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TÍTULO DO PROJETO:	RELATIVISTIC GAS DYNAMICS: LORENTZ INVARIANT RAPIDITY DISTRIBUTION

Abstract

We use the molecular dynamics method to model a 1D-relativistic gas. This system contains only two kinds of particles, where its rest mass ratio corresponds to $m_h/m_l = 2$. Our results show that the probability density function (PDF) corresponding to the relative rapidity between the particles is a Lorentz invariant quantity, in contraposition to the PDF for the velocity of the particles as measured in the Lab frame, which is not a Lorentz invariant. Hence, the 1D-relativistic gas temperature is also a conserved quantity under Lorentz transformations.

Relevance

This system allow us to model:

- Thermalisation in ultrarelativistic plasma beams [Dieckmann M., Drury L., and Shukla P. *N. J. Phys. 8, 40 (2006)*].
  Bulk plasma flowing at ultra-relativistic speeds is common in astrophysical settings. E.g. Collimated jets of Active Galactic Nuclei (AGN) and Gamma-Ray Bursts (GRBs).
- The Relativistic Sunyaev-Zel'dovich effect [Itoh N., Kohyama Y., and Nozawa S. *Astroph. J. 502, 7 (1998)*, Nozawa, S., Itoh, N., Suda, Y., and Ohhata, Y. *Il N. Cim. B, 121, 487 (2006)*].
  The distortions of the CMB radiation spectrum, due to the interaction of the CMB photons with energetic electrons in galaxy clusters.
  Connected with the measurement of H_0.
  These distortions depend sensitively on the assumed electron velocity distribution.

Introduction

There is a long discussion (around one hundred years) on the foundations of thermodynamics in the special theory of relativity.
An important and moreover fundamental question is:
What transformation rules are followed by the heat and the temperature between two different inertial observers?

[Einstein A. *Jahrb. Radioakt. Elektronik 4 411 (1907)*], [Planck M. *Ann. Phys., Lpz. 26, 1 (1908)*], [Pauli W. *Relativitätstheorie (Berlin: Teubner) (1921)*], and [von Laue M. *Die Relativitätstheorie Braunschweig: Friedrich Vieweg und Sohn (1921)*] though that

$$T = \gamma^{-1} T_0,$$

which could be interpreted as “a moving body appears to be cool” as [Landsberg P. *Nat. 5052, 571 (1966)*] appointed.

They thought that, because when consider that the entropy *S*, the pressure *p* and the inner product $U^\alpha P_\alpha$ are Lorentz invariant then

$$dS = \frac{U^\alpha P_\alpha}{\gamma c T} + \frac{p_0}{\gamma T} dV_0, \quad dS_0 = \frac{U_0^\alpha P_{0\alpha}}{c T_0} + \frac{p_0}{T_0} dV_0,$$

where the quantities with zero index are measured in the rest frame and the other in the inertial frame, *V* is volume, *U* four-velocity an *P* four-momentum, and γ is the Lorentz factor.
Consequently, the heat is transformed, between the rest frame and any inertial frame, as

$$dQ = TdS = \gamma^{-1} T_0 dS_0 = \gamma^{-1} dQ_0.$$

However we must consider some important facts:

- It’s not clear how the temperature of a moving body is to be measured.
- Temperature is a statistical concept arising from many events involved in the motion of particles relative to each other.
- *T* is not expected to change in value for an observer moving uniformly with respect to the centre of mass of the system. [Landsberg P. *Nat. 5052, 571 (1966)*], i.e., *T* must be Lorentz invariant.
- A strong analytical derivation given by [Curado E. and Soares I. D. Arxiv 1406.0777v1 (2014)] and a previous numerical exploration performed in [Curado E., Germani F. and Soares I. Physica A 444, 963 (2016)] shed lights into this important question

Results

$$P(\sigma) = A e^{-\frac{(\sigma - b)^2}{2s^2}} \quad (1)$$

$$s = \sqrt{\frac{k_B T}{m_0 c^3}} \quad (2)$$

$$\sigma = \tanh^{-1}\left(\frac{v}{c}\right) \quad (3)$$

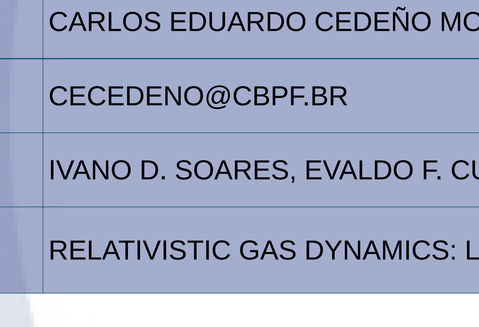

Figure 1. Probability density function for the relative rapidity σ, for light and heavy particles for low energy. The adjustment was done using (1), the relation between *s* and T is given by (2).

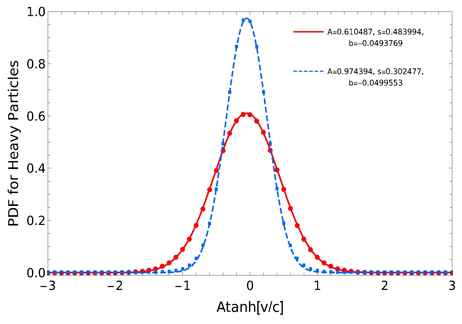

Figure 2. Probability density function for the relative rapidity σ, for light and heavy particles for high energy. The adjustment was done using (1), the relation between *s* and T is given by (2).

$$\gamma(v) = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (4)$$

$$P(\sigma) = A\gamma(\sigma) e^{-s\gamma(\sigma) + b} \quad (5)$$

$$A = \frac{m_0}{Z} \quad s = \frac{m_0^2 c^3}{2 k_B T} \quad (6)$$

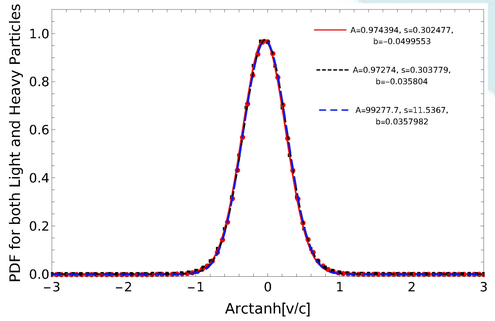

Figure 3. Probability density function for the relative rapidity σ for light and heavy particles for low energy, scaling by the root square of the ratio of the mass rest.
Gaussian adjustment in red and black (1) and (2) and Jüttner adjustment in blue (5) and (6), for low energy

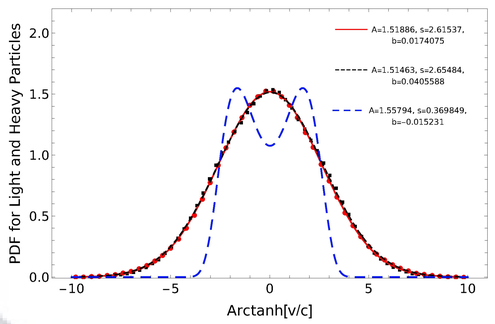

Figure 4. Probability density function for the relative rapidity σ for light and heavy particles for high energy, scaling by the root square of the ratio of the mass rest.
Gaussian adjustment in red and black (1) and (2) and Jüttner adjustment in blue (5) and (6), for high energy

Conclusions

- Our numerical simulations confirm the probability density function for the relative rapidity σ as described by Gaussian distributions, (1) and (2), in agreement with the central limit theorem of the statistical mechanics.
- Our simulations confirm σ-PDF as a Lorentz invariant distribution. Therefore, it implies that the temperature *T* of the unidimensional gas, inferred from the relative rapidity between the particles, is a Lorentz invariant.
- We show that Jüttner probability density function for rapidity, (5) and (6), can not reproduce adequately the distributions obtained from the simulations, except for low energies. This last disagreement is more evident when are considered higher energy levels.
- In the regime of high energies Jüttner probability density function for relative velocities, is not convenient to fit the PDF for the velocities obtained from the simulations.