New features of K-essential cosmologies

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Como

Einstein's (Cosmological) Equations

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

In comoving coordinates:

$$T_{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

Pure k-essence models

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$L(\phi, \partial\phi) = F((\partial\phi)^2) = F(X)$$

$$T_{\mu\nu} = (\partial_X F)\,\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\,F\,g_{\mu\nu}.$$

$$L(\phi, \partial\phi) = V_1(\phi)F(X) + V_2(\phi)$$

Cosmological Equations

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

Raychaudhuri Eq.

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p)$$

Friedmann Eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{K}{a^2}$$

Together imply
(for each component):

$$\dot{\rho}_i = -3\frac{\dot{a}}{a}(\rho_i + p_i)$$

Equation of state

$$p = f(\rho, s)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{K}{a^2} \qquad \rho = -\frac{3\dot{a}}{a}(\rho + f(\rho))$$

Linear case: $$p = w\,\rho$$

$$\dot{\rho}a + 3\dot{a}(\rho + w\rho) = 0 \quad \rightarrow \quad \rho a^{3(1+w)} = const$$

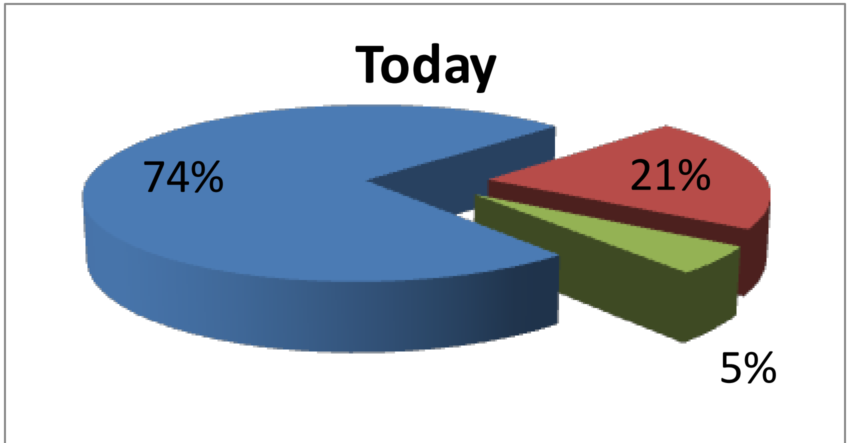
$$w$$

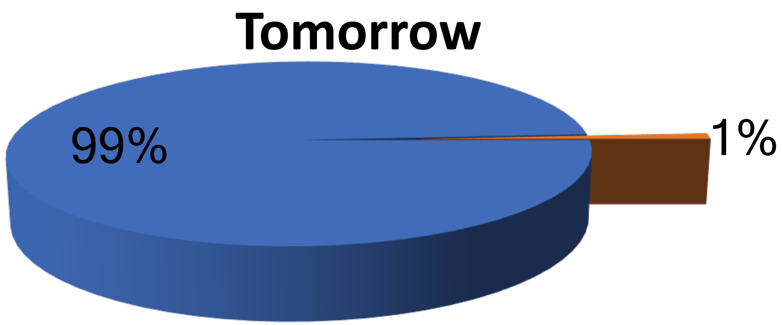
Lambda forever

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G(\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

$$p_M = 0, \quad p_R = \frac{1}{3}\rho_R, \quad p_\Lambda = -\rho_\Lambda,$$

$$H^2 = \cancel{\frac{\Omega_M}{a(t)^3}} + \cancel{\frac{\Omega_R}{a(t)^4}} + \Omega_\Lambda + \cancel{\frac{\Omega_K}{a(t)^2}}$$





Simple Dark Energy Models

Cosmic Acceleration

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p) \qquad \rho + 3p < 0$$

1) A perfect fluid $\quad w < -1/3$

$w = -1/3$ cosmic strings
$w = -2/3$ domain walls

2) Cosmological Constant $\quad w = -1$

3) Chaplygin Gas $\quad \rho = -\frac{A}{p^{\alpha}} > 0$

4) Dynamical scalar fields: Quintessence, k-essence, tachyons, f(R),…….

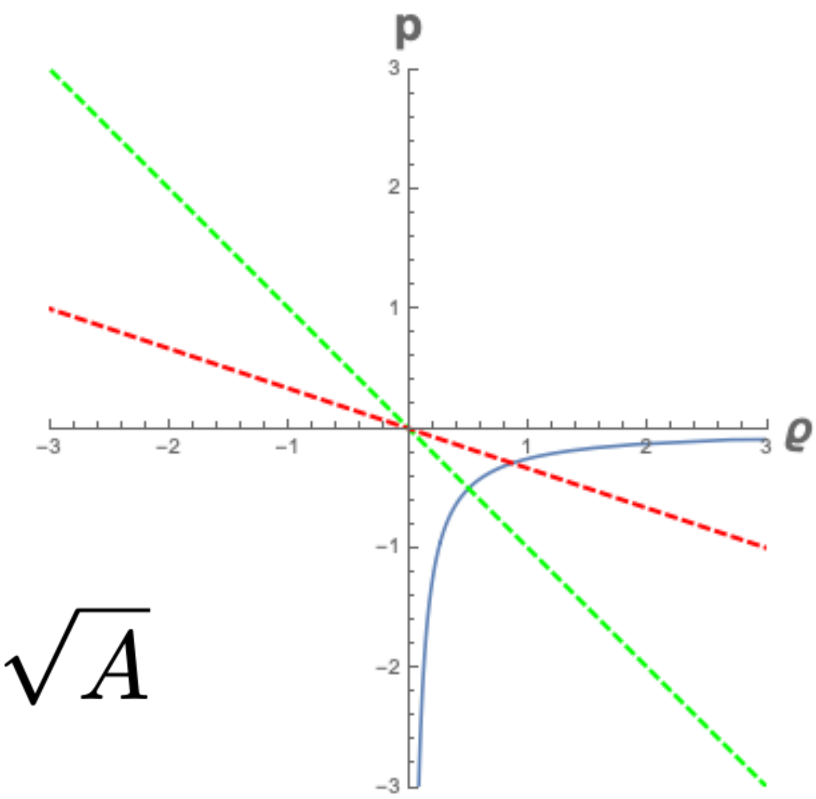
Chaplygin gas

$$p = -\frac{A}{\rho}$$

$$v_s^2 = \frac{\partial p}{\partial \rho} = \frac{A}{\rho^2},$$

$$\rho = -\frac{3\dot{a}}{a}(\rho + f(\rho))$$

$$\rho = \sqrt{A + \frac{B}{a^6}}$$



$$\rho \sim \frac{\sqrt{B}}{a^3}$$

$$\rho \sim \sqrt{A}$$

A string in three dimensions

$$\tau, \sigma \rightarrow x(\tau, \sigma) \qquad\qquad \Pi = \partial_\tau x$$

x is a transverse coordinate in the light-cone gauge

$$H = \frac{1}{2} \int [\Pi^2 + (\partial_\sigma x)^2] d\sigma$$

$$\partial_\tau^2 x - \partial_\sigma^2 x = 0$$

Integrability: hodograph transformation

Introduce the density $\dfrac{1}{\rho(x)} = (\partial_\sigma x)$

Introduce the velocity $v = (\partial_\tau x)$

$$dt = d\tau \qquad \partial_\tau = \partial_t + v\partial_x$$

$$dx = \frac{\partial x}{\partial \tau}\, d\tau + \frac{\partial x}{\partial \sigma}\, d\sigma = v\, d\tau + \frac{1}{\rho}\, d\sigma \qquad \partial_\sigma = \frac{1}{\rho}\partial_x$$

$$\partial_\tau(\partial_\tau x) = \partial_\sigma(\partial_\sigma x) \Rightarrow (\partial_t + v\partial_x)v = \frac{1}{\rho}\partial_x\frac{1}{\rho}$$

$$\rho(\partial_t + v\partial_x)v - \partial_x\frac{1}{\rho} = 0 \qquad p = -\frac{1}{\rho}$$

Braneworld

$$x^M = (x^\mu, y^4) \ \text{Induced metric: } \tilde{g}_{\mu\nu} = g_{\mu\nu} - \phi_{,\mu}\phi_{,\nu}$$

ϕ is a scalar field describing the embedding of the brane in the bulk

$$S_{eff} = \int d^4x\sqrt{-\tilde{g}}(-\kappa + \cdots) = \int d^4x\sqrt{-g}\sqrt{1 - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}(-\kappa + \cdots)$$

$$T_{\mu\nu} = \kappa\left(\frac{\phi_{,\mu}\phi_{,\nu}}{\sqrt{1 - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}} + g_{\mu\nu}\sqrt{1 - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}\right)$$
$$= (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$$

$$u_\mu = \frac{\phi_{,\mu}}{\sqrt{g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}} \qquad p = -\kappa\sqrt{1 - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}} = -\frac{\kappa^2}{\rho}$$

Pure k-essence models. One scalar field

$$L = -\sqrt{1 - (\partial\phi)^2} \qquad L(\phi, \partial\phi) = F(X)$$

$$X = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = (\partial\phi)^2$$

$$T_{\mu\nu} = (\partial_X F)\,\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\,F\,g_{\mu\nu}.$$

Flat k-essence: cosmological equations

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right) \qquad \phi = \phi(t) \quad X = \dot{\phi}^2 \geq 0$$

$$\varrho(X) = \frac{3\dot{a}^2}{a^2} = X\partial_X F - \frac{1}{2}F \qquad p(X) = \frac{1}{2}F$$

$$\text{Field equations} \quad (\partial_X F)\sqrt{X} = \pm\left(\frac{a_0}{a}\right)^3, \; a_0 \neq 0$$

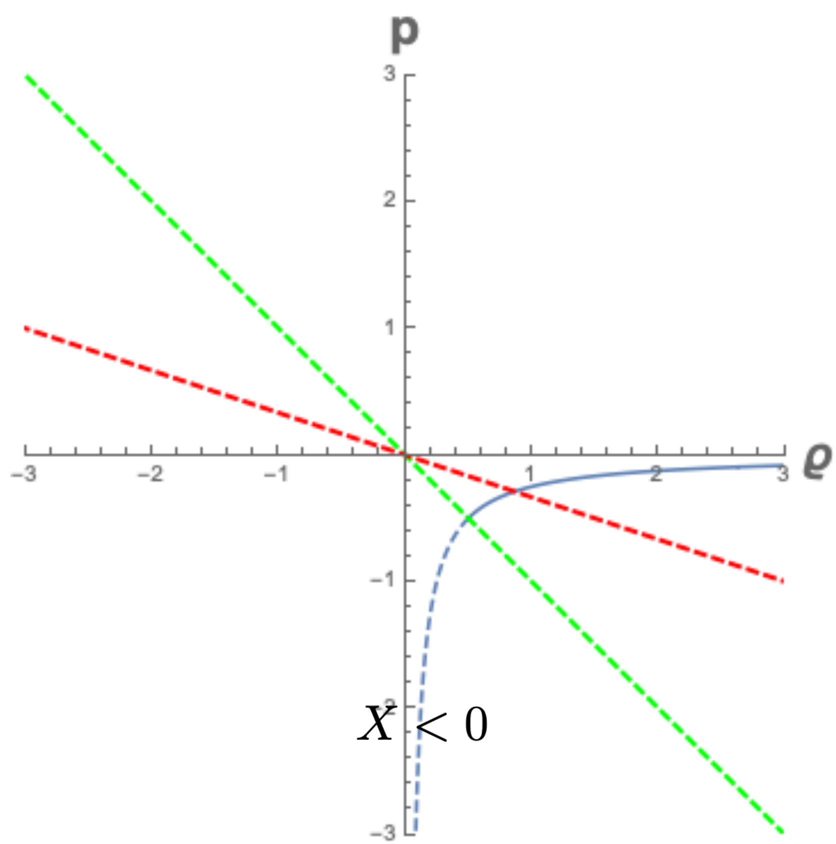
Dependence on the cosmic time

$$\frac{dX}{dt} = \mp\frac{2\sqrt{3}\,X\,F_X\,\sqrt{XF_X - \frac{1}{2}F}}{2XF_{XX} + F_X} = \mp\frac{(\rho(X) + p(X))\,\sqrt{3\rho(X)}}{\rho'(X)}$$

The Chaplygin Gas again

$$p = \frac{1}{2}F = -\frac{1}{2}\sqrt{1-X}$$

$$\varrho = \frac{1}{2}\frac{1}{\sqrt{1-X}}$$



$$p = -\frac{1}{4\varrho}$$

Solving the FE : the Chaplygin Gas

$$p = \frac{1}{2}F = -\frac{1}{2}\sqrt{1-X}$$

$$\text{Field equations} \quad (\partial_X F)\sqrt{X} = \pm\left(\frac{a_0}{a}\right)^3, \; a_0 \neq 0$$

$$\frac{\sqrt{X}}{2\sqrt{1-X}} = \pm\left(\frac{a_0}{a}\right)^3$$

$$X = \frac{4a_0^6}{a^6 + 4a_0^6}$$

$$\varrho = \frac{1}{2}\frac{1}{\sqrt{1-X}} = \frac{1}{2}\sqrt{1+\frac{4a_0^6}{a^6}}$$

$$\varrho(a) \sim \left(\frac{a_0}{a}\right)^3 \qquad \varrho(a) \sim \frac{1}{2}$$

Barotropic fluids

$$F(X) = \operatorname{sgn}\left(\gamma - \frac{1}{2}\right)(\partial\phi)^{2\gamma} = 2p \qquad \varrho = \left|\gamma - \frac{1}{2}\right|(\partial\phi)^{2\gamma}$$

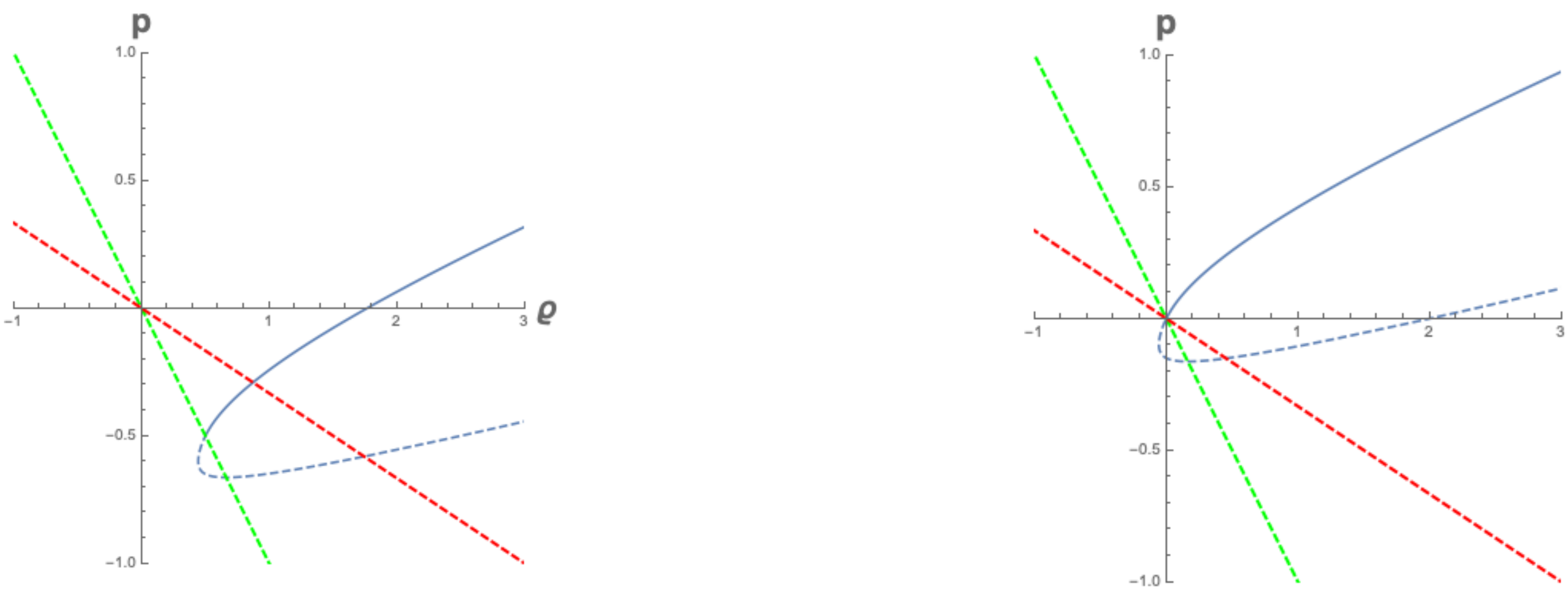
$$p = w\rho = \frac{\rho}{2\gamma - 1}$$

- $\gamma = 0$ corresponds to the cosmological constant
- $\gamma = 1$ corresponds to stiff matter
- $\gamma = 2$ corresponds to radiation
- The limits $\gamma \to \pm\infty$ corresponds to dust $(p = 0)$
- The limit $\gamma \to \frac{1}{2}$ corresponds to empty space

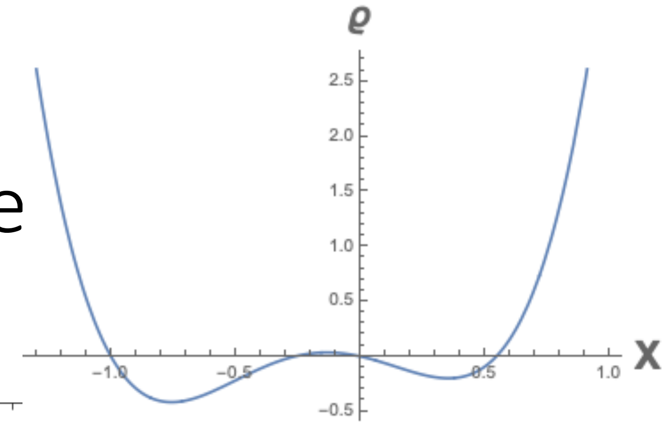
Polynomial interactions: standard case

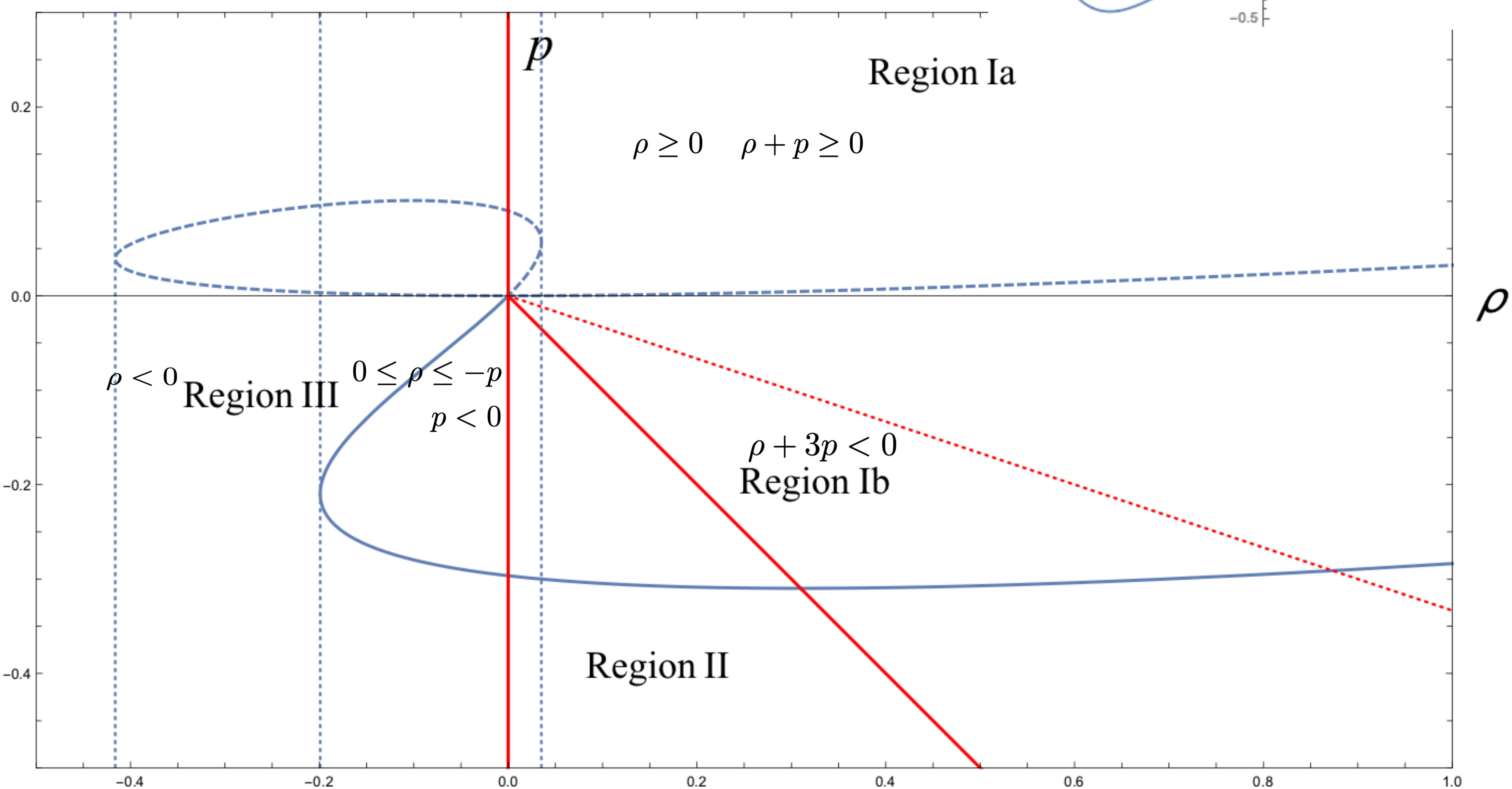
$$F(X) = \sum_{n=0}^{N} c_n X^n. \qquad c_0 \leq 0, \qquad c_n \geq 0 \ \text{ for } \ n \geq 1.$$

$$\rho(a) \sim \left(N - \frac{1}{2}\right) c_N^{\frac{1}{1-2N}} \left(\frac{a^3 N}{A}\right)^{\frac{2N}{1-2N}}$$



Polynomial interactions: ge





A completely soluble model: lessons form the quartic self-interaction

$$F = -\Lambda + \mu(\partial\varphi)^2 + \lambda(\partial\varphi)^4$$

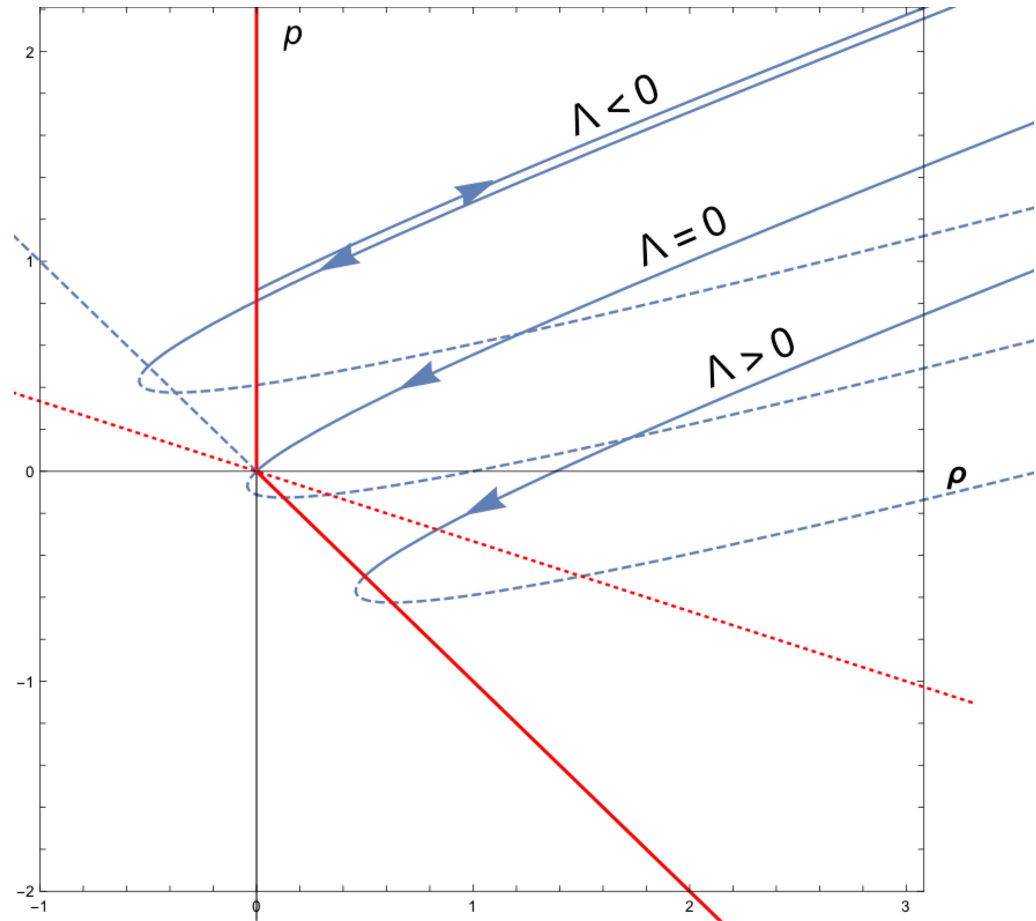
$$F = -\Lambda + \mu X + \lambda X^2$$

K-essence with a cosmological constant

$$F = -\Lambda + \mu X + \lambda X^2 \qquad \mu > 0, \quad \lambda > 0$$

$$\rho(X) = \frac{1}{2}\left(3X^2 + X + \Lambda\right)$$

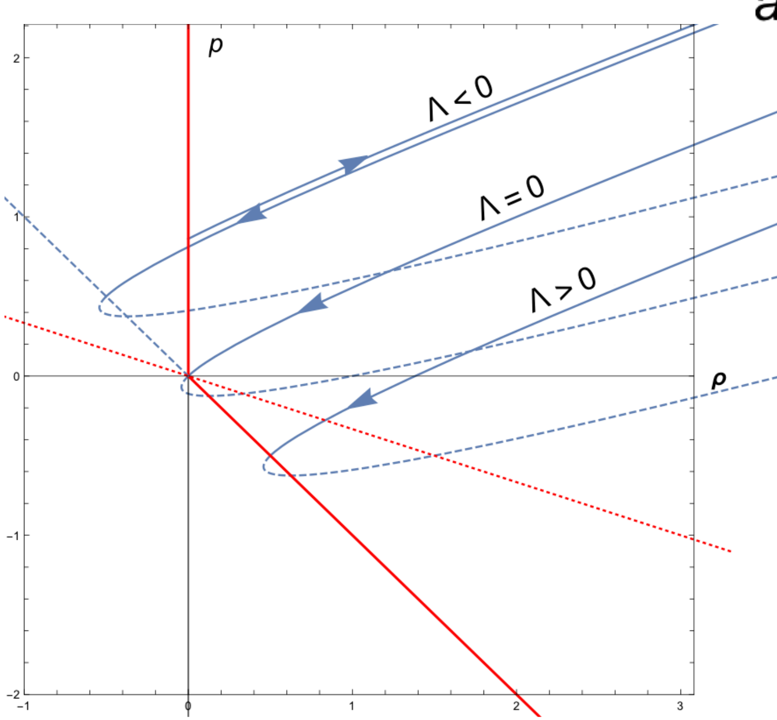
$$p(X) = \frac{1}{2}\left(X^2 + X - \Lambda\right)$$

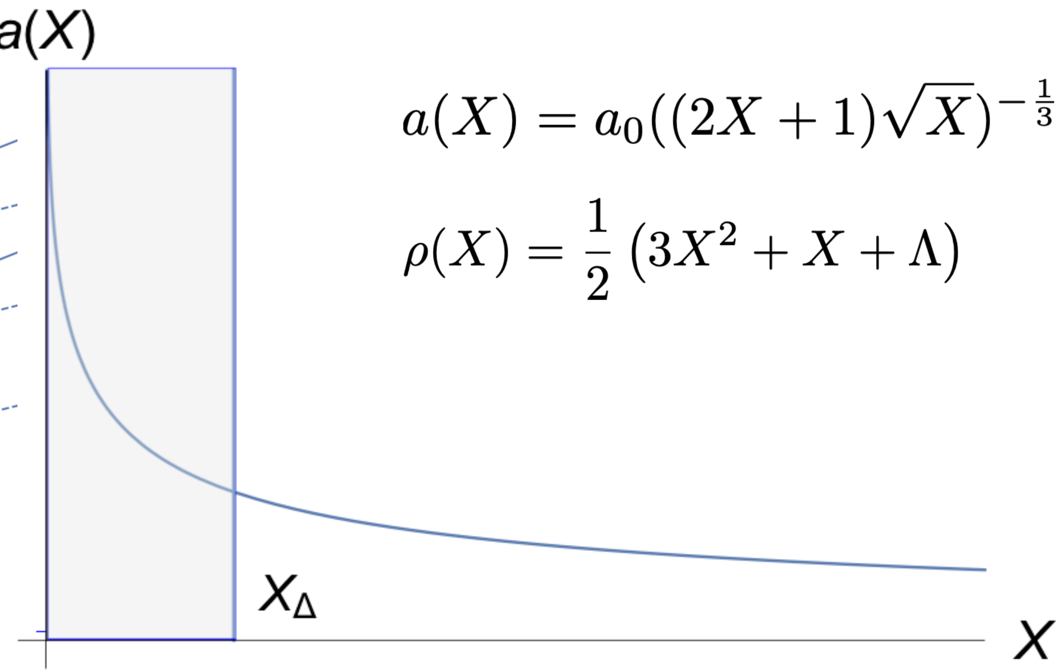


K-essence with a cosmological constant

$$F = -\Lambda + X + X^2$$

Field equations $(\partial_X F)\sqrt{X} = \pm\left(\frac{a_0}{a}\right)^3, \; a_0 \neq 0$





$$a(X) = a_0((2X+1)\sqrt{X})^{-\frac{1}{3}},$$

$$\rho(X) = \frac{1}{2}\left(3X^2 + X + \Lambda\right)$$

$$\frac{dX}{dt} = \mp\frac{2\sqrt{3}\,X\,F_X\sqrt{XF_X - \frac{1}{2}F}}{2XF_{XX} + F_X}$$

$$t(X) = \log\left(\frac{1}{2X} + 1\right)^{\frac{1}{3}\sqrt{2}}$$

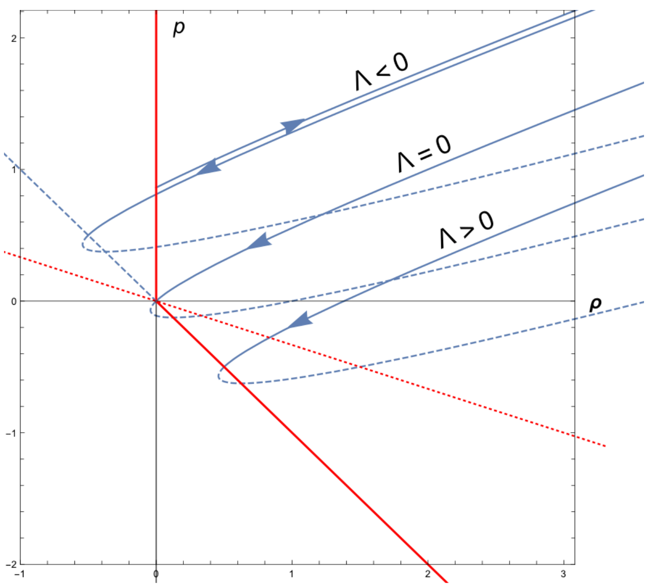
$$\Lambda = 1/12$$

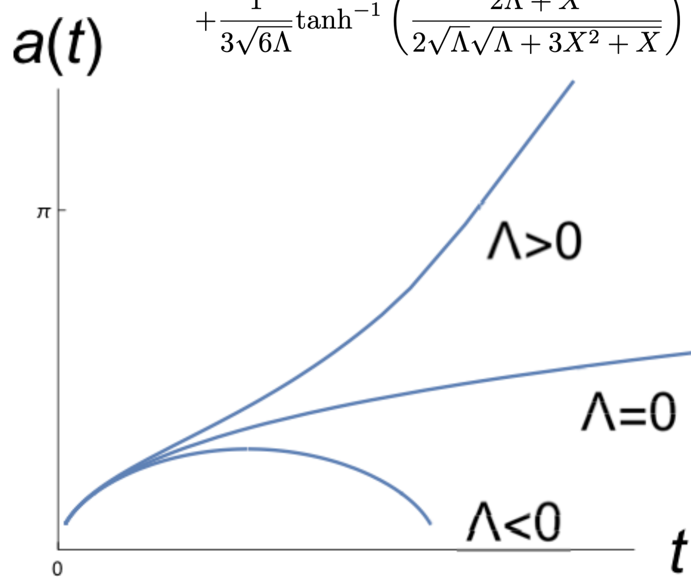
With the cosmological constant

$$F = -\Lambda + X + X^2$$

$$a(X) = a_0((2X+1)\sqrt{X})^{-\frac{1}{3}},$$

$$t(X) = \frac{2}{3}\sqrt{\frac{2}{12\Lambda+3}}\left[\tanh^{-1}\left(\frac{2}{\sqrt{12\Lambda+3}}\right) - \tanh^{-1}\left(\frac{2X-2\Lambda+\frac{1}{2}}{\sqrt{4\Lambda+1}\sqrt{\Lambda+3X^2+X}}\right)\right] + \\ + \frac{1}{3\sqrt{6\Lambda}}\tanh^{-1}\left(\frac{2\Lambda+X}{2\sqrt{\Lambda}\sqrt{\Lambda+3X^2+X}}\right) - \frac{1}{3\sqrt{6\Lambda}}\coth^{-1}\left(2\sqrt{3\Lambda}\right)$$



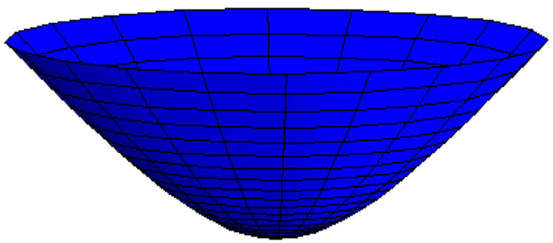


$\Delta > 1/4$
$$t_\Delta = \frac{\pi + 2\tan^{-1}\left(\frac{1}{2\sqrt{3\Delta}}\right)}{6\sqrt{6\Delta}} + \frac{\left(\pi - 2\tan^{-1}\left(\frac{2}{\sqrt{12\Delta-3}}\right)\right)}{3\sqrt{6\Delta-\frac{3}{2}}}$$

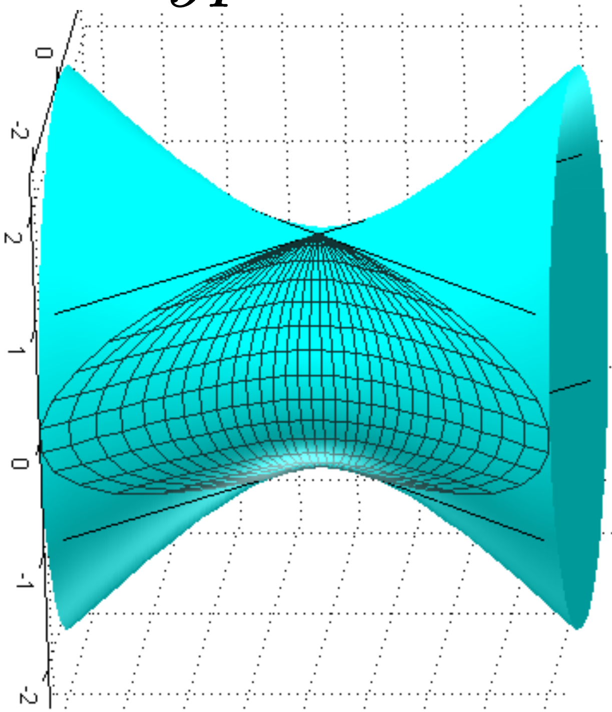
$\Delta < 1/4$
$$t_\Delta = \frac{\pi + 2\tan^{-1}\left(\frac{1}{2\sqrt{3\Delta}}\right)}{6\sqrt{6\Delta}} + \frac{2\log\left(\frac{2+\sqrt{3}\sqrt{1-4\Delta}}{2-\sqrt{3}\sqrt{1-4\Delta}}\right)}{3\sqrt{6-24\Delta}}$$

Only Λ<0

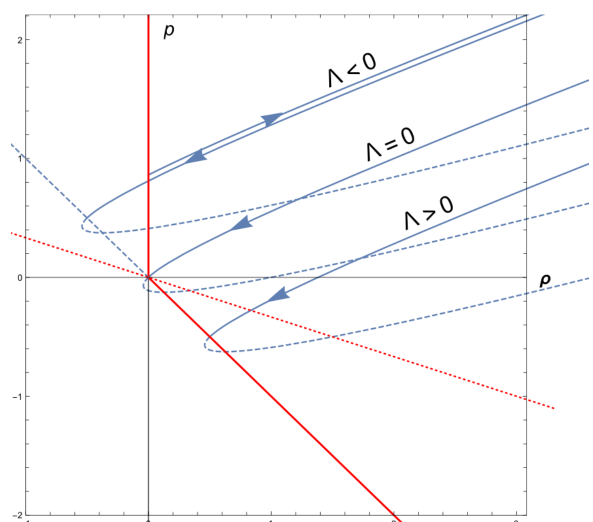
$$\ddot{a} = -\frac{1}{3}|\Lambda|\, a \qquad \dot{a}^2 = -\frac{1}{3}|\Lambda|\, a^2 - K$$



$$K = -1 \qquad hyperbolic$$



$$a(t) = \sqrt{\frac{3}{|\Lambda|}} \sin \sqrt{\frac{|\Lambda|}{3}}\, t$$



Without the cosmological constant

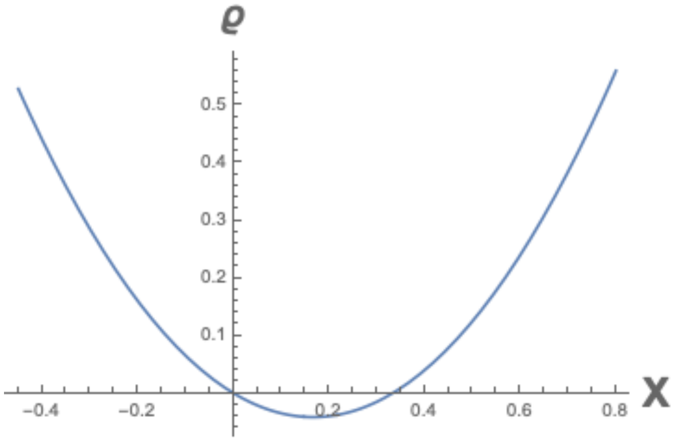
$$\Lambda = 0, \quad \mu < 0, \quad \lambda > 0$$

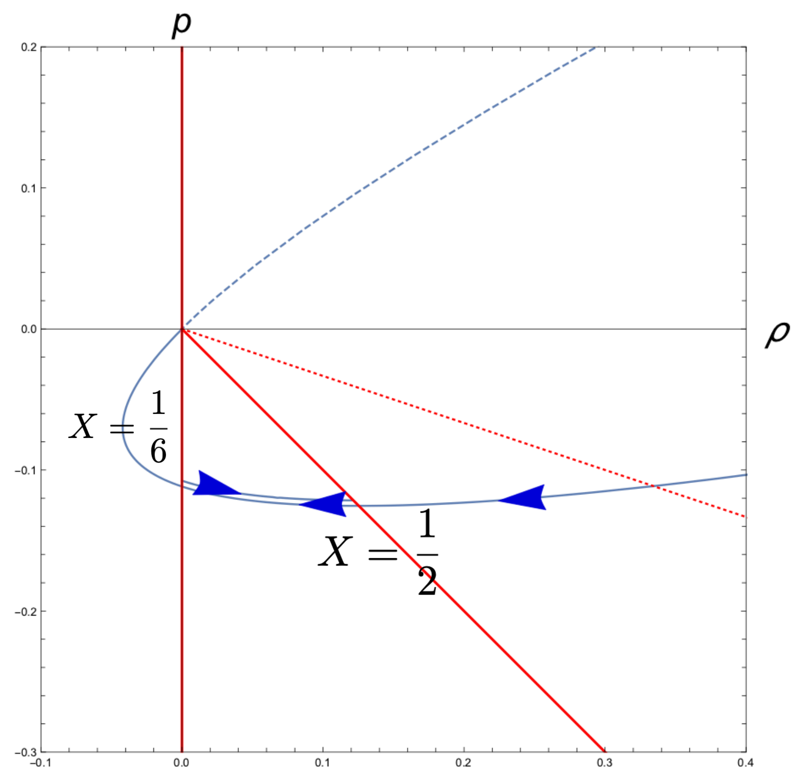
$$F = -X + X^2 = -(\partial\phi)^2 + (\partial\phi)^4$$

$$\rho(X) = \frac{1}{2}\left(3X^2 - X\right)$$

$$p(X) = \frac{1}{2}\left(X^2 - X\right)$$

$$c_s^2 = \frac{1-2X}{1-6X}$$

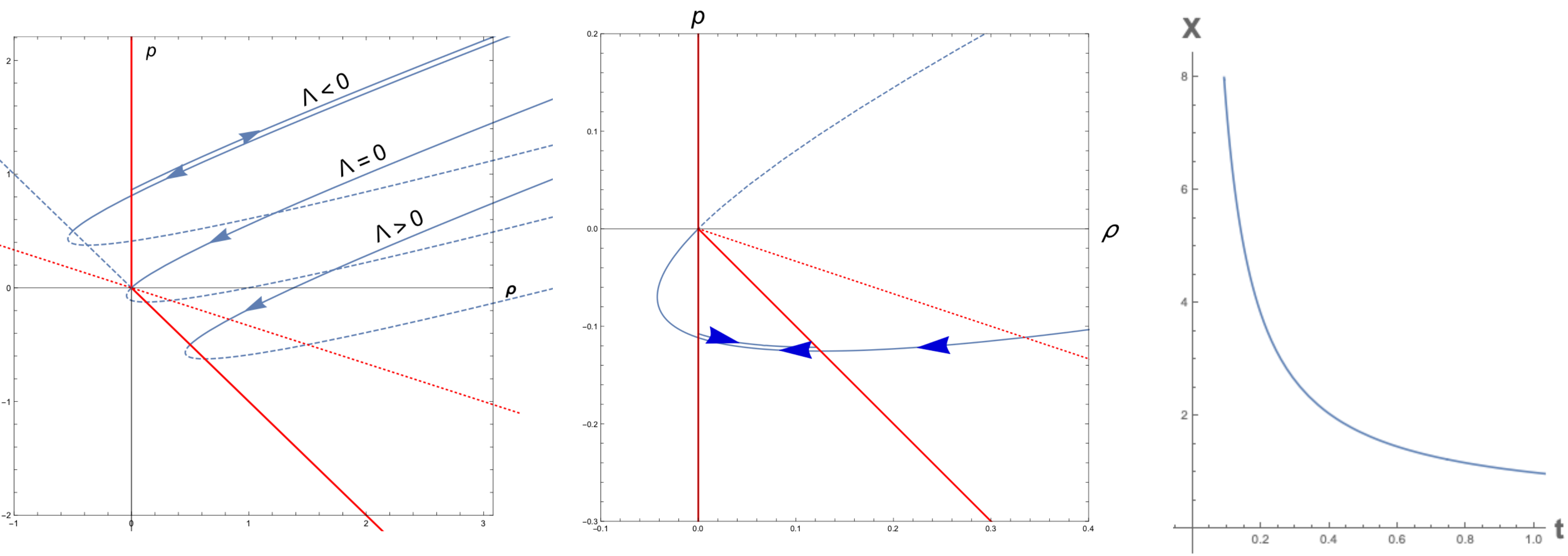




$$F = -X + X^2 = -(\partial\phi)^2 + (\partial\phi)^4$$

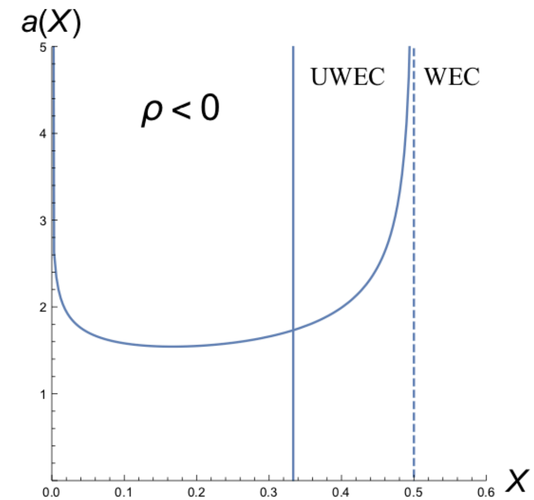
$$\rho(X) = \frac{1}{2}\left(3X^2 - X\right) \qquad a(X) = a_0((2X-1)\sqrt{X})^{-\frac{1}{3}}$$

$$t(X) = \frac{1}{3}\sqrt{2} - \frac{1}{3}\sqrt{2 - \frac{2}{3X}} + \frac{4}{3}\sqrt{\frac{2}{3}}\left(\tanh^{-1}\left(\frac{\sqrt{X}}{\sqrt{3X-1}}\right) - \tanh^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$$



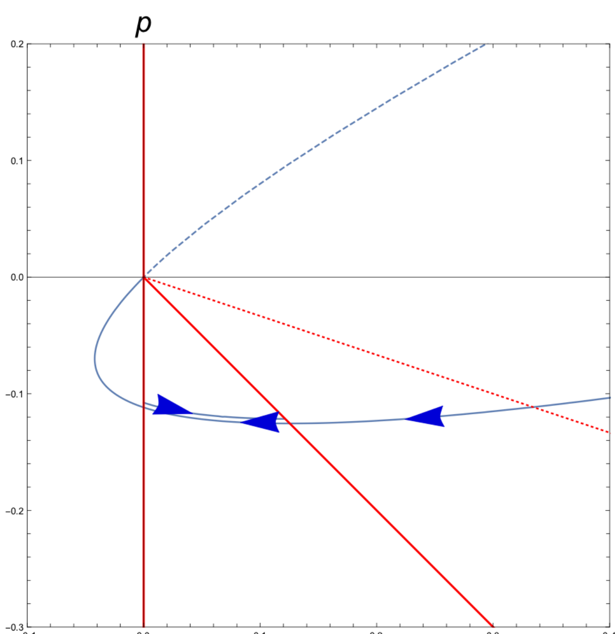
$$F = -X + X^2 = -(\partial\phi)^2 + (\partial\phi)^4$$

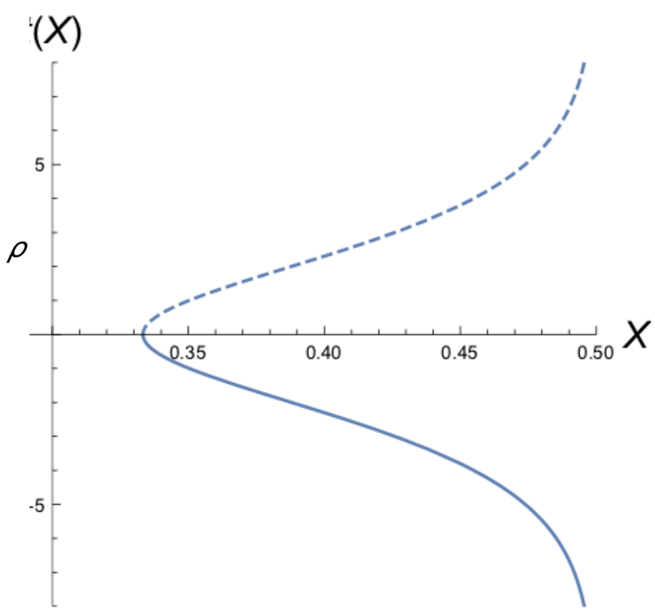
$$a(X) = a_0((1-2X)\sqrt{X})^{-\frac{1}{3}}$$

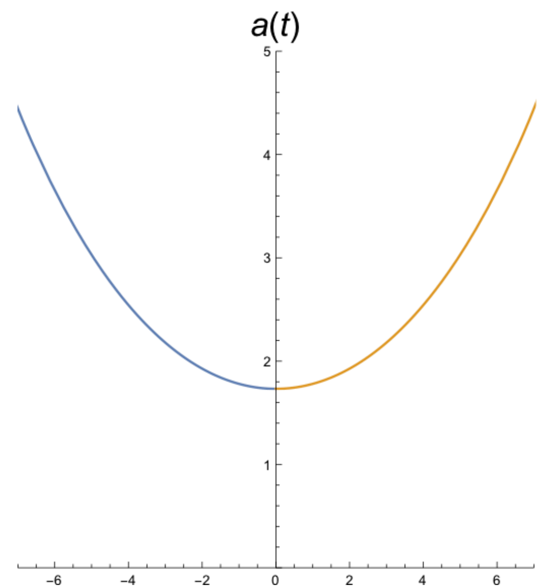


$$\frac{dX}{dt} = \mp \frac{2\sqrt{3}\, X\, F_X \sqrt{X F_X - \frac{1}{2} F}}{2 X F_{XX} + F_X}$$

$$t(X) = \frac{1}{3}\sqrt{\frac{2(3X-1)}{3X}} - \frac{2}{3}\sqrt{\frac{2}{3}}\log\left(\frac{4X + 2\sqrt{X(3X-1)} - 1}{1-2X}\right).$$





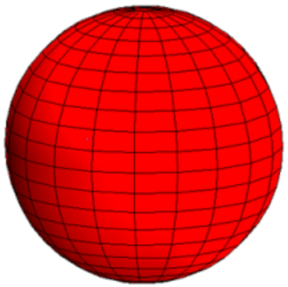


Vacuum Cosmological Equations

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

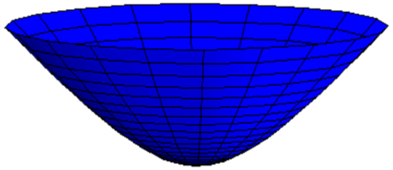
$$\ddot{a} = \frac{1}{3}\Lambda a \,, \qquad \dot{a}^2 = \frac{1}{3}\Lambda a^2 - K$$

$$K = 1 \quad \rightarrow \quad a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}} t$$

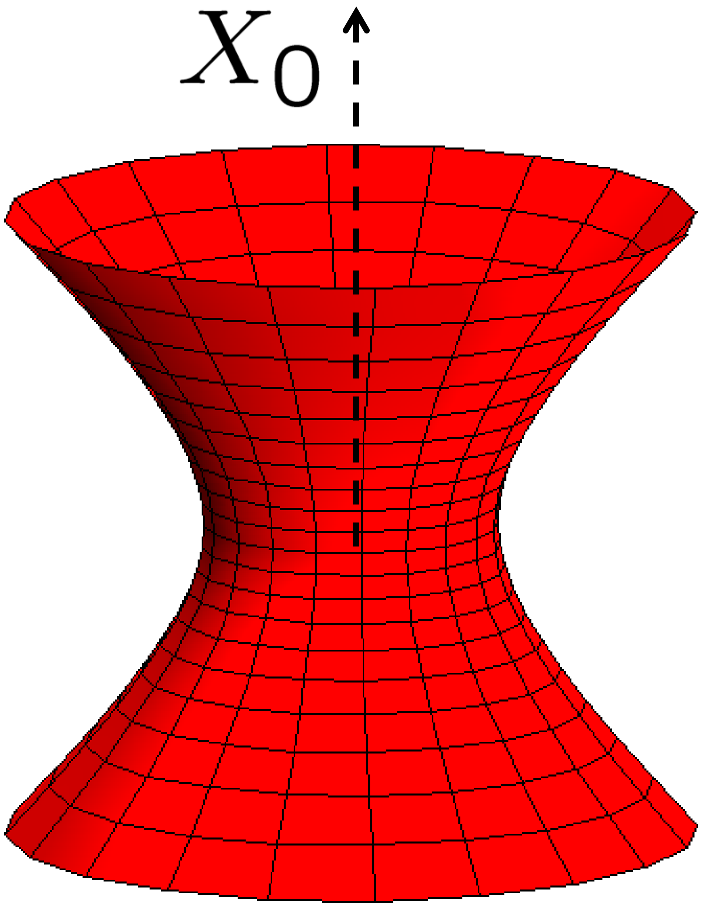


$$K = 0 \quad \rightarrow \quad a(t) = \exp \sqrt{\frac{\Lambda}{3}} t$$

$$K = -1 \quad \rightarrow \quad a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}} t$$



Spherical de Sitter model



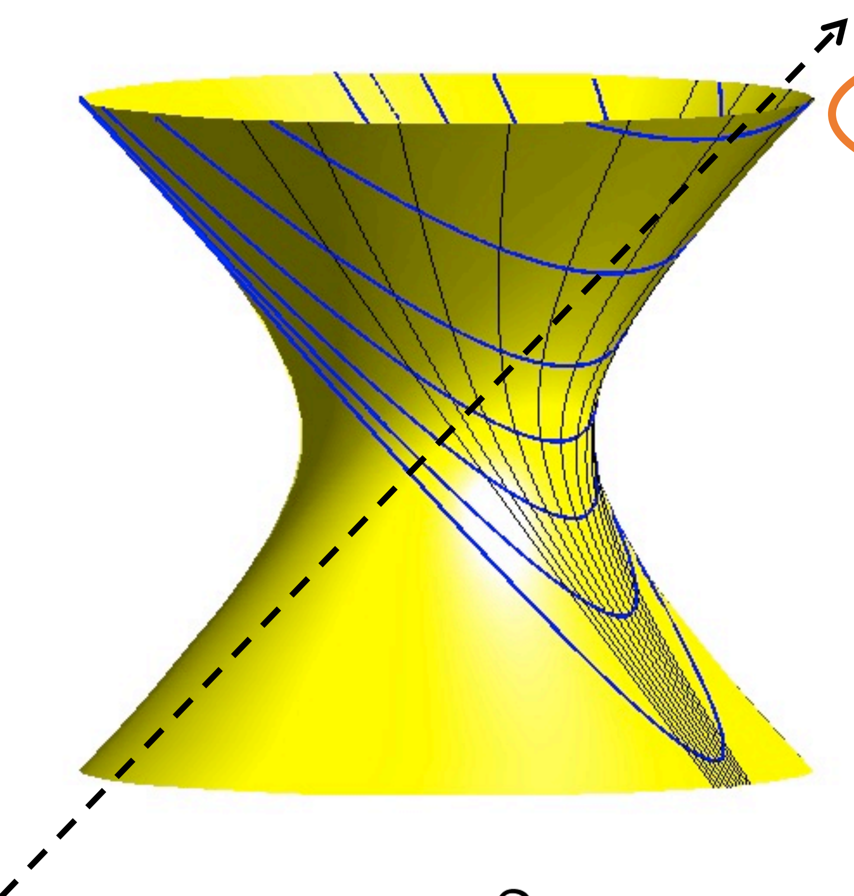
$$\begin{cases} X_0 &= R\sinh(t/R) \\ X_1 &= R\cosh(t/R)\sin\theta\sin\chi\sin\phi \\ X_2 &= R\cosh(t/R)\sin\theta\sin\chi\cos\phi \\ X_3 &= R\cosh(t/R)\sin\theta\cos\chi \\ X_4 &= R\cosh(t/R)\cos\theta \end{cases}$$

$$R = \sqrt{\frac{3}{\Lambda}}$$

$$ds^2 = dX_0^2 - dX_1^2 - \ldots dX_4^2\Big|_{dS} =$$

$$= dt^2 - R^2\cosh^2\frac{t}{R}\left(d\theta^2 + \sin^2\theta(d\chi^2 + \sin^2\chi d\phi^2)\right)$$

Flat de Sitter model (Lemaître, 1924)

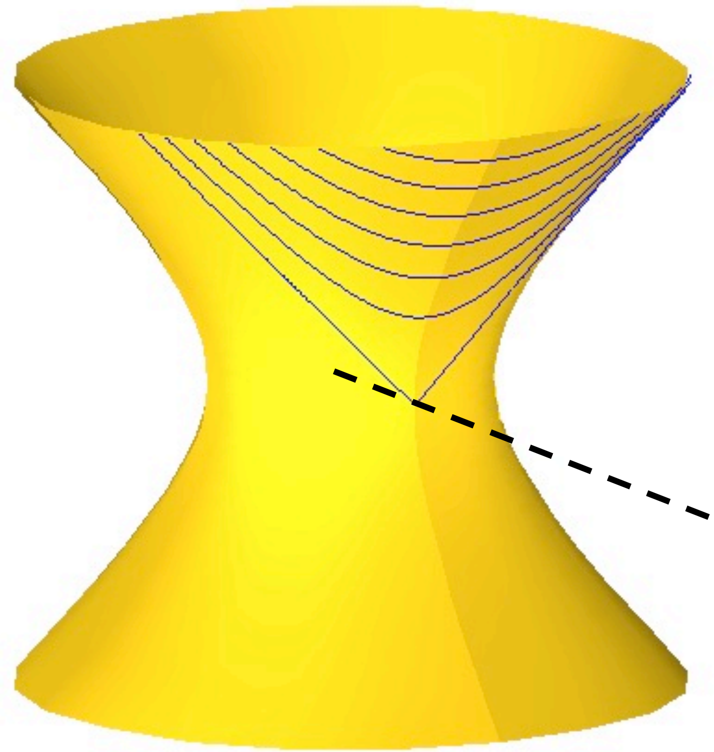


$$X_0 + X_4 = R \exp \frac{t}{R}$$

$$\begin{cases} X_0 &= R \sinh \frac{t}{R} + \frac{1}{2R} e^{\frac{t}{R}} |\vec{x}|^2 \\ X_1 &= \exp\left(\frac{t}{R}\right) x_1 \\ X_2 &= \exp\left(\frac{t}{R}\right) x_2 \\ X_3 &= \exp\left(\frac{t}{R}\right) x_3 \\ X_4 &= R \cosh \frac{t}{R} - \frac{1}{2R} e^{\frac{t}{R}} |\vec{x}|^2 \end{cases}$$

$$ds^2 = dX_0^2 - dX_1^2 - \ldots dX_4^2 \Big|_{dS} =$$
$$= dt^2 - \exp \frac{2t}{R} \left(dx_1^2 + dx_2^2 + dx_3^2 \right)$$

Open de Sitter model (de Sitter 1917)



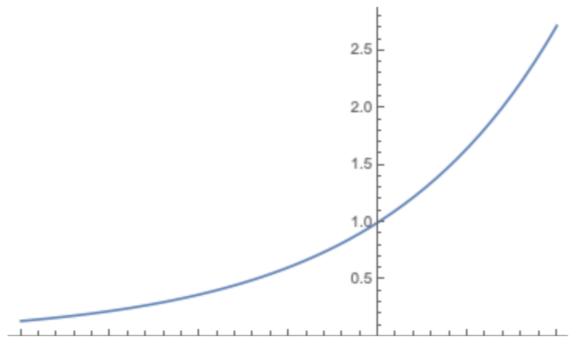
$$\begin{cases} X_0 = R \sinh \frac{t}{R} \cosh \chi \\ X_1 = R \sinh \frac{t}{R} \sinh \chi \sin \theta \sin \phi \\ X_2 = R \sinh \frac{t}{R} \sinh \chi \sin \theta \cos \phi \\ X_3 = R \sinh \frac{t}{R} \sinh \chi \cos \theta \\ X_4 = R \cosh \frac{t}{R} \end{cases}$$

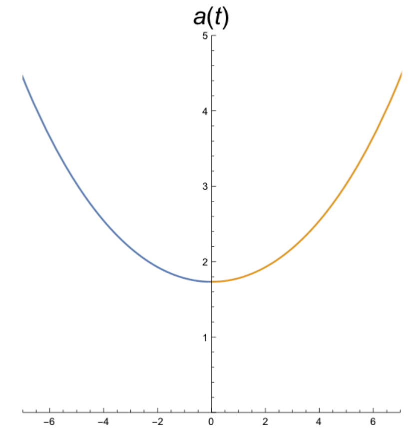
$$ds^2 = dX_0^2 - dX_1^2 - \ldots dX_4^2 \Big|_{dS} =$$

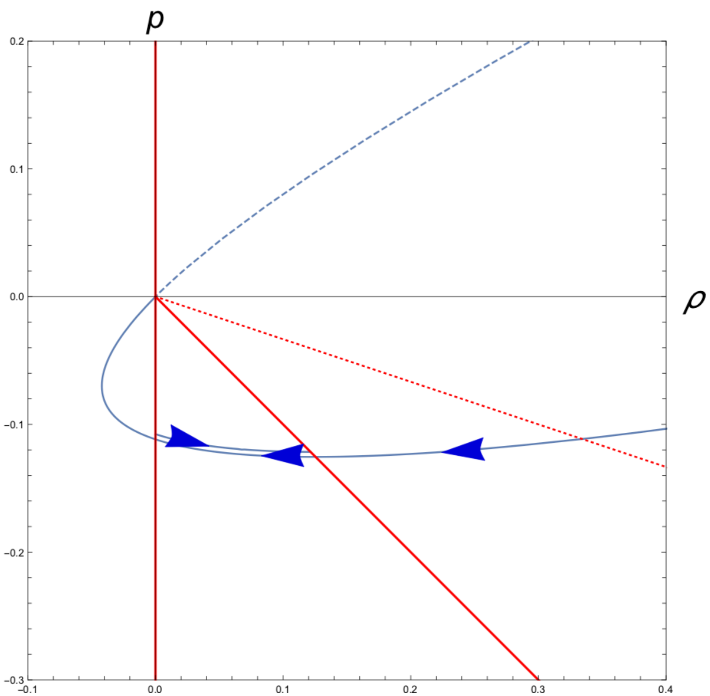
$$= dt^2 - R^2 \sinh^2 \frac{t}{R} \left(d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

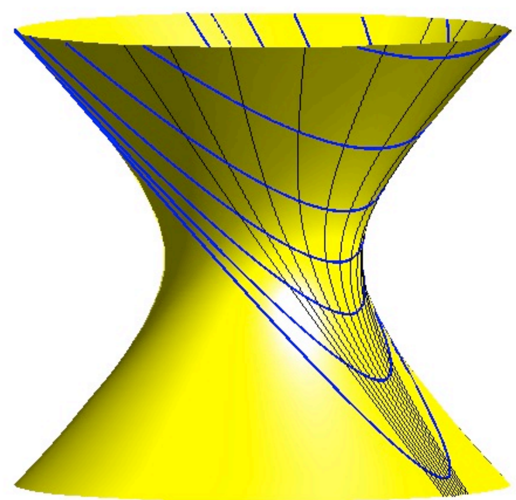
$$a(X) = a_0((1-2X)\sqrt{X})^{-\frac{1}{3}}$$

$$t(X) = \frac{1}{3}\sqrt{\frac{2(3X-1)}{3X}} - \frac{2}{3}\sqrt{\frac{2}{3}}\log\left(\frac{4X+2\sqrt{X(3X-1)}-1}{1-2X}\right).$$

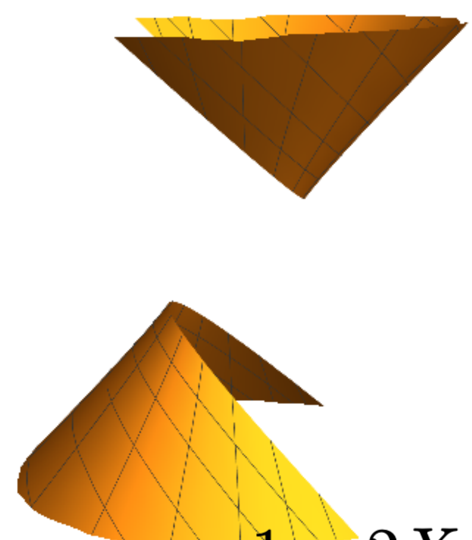








$$c_s^2 = -1$$

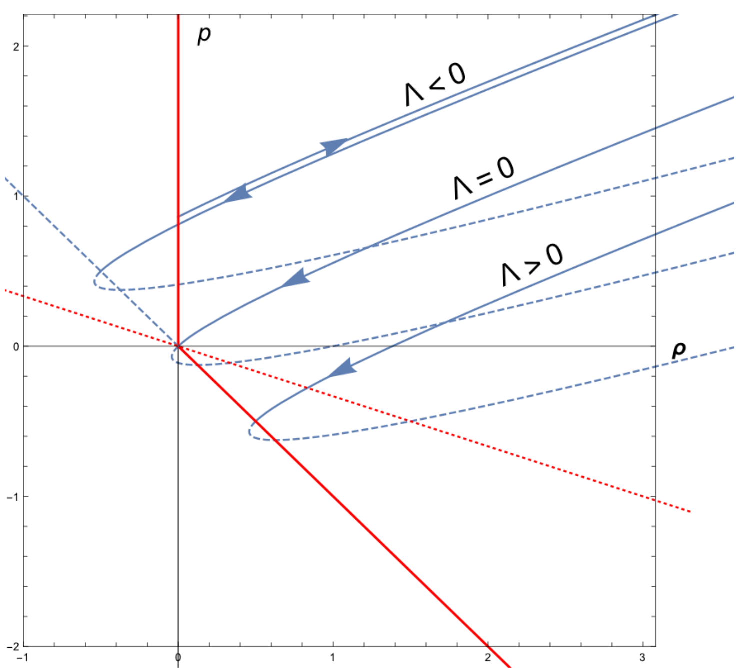


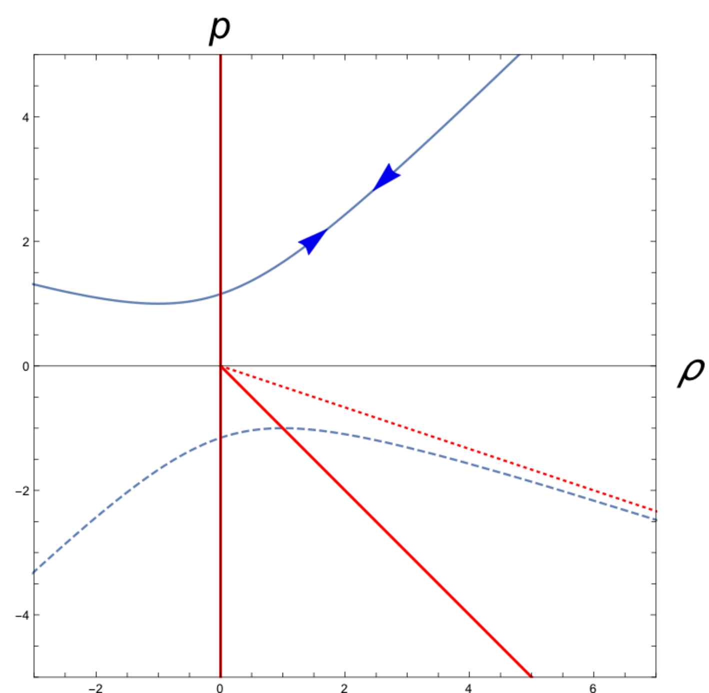
$$c_s^2 = \frac{1-2X}{1-6X}$$

Mimicking a negative cosmological constant

$$F = X + \frac{1}{X} = 2p \qquad \rho(X) = \frac{X^2 - 3}{2X}$$

$$a(X) = \frac{a_0\sqrt{X}}{\sqrt[3]{X^2-1}}, \qquad t(X) = \frac{1}{\sqrt{6}}\int_{\sqrt{3}}^{X} \frac{x^2+3}{\sqrt{x^3-3x}\,(x^2-1)}dx.$$





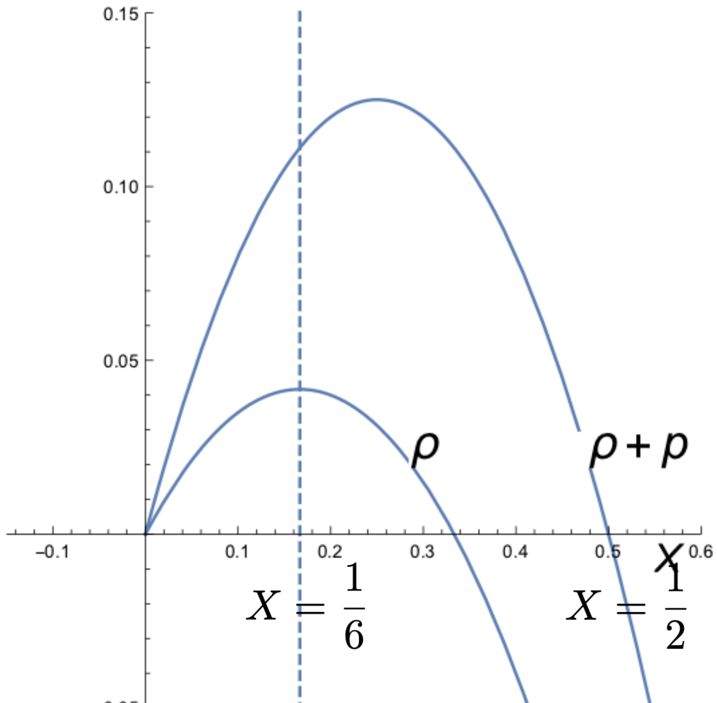
Phenomenology of a branching point

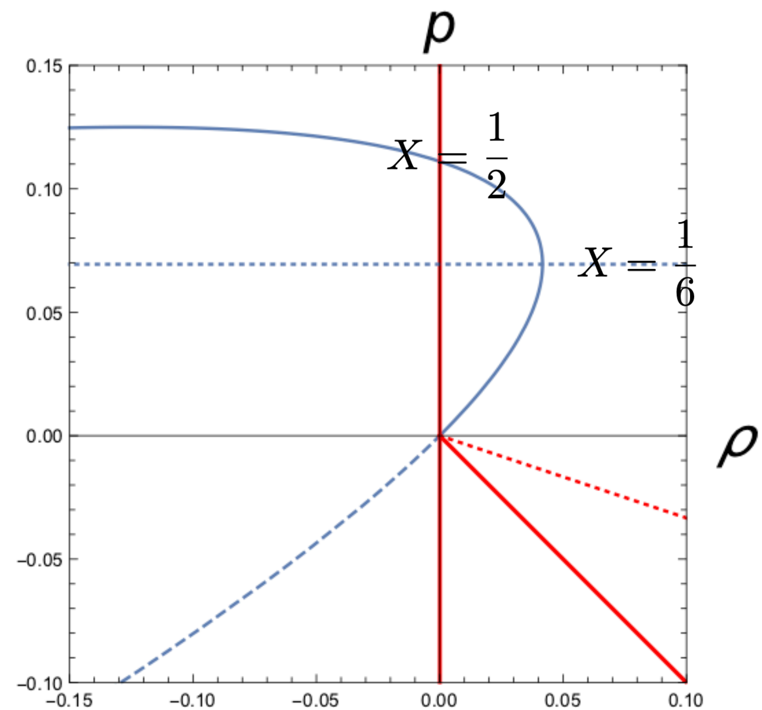
$$\Lambda = 0, \quad \mu < 0, \quad \lambda > 0$$

$$F = X - X^2$$

$$\rho = \frac{1}{2}\left(X - 3X^2\right)$$

$$c_s^2 = \frac{1-2X}{1-6X}$$





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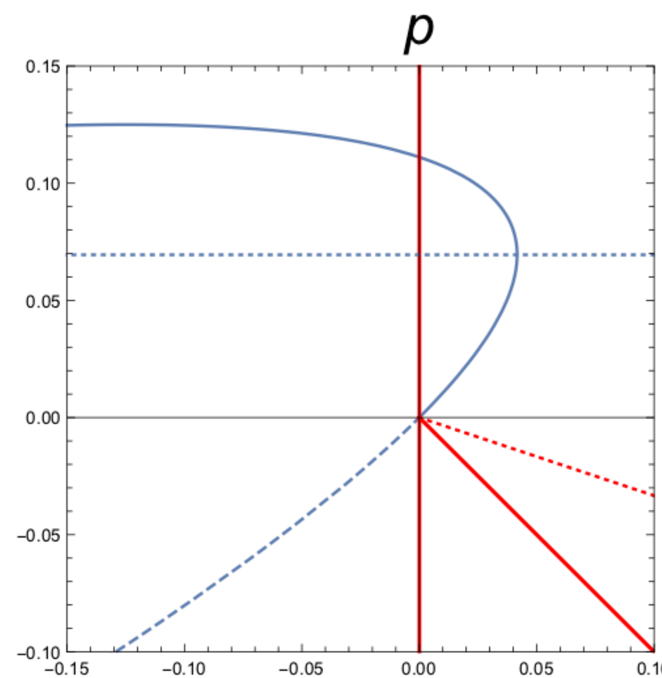


The presence of a branching point in the physical region renders the dynamical behaviour subtler
The following equations are the starting point:

$$\begin{aligned} a(X) &= a_0 \left(\sqrt[3]{\sqrt{X}(1-2X)} \right)^{-1}, \\ \dot{X} &= \pm \frac{3\sqrt{6}X(2X-1)\sqrt{X-3X^2}}{6X-1} \end{aligned}$$

the final task is to describe
the scale factor $a(t)$ as a function of the cosmic time.

Phenomenology of a branching point

$$F = X - X^2$$

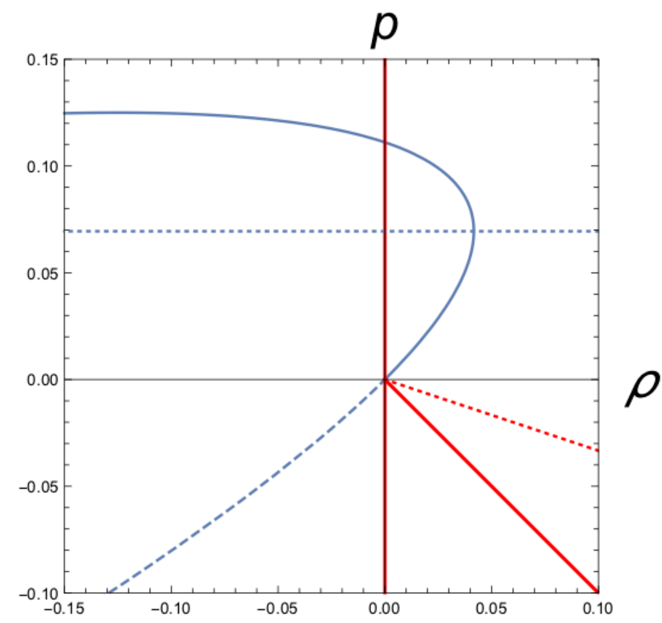
$$\Lambda = 0, \quad \mu < 0, \quad \lambda > 0$$

Initial conditions at $X = 0$ (which means $a \sim \infty$).

$$\dot{a}(X) = \frac{\partial a}{\partial X}\dot{X} = -\frac{\sqrt{3X - 9X^2}}{\sqrt{2}\sqrt[3]{(1-2X)\sqrt{X}}} = -\sqrt{3\rho(X)}a(X)$$

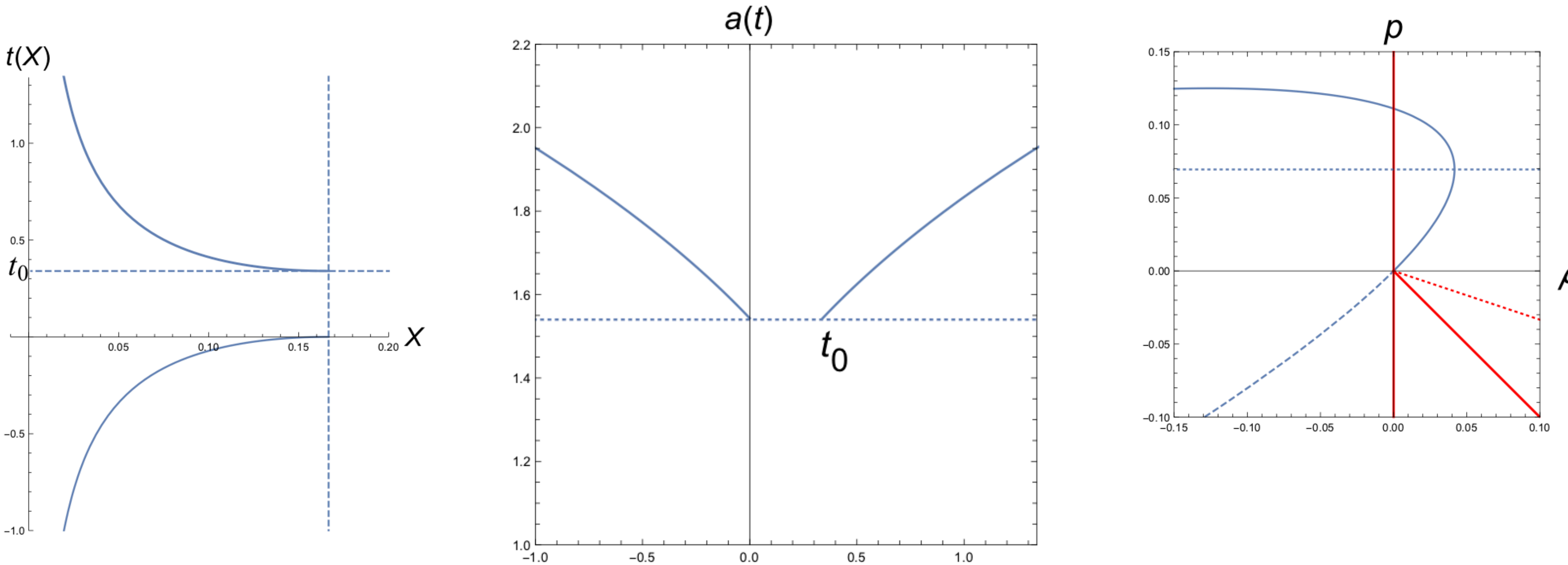
$$\ddot{a}(X) = \frac{\partial \dot{a}}{\partial X}\dot{X} = \frac{3X(3X-2)}{2\sqrt[3]{(1-2X)\sqrt{X}}} = -\frac{3a(X)}{2}(\rho + 3p) < 0.$$

$$t(X) = \frac{1}{27}\sqrt{2}\left(-\frac{3\sqrt{3-9X}}{\sqrt{X}} - 12\sqrt{3}\tan^{-1}\left(\frac{\sqrt{X}}{\sqrt{1-3X}}\right) + 2\sqrt{3}\pi + 9\right),$$



$$a(X_c) = \frac{\sqrt{3}}{\sqrt[6]{2}}, \qquad \dot{a}(X_c) = -\frac{\sqrt{3}}{2^{5/3}}, \qquad \ddot{a}(X_c) = -\frac{3\sqrt{3}}{8\sqrt[6]{2}}.$$

Phenomenology of a branching point



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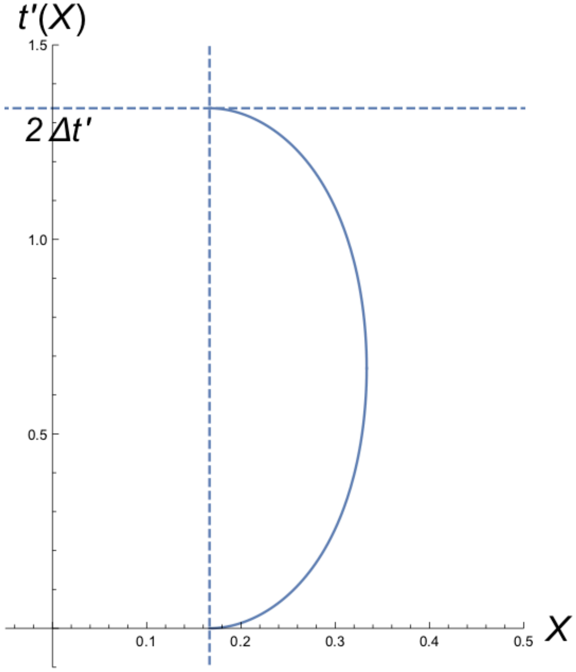
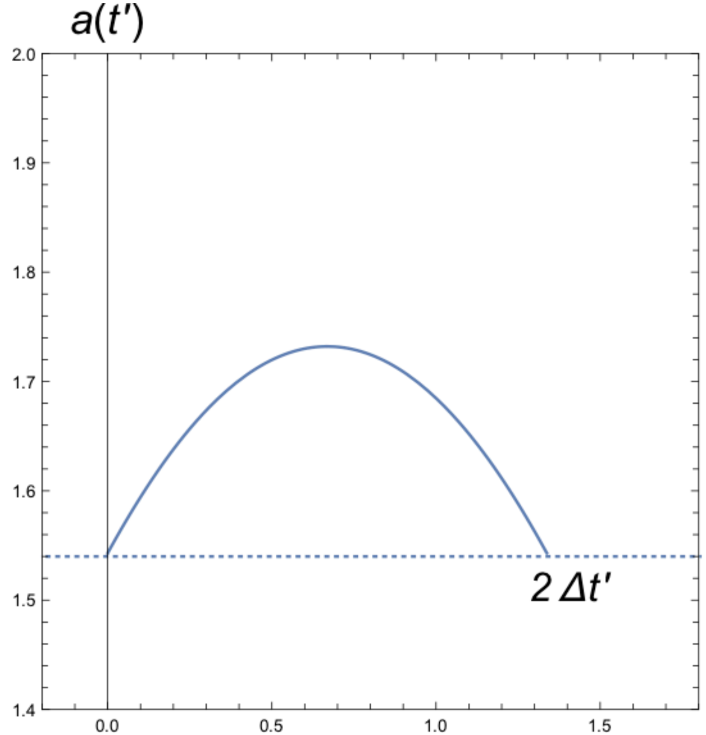
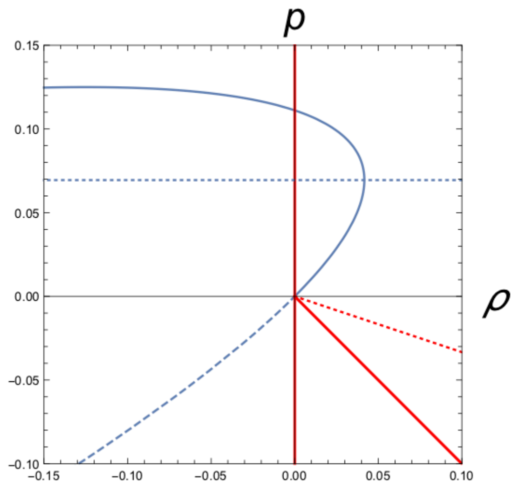
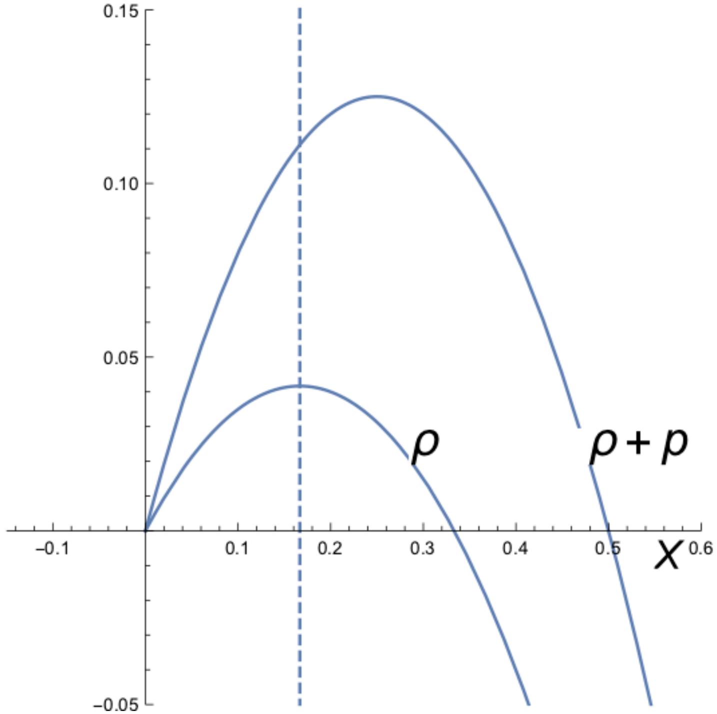


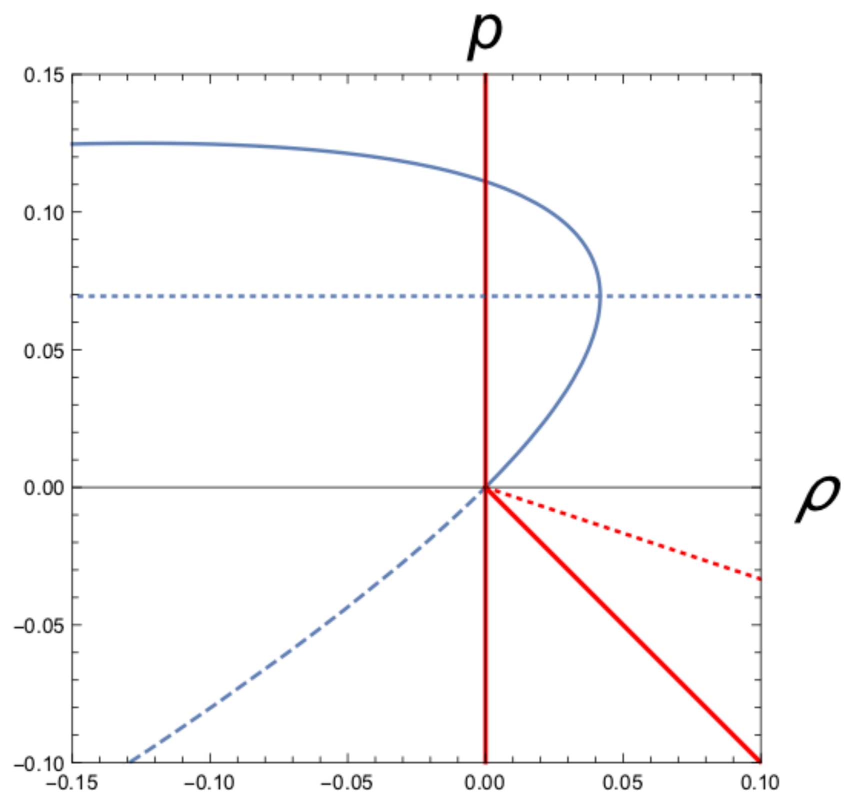




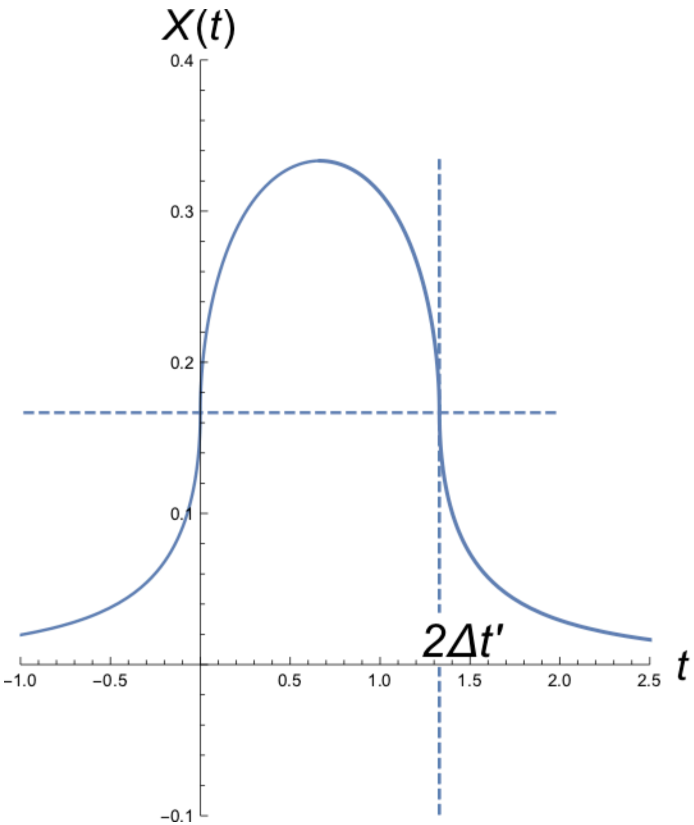
$$t'(X) = \frac{1}{27}\sqrt{2}\left(\frac{3\sqrt{3-9X}}{\sqrt{X}} + 12\sqrt{3}\tan^{-1}\left(\frac{\sqrt{X}}{\sqrt{1-3X}}\right) - 2\sqrt{3}\pi - 9\right)$$

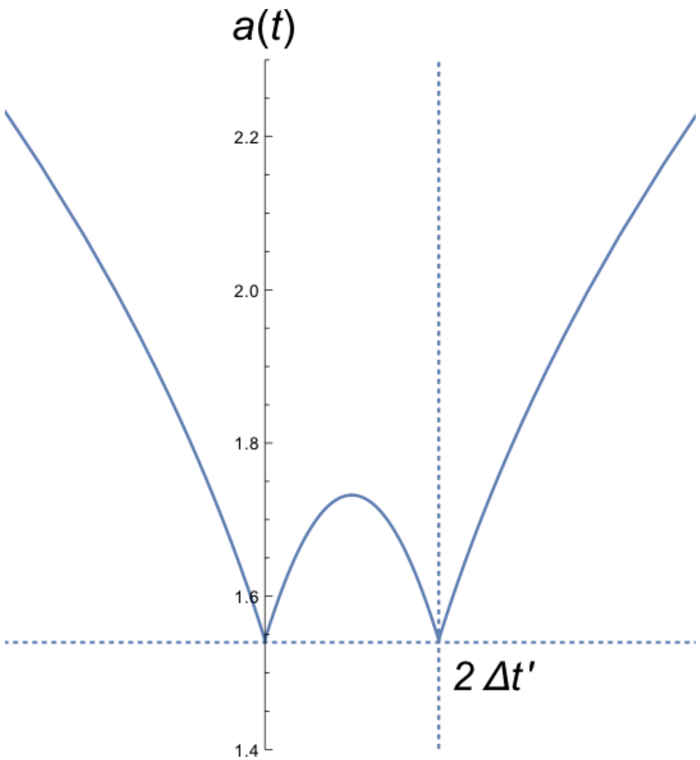
So what happens at the branching point?





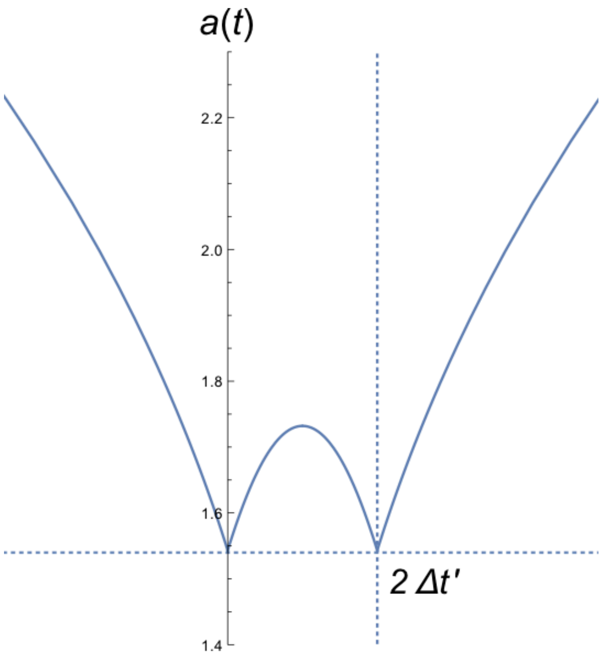
First scenario



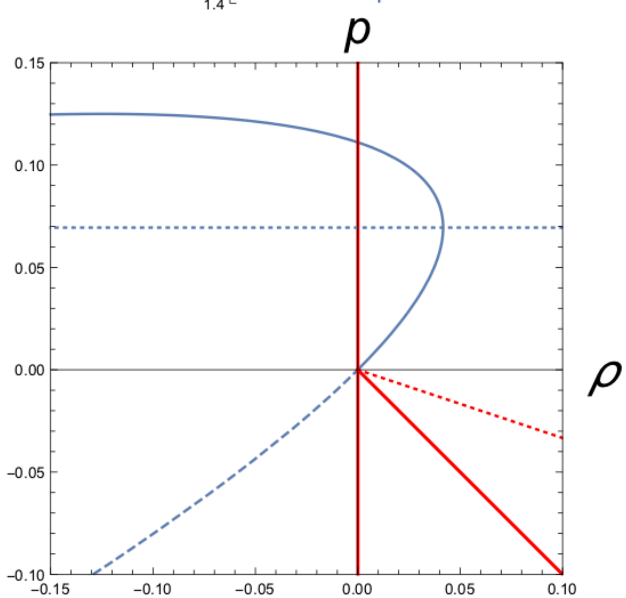


First interpretation

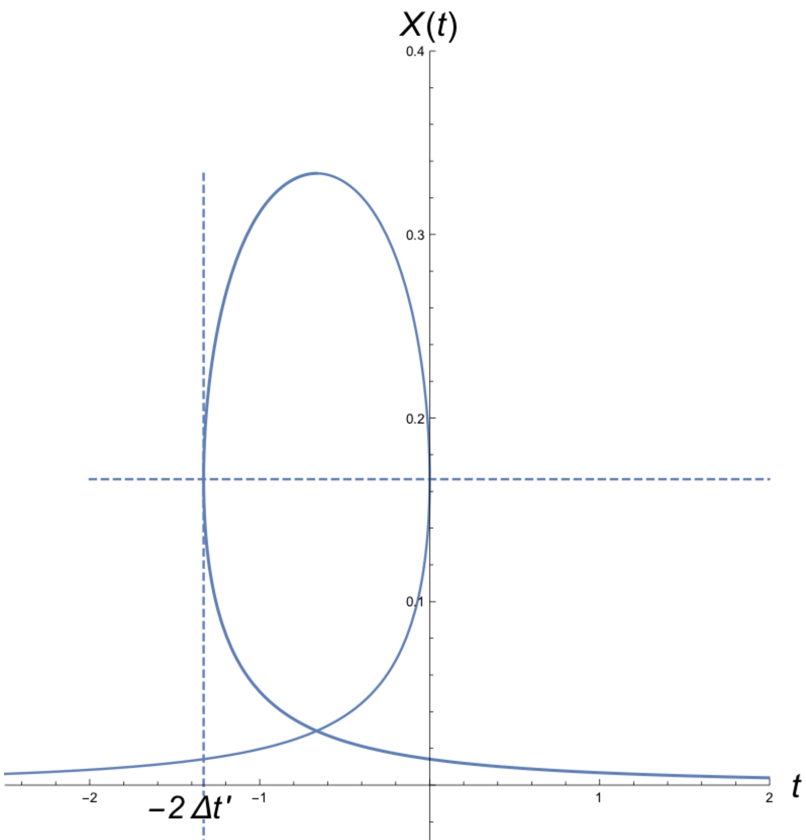
When the field gets at the critical point the density reaches a maximum and the universe cannot shrink anymore. Like in the elastic collision the sign of the velocity is reverted. The expansion following the collision keeps decelerating as the acceleration in this model is always negative.

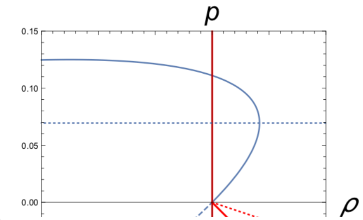


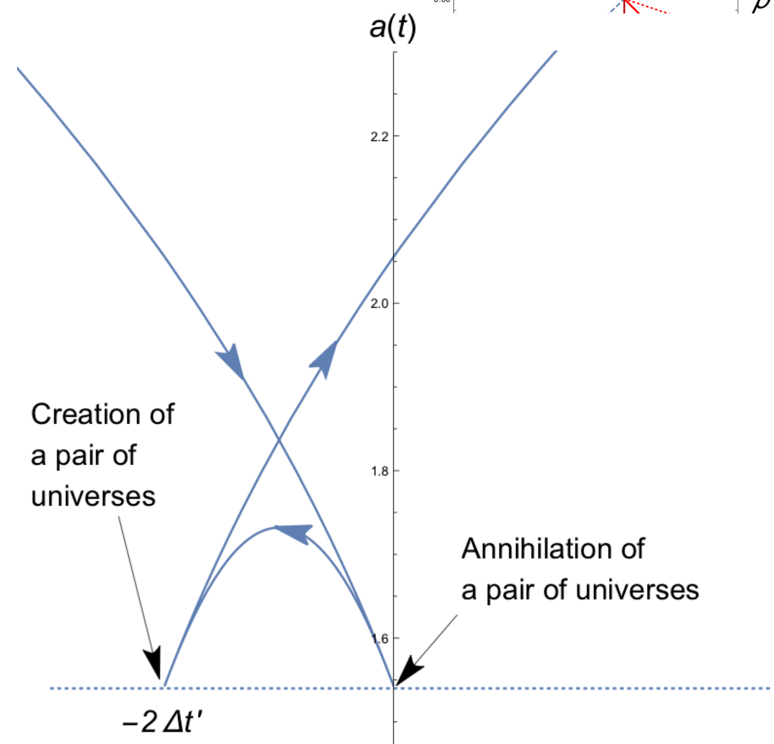
1. The Universe starts at $t = -\infty$ with infinite radius and zero velocity. It is a flat Minkowski spacetime.
2. The evolution of the universe makes the radius shrink with a negative and decreasing velocity $\dot{a}$ up to $t = 0$ when the universe hits the branching point.
3. *The (unstable) bulged bounce.* The branching point acts like a wall and the universe undergoes an elastic collision where the velocity is reverted and becomes positive. The universe enters in a phase of decelerated expansion. At $t = \Delta t'$ the expansion stops and the universe starts again to shrink up to $t = 2\Delta t'$ where it hits the branching point for a second time.
4. At the branching point the velocity is reverted again. The universe enters in a phase of everlasting decelerated expansion that will drive it back to Minkowski space at $t = \infty$.
5. There is no singularity at $t = \infty$.



Second interpretation

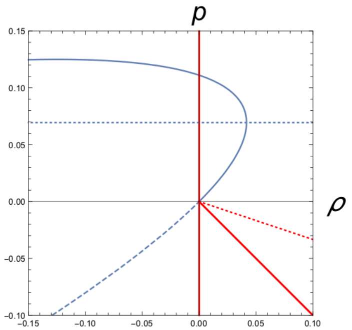






$X(t)$ and $a(t)$ are plotted in *trompe l'œil* . What is the meaning of such diagrams? They represents the clever solution that the universe gives to apparently unsolvable problem it has to face when arriving at the branching point: how could it go from $a(X_c) = \sqrt{3}/\sqrt[6]{2}$ to $a(1/3) = \sqrt{3}$ with a negative velocity and from $\dot{a}(X_c) = -\sqrt{3}/2^{5/3}$ to $\dot{a}(1/3) = 0$ with a negative acceleration? Running backward in time!

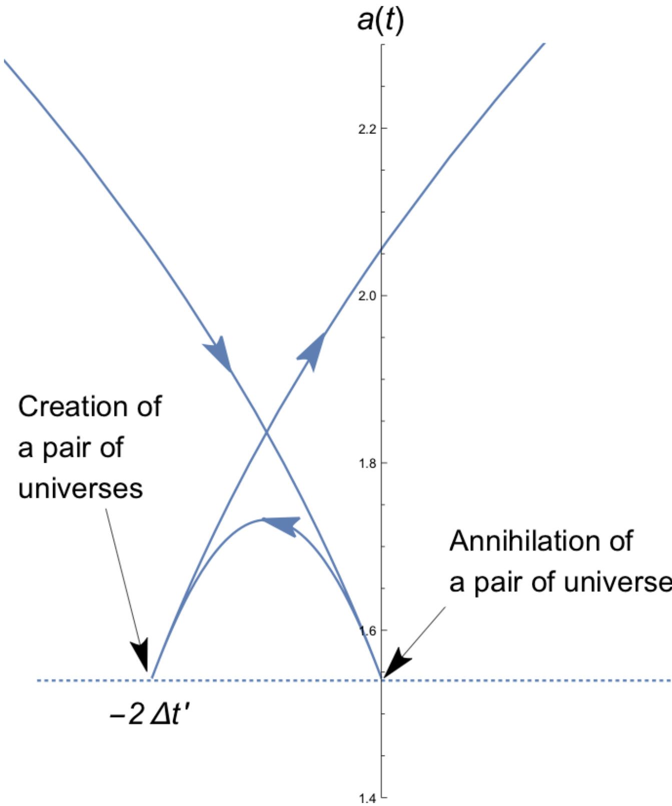
Second interpretation



The ramification point in the equation of state is encountered twice during the time evolution. When the universe gets at the ramification point the time starts flowing backward till the universe gets again to the ramification point, when the usual forward orientation of time is recovered. The phenomenology is essentially the same as in the previous conservative description but the inversion of the velocity is caused by an inversion of the sense of the flow of time, inversion that is short-lived.

Borrowing from relativistic quantum mechanics ideas it is tempting to say that a universe pair is annihilated when the ramification point is first encountered; a pair of universes is created when the ramification point is encountered a second time.

There is only one possible difference. Since the time runs backward the instability due to the negative squared velocity of sound needs to be reconsidered.



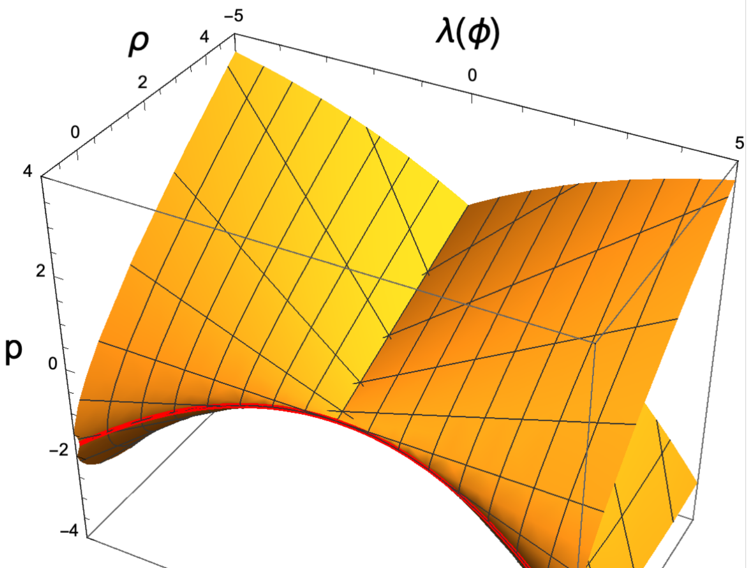
General models (non pure)

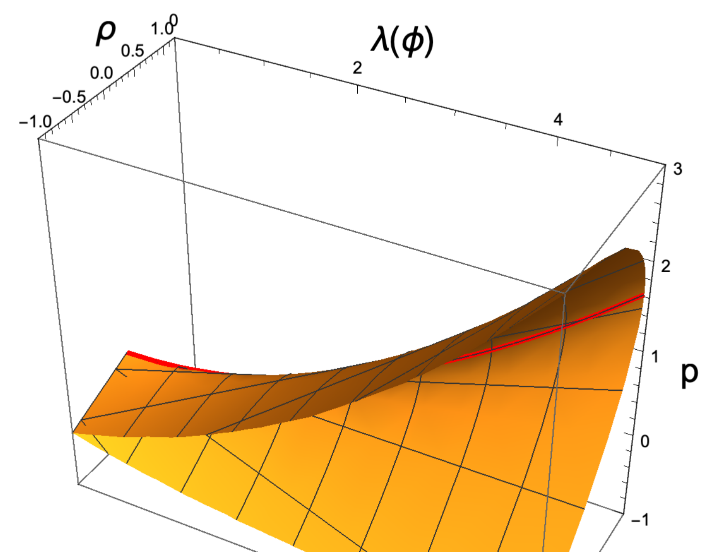
$$F(\varphi, \partial\varphi) = \lambda(\varphi)(\partial\varphi)^2 + \mu(\varphi)(\partial\varphi)^4.$$

$$F(\varphi, \partial\varphi) = \lambda(\varphi)(\partial\varphi)^2 \pm (\partial\varphi)^4.$$

$$p(\rho, \varphi) = \frac{1}{18}\left(6\rho + \lambda(\varphi)^2 \pm \lambda(\varphi)\sqrt{\lambda(\varphi)^2 - 24\rho}\right).$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3} \mp \frac{2\lambda(\varphi)}{3\sqrt{\lambda(\varphi)^2 - 24\rho}}$$





General models (non pure)