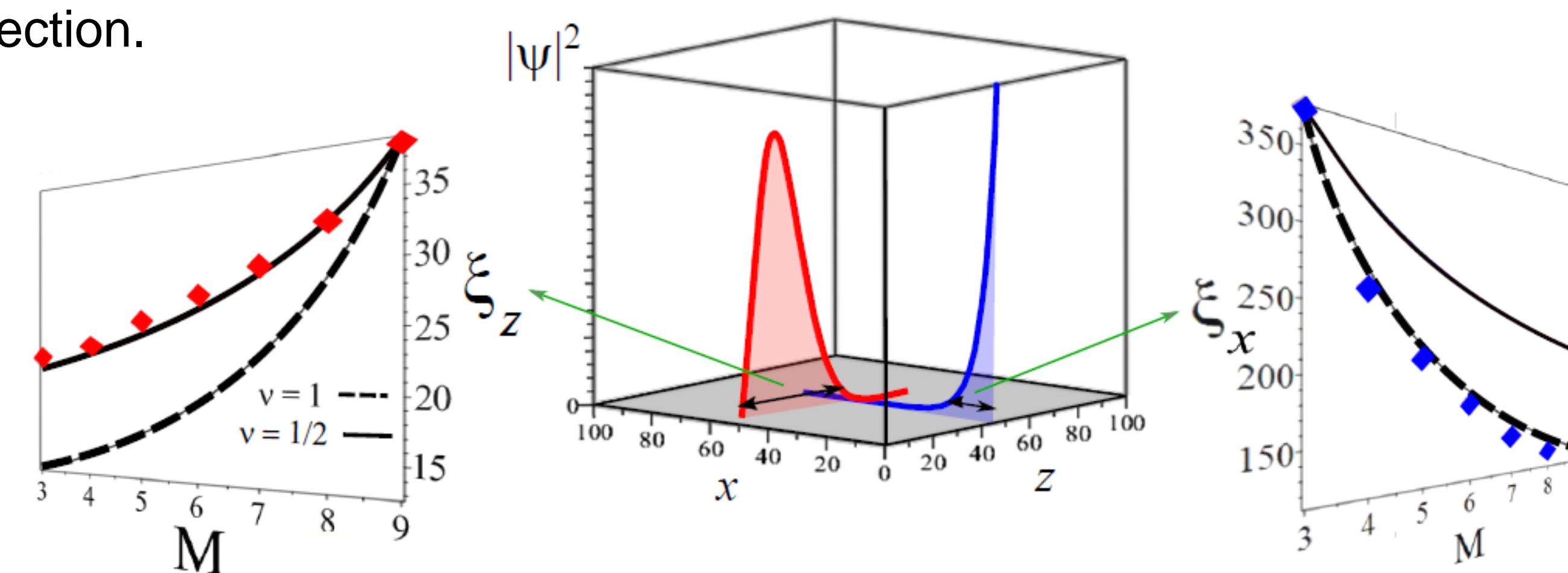
In Fig.3 we obtain the critical exponent ν associated to the spacial length correlation, where at the QCP $\xi_{-} \rightarrow \infty$ and the system is completely correlated.

Fig3. plan $x - |\psi|^2$: Probability density decay into the bulk along x direction. Plan $M - \xi$: Fit between the ξ_{-} and characteristic length indicating $\nu = 1$. Each color represents a value of the distance to the critical point M . This result of $\nu = 1$ is the same for directions x and y .



Model: Topological Insulator AIII

We propose the Hamiltonian of Eq.(1) in presence of additional term $t_a \sin k_z \tau_x$ that reproduces a TI in class AIII and allow us to obtain different ν for each direction.



In Fig.4 we obtain different critical exponent for each direction. The red color is associated with surface states with open boundary conditions along z direction, ξ_z and $\nu = \frac{1}{2}$. Attached to the blue color, we have open boundary conditions along x , ξ_x and $\nu = 1$.

Model: Hopf Insulator

This kind of non-symmetry protected topological phase describes a two-band model in 3D, and is given by:

$$\mathcal{H}(k) = h(k) \cdot \tau, \text{ where } h(k) = (\text{Re}[2\eta_{\uparrow}^p \eta_{\downarrow}^q], \text{Im}[2\eta_{\uparrow}^p \eta_{\downarrow}^q], [|\eta_{\uparrow}|^{2p} - |\eta_{\downarrow}|^{2q}])$$

$$\text{and } \eta_{\uparrow}(k) = \sin k_x + i \sin k_y,$$

$$\eta_{\downarrow}(k) = \sin k_z + i(\cos k_x + \cos k_y + \cos k_z + h)$$

Here, p e q comes from a mapping $\mathbb{T}^3 \rightarrow \mathbb{S}^2$ and can represents a tight-binding model in real space. Following the same previors steps, we found from energy analysis[3]:

p	q	Z (dynamical critical exponente)	ν
1	1	$Z_x=Z_y=Z_z=2$	$\nu_x=\nu_y=\nu_z=1$
2	1	$Z_x=Z_y=4; Z_z=2$	$\nu_x=\nu_y=1/2; \nu_z=1$
3	1	$Z_x=Z_y=4; Z_z=2$	$\nu_x=\nu_y=1/2; \nu_z=1$

Conclusion

The correspondence between the penetration depth and the correlation length is valid even for 3D systems and for topological materials beyond the standard table of TI. Furthermore, is able to predict different critical exponent for different directions, which in a last analysis could be useful to develop an independent table for the topological phases not included in the first one.

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