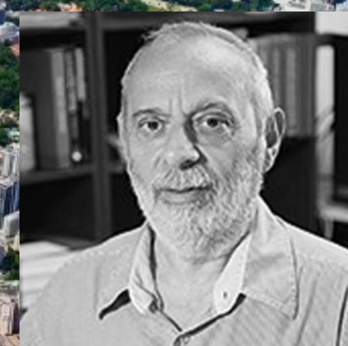
Gravitational Particle Production and Dark Matter — Four Lectures

1. Dark Matter: Evidence and the Standard WIMP
2. Gravitational Particle Production (Schrödinger's Alarming Phenomenon)
3. GPP of Scalar Fields
4. Beyond Scalar Fields



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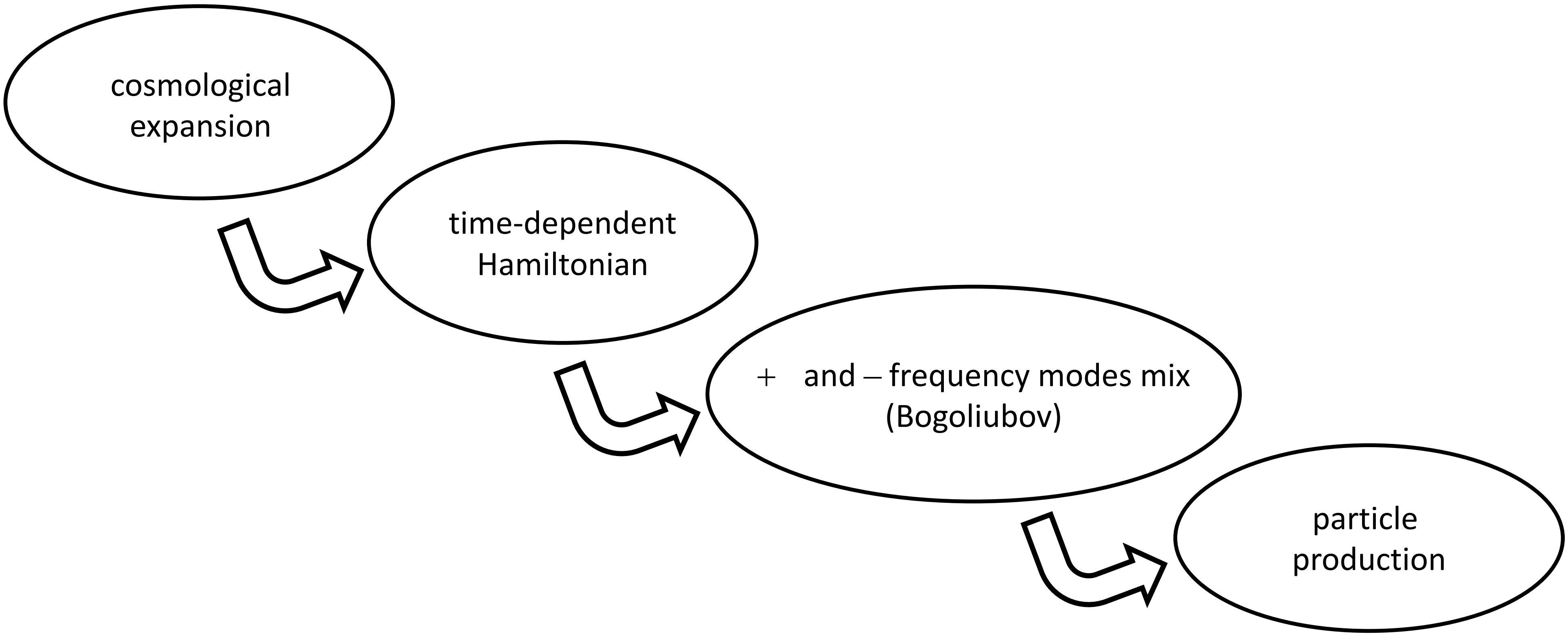
CBPF 9/2022

Cosmological Gravitational Particle Production (CGPP)

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a "particle" is approximate.

Schrodinger (1939); Parker (1965, 68); Fulling, Ford, & Hu; Zel'dovich; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



Scalar field ϕ in FRW background

covariant action

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

Gravity enters the picture

in spatially flat FRW background : $ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$ (η is conformal time)

$$S[\varphi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}a^2(\partial_\eta\varphi)^2 - \frac{1}{2}a^2(\nabla\varphi)^2 - \frac{1}{2}a^4 m^2\varphi^2 + \frac{1}{2}a^4\xi R\varphi^2\right]$$

field rescaling

$$\phi(\eta, \boldsymbol{x}) = a(\eta)\varphi(\eta, \boldsymbol{x})$$

$aH \to 0$ to zero at $\eta = \pm\infty$

action for canonically-normalized field

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\text{eff}}^2\phi^2 - \frac{1}{2}\partial_\eta(aH\phi^2)\right]$$

time-dependent effective mass

$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background ⇒
time-dependent Hamiltonian for spectator fields

Scalar field ϕ in FRW background

Fourier mode decomposition:

$$\widehat{\phi}(\eta, \boldsymbol{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\widehat{a}_{\mathbf{k}} \chi_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \widehat{a}^\dagger_{\mathbf{k}} \chi^*_{\mathbf{k}}(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

Mode functions satisfy wave equation:

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$$

but with a time-dependent dispersion relation

$$\omega_k^2(\eta) = k^2 + m_{\text{eff}}^2(\eta) \qquad m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right]$$

Vacuum state $|0\rangle$ defined as state where $\widehat{a}_{\mathbf{k}}|0\rangle = 0$

Scalar field ϕ in FRW background

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta) d\eta}$$

Assume start with only + frequency term: $\chi_k(\eta) = \frac{1}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta}$

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2 \left[1 + 3\left(\frac{\partial_\eta \omega_k}{2\omega_k^2}\right)^2 - \frac{\partial_\eta^2 \omega_k}{2\omega_k^3}\right] \chi_k(\eta) = 0$$

Mixing of + and – frequency terms depends on ”Adiabaticity parameter” A_k:

$$A_k \equiv \frac{\partial_\eta \omega_k}{\omega_k^2}$$

$A_k \ll 1,$ + frequency solution remains good
$A_k \gg 1,$ + and – frequency terms mix

Scalar field ϕ in FRW background

Mode functions satisfy wave equation:

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$$

but with a time-dependent dispersion relation

$$\omega_k^2(\eta) = k^2 + m_{\text{eff}}^2(\eta)$$

Vacuum state $|0\rangle$ defined as state where $\widehat{a}_{\mathbf{k}}|0\rangle = 0$

+ & – frequency modes

$$\chi_\pm^{(\text{early})}(\eta) \xrightarrow{\eta\to-\infty} \frac{1}{\sqrt{2k}} e^{\mp ik\eta}$$

$$\chi_\pm^{(\text{late})}(\eta) \xrightarrow{\eta\to+\infty} \frac{1}{\sqrt{2am}} e^{\mp i\int^\eta d\eta' am}$$

leads to mode mixing

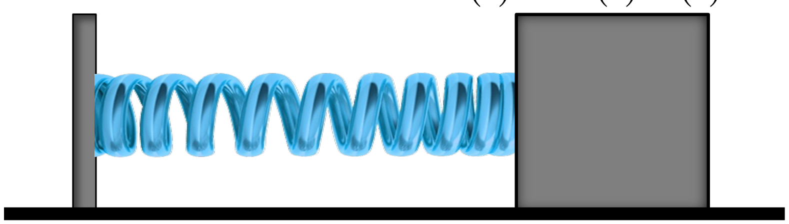
$$\begin{pmatrix} \chi_{k+}^{(\text{late})}(\eta) \\ \chi_{k-}^{(\text{late})}(\eta) \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta_k^* & \alpha_k^* \end{pmatrix} \begin{pmatrix} \chi_{k+}^{(\text{early})}(\eta) \\ \chi_{k-}^{(\text{early})}(\eta) \end{pmatrix}$$

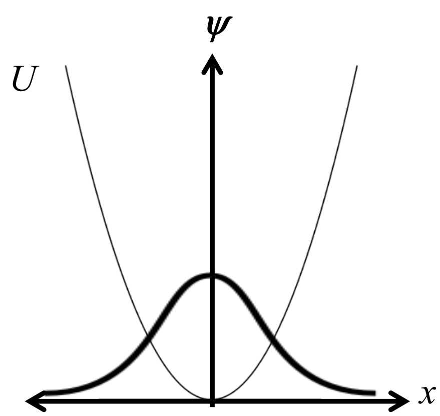
time-dependent Hamiltonian ⇒ mode mixing
⇒ – frequency modes from + frequency modes

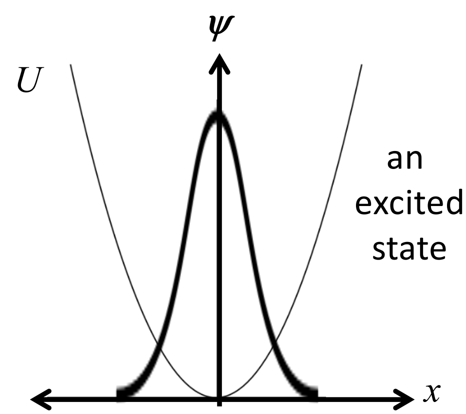
Schrödinger's Alarming Phenomenon

Expansion of the universe causes explicit time dependence in action for "spectator" fields.
Initial ~ de Sitter (early-time) vacuum may not evolve to final ~ Minkowski (late-time) vacuum, but to an excited state populated by particles.

$$\ddot{x}(t) + \omega^2(t)\; x(t) = 0$$







Scalar field ϕ in FRW background

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta) d\eta}$$

If start with only outgoing waves, $\beta_k(\eta) = 0$,
will generate incoming waves, $\beta_k(\eta) \neq 0$.

a_e

Comoving number density of particles at late time is

$$n\, a^3 = \frac{1}{(2\pi)^3} \int d^3k\, |\beta_k(\eta)|^2 = \frac{1}{2\pi^2} \int \frac{dk}{k}\, k^3 |\beta_k(\eta)|^2$$

$$n_k \equiv \frac{1}{2\pi^2} k^3 |\beta_k(\eta)|^2$$

Spectral density

Scalar field φ in FRW background

$$m_{\rm eff}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right]$$

Abrupt changes in $a(\eta)$ leads to nonadiabatic changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to of particle creation in the expanding universe.

Nonadiabaticity proportional to $\frac{\partial_\eta \omega_k}{\omega_k^2}$

Adiabatic deep in quasi-de Sitter phase
Adiabatic at late time after inflation

Nonadiabatic:

- $\xi = 0$:
  - $k/a \ll H$ when mode exits horizon during inflation. Super-Hubble.
  - $k/a \gg H$ at end of inflation. Sub-Hubble radius.
- $\xi = 1/6$: at end of inflation. Sub-Hubble radius.

Standard Inflationary Picture

Quasi-de Sitter Phase driven by vacuum energy of inflaton displaced from potential minimum, expansion rate H_e roughly constant

Matter-dominated phase due to inflaton oscillations about minimum of potential

Inflaton decays and leads to radiation-dominated phase characterized by a reheat temperature $T_{\rm RH}$

Conformal time $-\infty < \eta < 0$ de Sitter and $0 < \eta < \infty$ matter-dominated → radiation-dominated

<u>**Analytic**</u> inflation matter-dominated (a_e , H_e are values at end of inflation)

Inflation:

$$a = \frac{a_e}{1-\eta}$$

$$H = H_e$$

Matter-dominated:

$$a = a_e(1+\tfrac{1}{2}\eta)^2$$

$$H = \frac{H_e}{(1+\frac{1}{2}\eta)^3}$$

Standard Inflationary Picture

We can normalize the scale factor a such that $a_e H_e = 1$

A momentum mode of comoving value k. It has *physical* wavenumber k/a

At the end of inflation it is k/a_e .

Ratio of the physical wavenumber to H_e is $k/a_e H_e$, or simply k.

$k < 1$ corresponds to modes that are super-Hubble radius at the end of inflation.

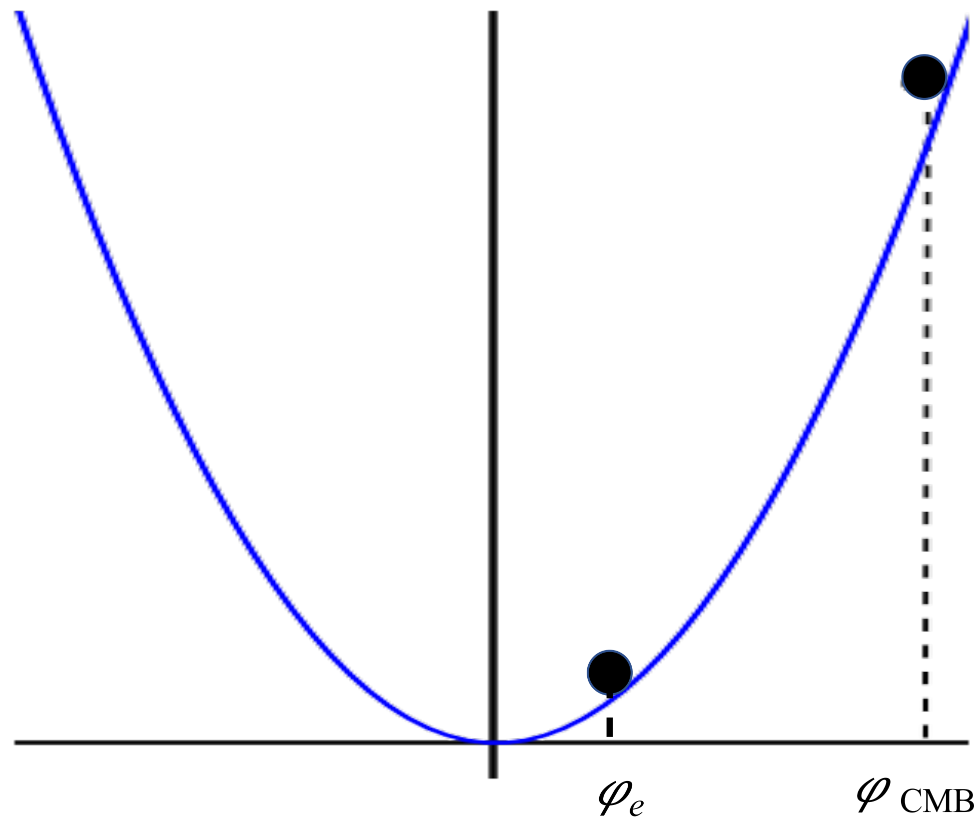
$k > 1$ corresponds to modes that are sub-Hubble radius at the end of inflation.

$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

But there is a "simple" inflationary model: single-field with quadratic inflaton potential:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2$$

Simple model ruled out by CMB measurements. *But* CMB measurements probe inflaton potential 60 or so e-folds before the end of inflation. For our studies we will often be interested in inflaton potential near the end or after inflation ends when φ is close to the minimum of its potential and quadratic description may be a good approximation.



$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

But there is a “simple” inflationary model:
single-field with quadratic inflaton potential:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2$$

EOM: $\ddot{\varphi} + 3H\dot{\varphi} + \partial_\varphi V(\varphi) = 0$

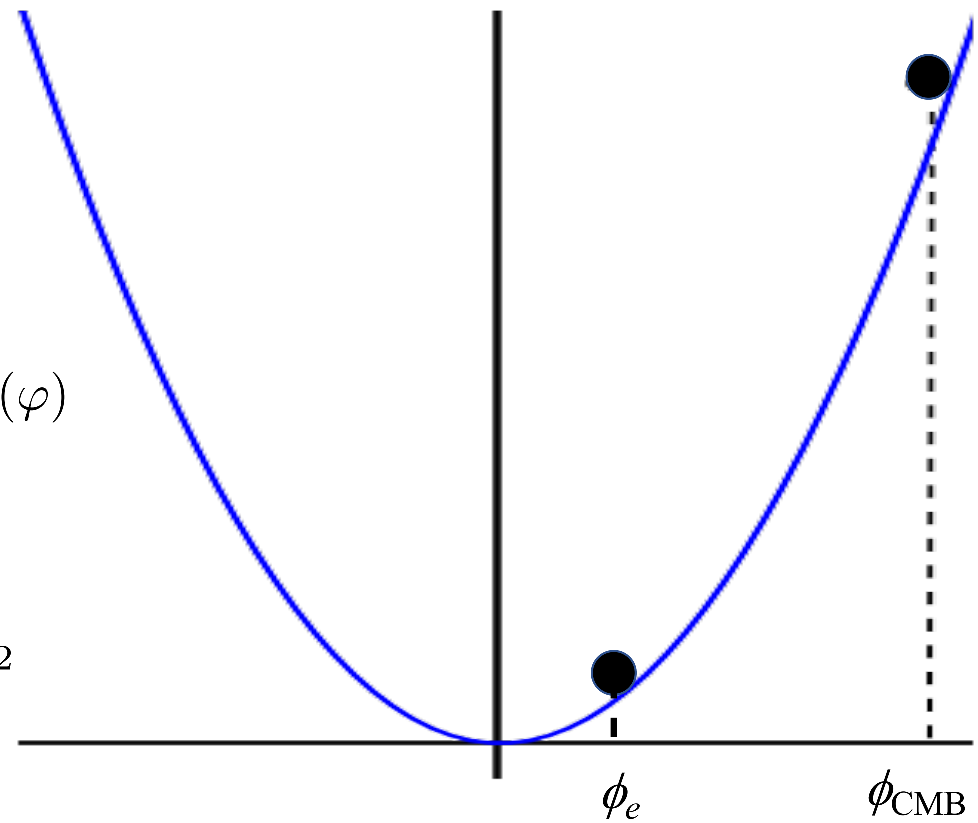
Slow roll during inflation ($\ddot{\varphi} = 0$): $3H\dot{\varphi} = -\partial_\varphi V(\varphi)$

Inflation is accelerated expansion: $\ddot{a} \propto -(\rho + 3p)$

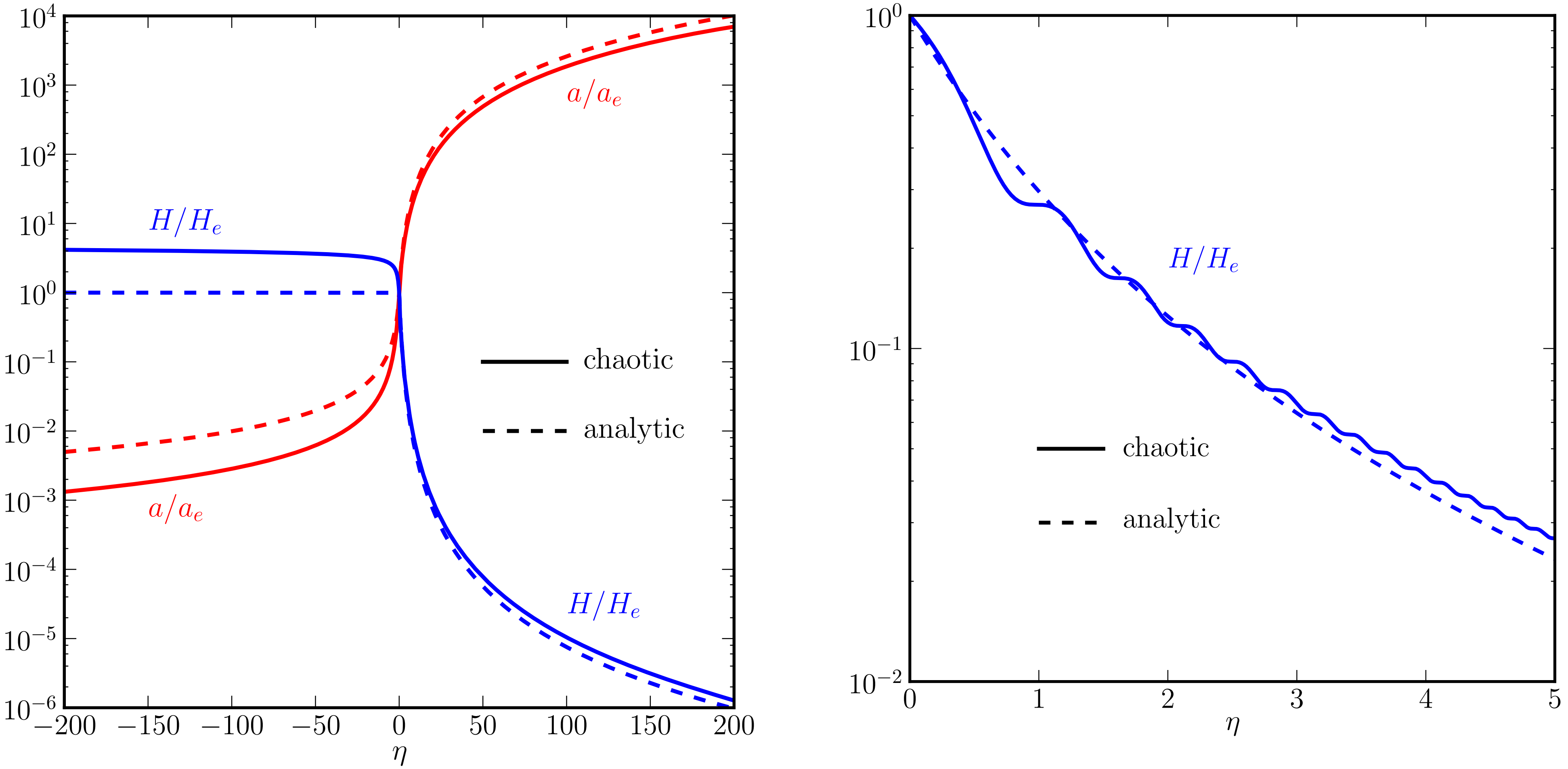
$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$

$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$

$\ddot{a} \propto -(\rho + 3p) \propto V(\varphi) - \dot{\varphi}^2$

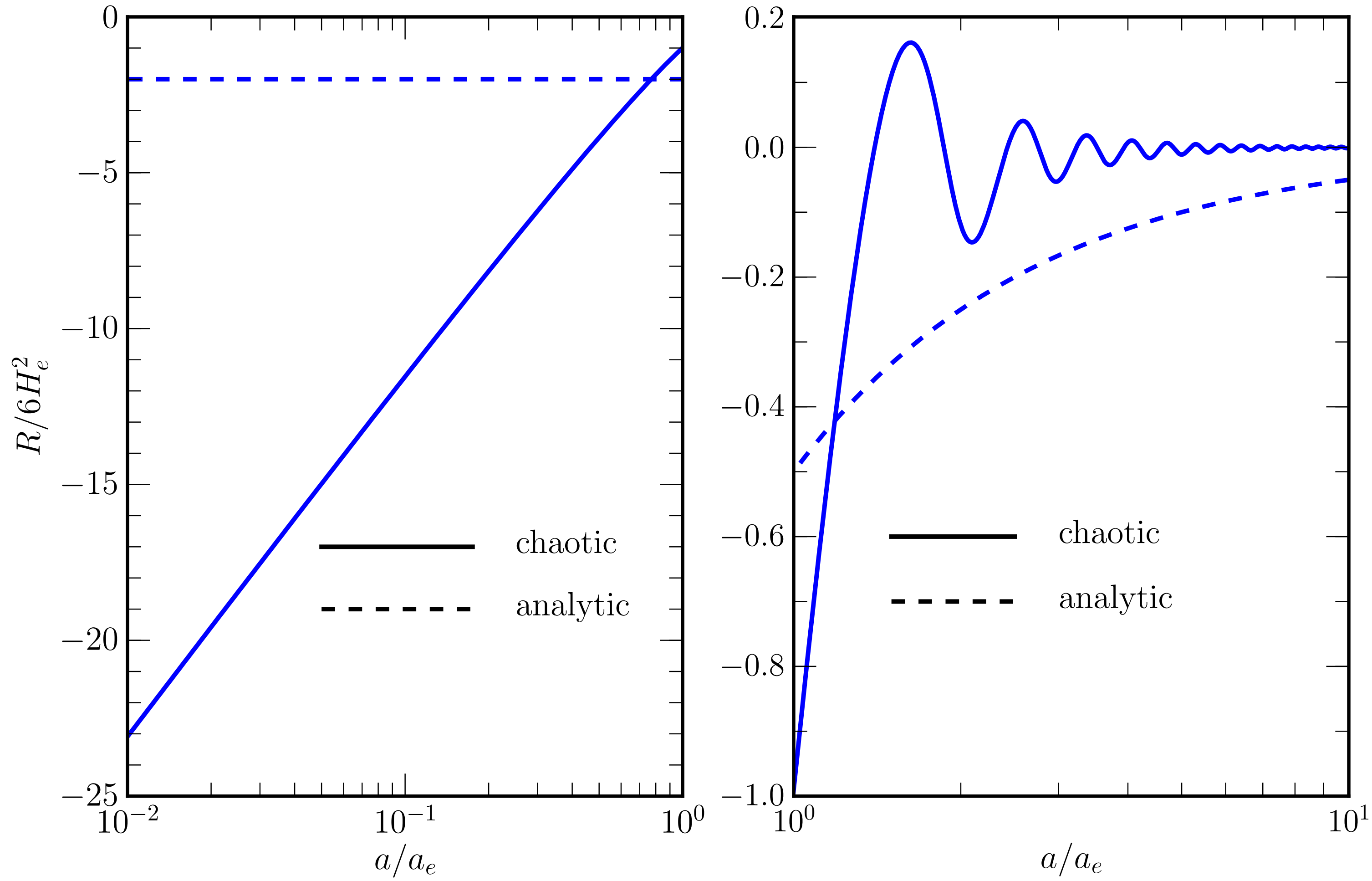


Chaotic Model Analytic vs. Numerical



Ricci Scalar

$$R = -M_{\rm Pl}^{-2}(\rho - 3p) = -6\left[\frac{\ddot{a}}{a} + H^2\right] = -6\frac{a''}{a^3}$$



Adiabticity Parameter A_k

Mixing of + and – frequency terms depends on "Adiabaticity parameter" A_k :

$$A_k \equiv \frac{\partial_\eta \omega_k}{\omega_k^2}$$

$A_k \ll 1,$ + frequency solution remains good

$A_k \gg 1,$ + and – frequency terms mix

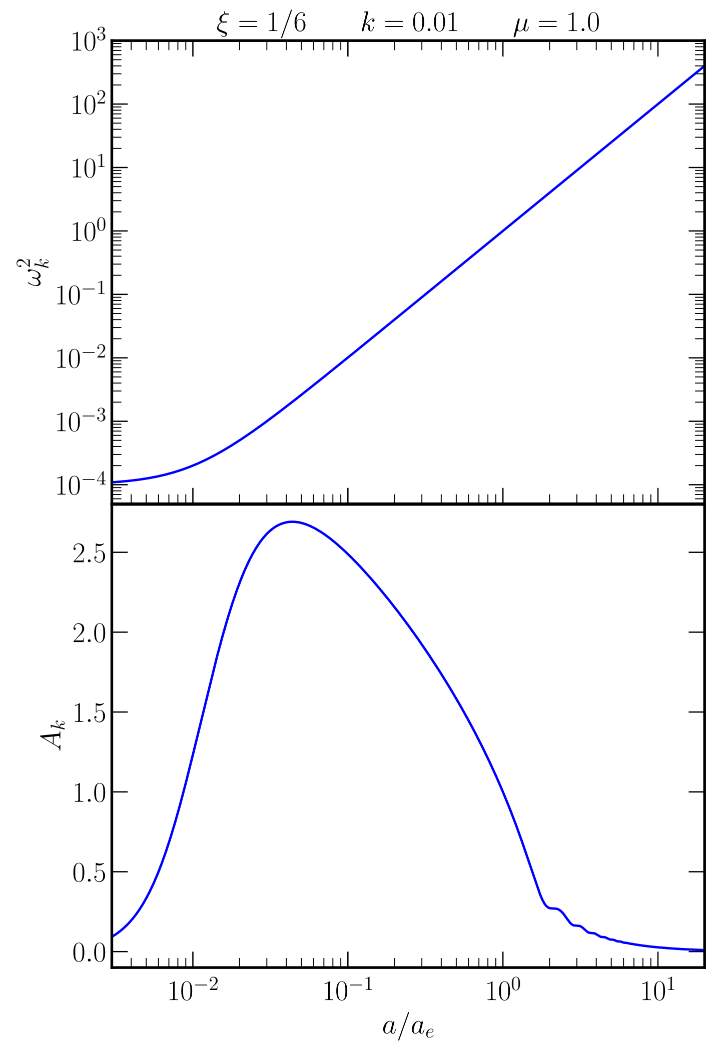
$$\omega_k^2 = k^2 + a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right]$$

Define some dimensionless parameters

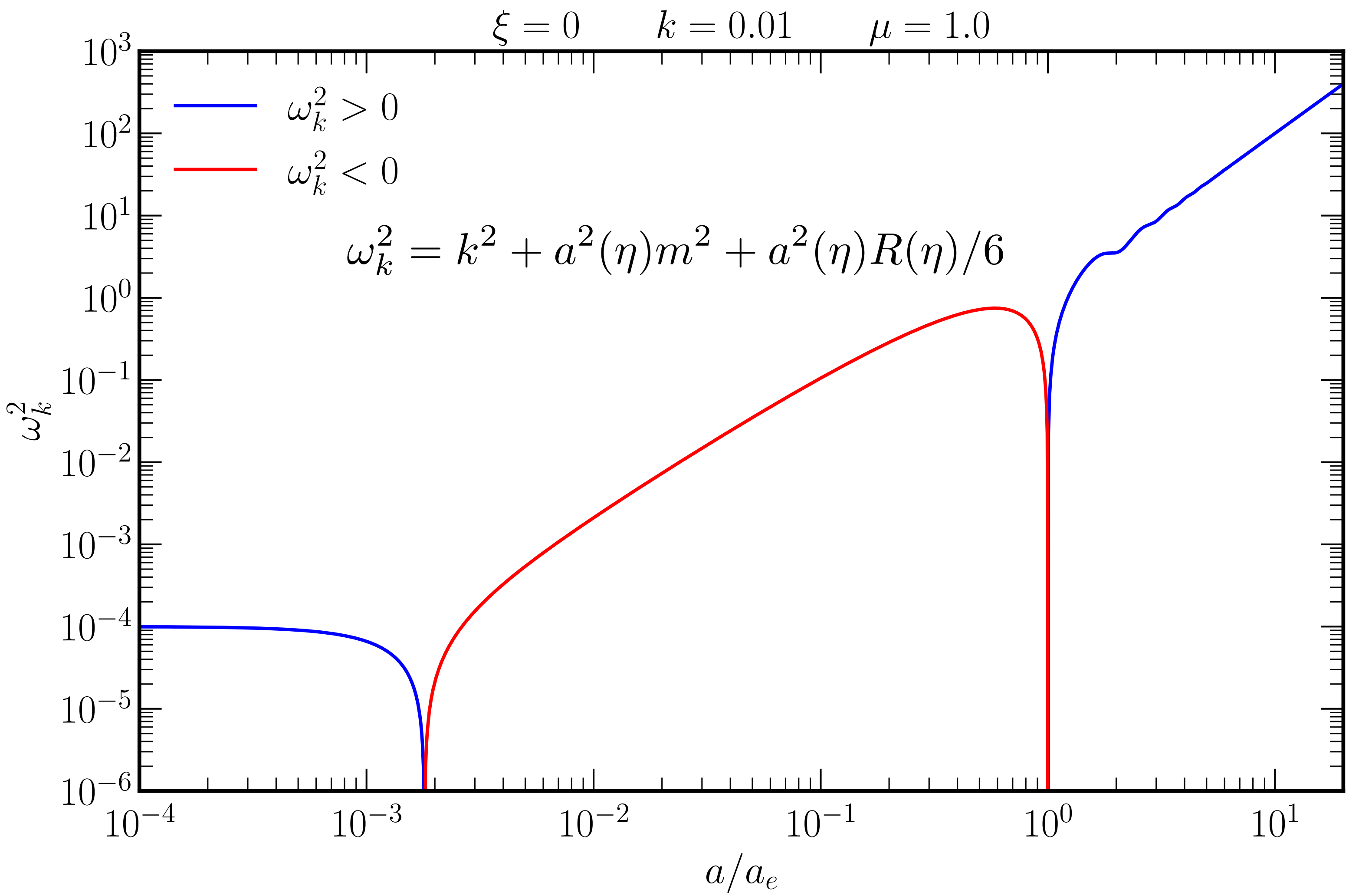
$$\alpha \equiv a/a_e$$
$$\mu \equiv m/H_e$$
$$h \equiv H/H_e$$

$$A_k = \frac{\alpha^3\mu^2 h + \alpha^3 h(R/H_e^2)(1/6-\xi) - \frac{1}{2}\alpha^2(R'/H_e^2)(1/6-\xi)}{[k^2 + \alpha^2\mu^2 + \alpha^2(R/H_e^2)(1/6-\xi)]^{3/2}}$$

Adiabaticity Parameter A_k



Adiabaticity Parameter A_k



Conformal and Minimal Couplings Very Different

Mode equation:

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$$

$\omega_k^2 < 0$ Possible for minimal coupling—expect growth

Why should ξ be constant? There should be an RG flow for ξ.

Might set ξ to 0 or $1/6$ at some scale (say $M_{\rm Pl}$?) but at other scales there should be log corrections.

Energy Density & Number Density in Terms of Mode Functions

$\rho = T^0_{\ 0}$ expressed in conformal time and in terms of field variable $\chi = a\phi$

$$2a^4\rho = (\partial_\eta\chi)^2 + (\nabla\chi)^2 + a^2m^2\chi^2 - 4\xi\left[\chi\nabla^2\chi + (\nabla\chi)^2\right]$$
$$+ (1-6\xi)\left\{a^2H^2\chi^2 - aH\left[\chi\partial_\eta\chi + (\partial_\eta\chi)\,\chi\right]\right\}$$

Even though the kinetic term in the action is canonically normalized for the comoving field χ the energy density still has a mixed term.

Using the mode expansion and the commutation relations for $\widehat{a}$ and $\widehat{a}^\dagger$, we can express the energy density $\rho = \langle 0|\widehat{T}^0{}_0|0\rangle$ in terms of the mode functions. For instance, χ^2 term:

$$\left\langle 0\left|\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\left[\hat{a}_k\,\chi_k(\eta)\,e^{ik\cdot x} + \hat{a}_k^\dagger\,\chi_k^*(\eta)\,e^{-ik\cdot x}\right]\left[\hat{a}_q\,\chi_q(\eta)\,e^{iq\cdot x} + \hat{a}_q^\dagger\,\chi_q^*(\eta)\,e^{-iq\cdot x}\right]\right|0\right\rangle$$

Energy Density & Number Density in Terms of Mode Functions

$$\left\langle 0 \left| \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \left[\hat{a}_k \, \chi_k(\eta) \, e^{ik\cdot x} + \hat{a}_k^\dagger \, \chi_k^*(\eta) \, e^{-ik\cdot x} \right] \left[\hat{a}_q \, \chi_q(\eta) \, e^{iq\cdot x} + \hat{a}_q^\dagger \, \chi_q^*(\eta) \, e^{-iq\cdot x} \right] \right| 0 \right\rangle$$

Has terms proportional to $\langle 0 \,| \hat{a}_k \hat{a}_q |\, 0 \rangle$, $\left\langle 0 \left| \hat{a}_k^\dagger \hat{a}_q \right| 0 \right\rangle$, $\left\langle 0 \left| \hat{a}_k^\dagger \hat{a}_q^\dagger \right| 0 \right\rangle$, $\langle 0 \,| \hat{a}_k \hat{a}_q^\dagger |\, 0 \rangle$

Only last term nonvanishing. Normal order: $\left\langle 0 \left| \hat{a}_k \hat{a}_q^\dagger \right| 0 \right\rangle \to (2\pi)^3 \delta^3(k-q)$ with result

$$\int \frac{d^3k}{(2\pi)^3} |\chi_k|^2$$

Following similar procedure for the rest of the terms yields

$$2a^4\rho = \int \frac{d^3k}{(2\pi)^3} \left\{ |\partial_\eta \chi_k|^2 + \omega_k^2 \, |\chi_k|^2 + (1-6\xi) \left[\left(a^2H^2 - \frac{1}{6} a^2 R \right) |\chi_k|^2 - 2aH \; \mathrm{Re} \left[\chi_k \, \partial_\eta \chi_k^* \right] \right] \right\}$$

Energy Density & Number Density in Terms of Mode Functions

$$2a^4\rho = \int \frac{d^3k}{(2\pi)^3} \left\{ |\partial_\eta \chi_k|^2 + \omega_k^2 |\chi_k|^2 + (1-6\xi)\left[\left(a^2H^2 - \frac{1}{6}a^2R\right)|\chi_k|^2 - 2aH\ \mathrm{Re}\left[\chi_k\,\partial_\eta\chi_k^*\right]\right]\right\}$$

Infinite before renormalization (duh, it's field theory). Introduce UV cutoff Λ_{UV}

Leads to time-dependent and time-independent counterterms $\Lambda_{\mathrm{UV}}^4, H^2\Lambda_{\mathrm{UV}}^2,\ \&\ H^4 \log \Lambda_{\mathrm{UV}}$

Renormalize divergences by requiring $\left\langle 0|\hat{T}_{00}|0\right\rangle = 0$ in Minkowski vacuum.

Since we will be interested in the asymptotic value of ρ where H and R vanish

$$\lim_{\eta\to\infty} a^4\rho^{\mathrm{ren}} = \int \frac{dk}{k}\ \frac{k^3}{2\pi^2}\ \lim_{\eta\to\infty}\left(\tfrac{1}{2}|\partial_\eta\chi_k|^2 + \tfrac{1}{2}\omega_k^2|\chi_k|^2 - \tfrac{1}{2}\omega_k\right)$$

Energy Density & Number Density in Terms of Mode Functions

$$\lim_{\eta\to\infty} a^4 \rho^{\text{ren}} = \int \frac{dk}{k}\ \frac{k^3}{2\pi^2}\ \lim_{\eta\to\infty} \left(\tfrac{1}{2} \left|\partial_\eta \chi_k\right|^2 + \tfrac{1}{2}\omega_k^2 \left|\chi_k\right|^2 - \tfrac{1}{2}\omega_k \right)$$

At late time in the NR limit $\omega_k \to am$ and $\rho = m\, n$

$$\lim_{\eta\to\infty} a^3 n = \int \frac{dk}{k}\ n_k = \int \frac{dk}{k}\ \frac{k^3}{2\pi^2}\ \left[\frac{1}{am} \lim_{\eta\to\infty} \left(\tfrac{1}{2} \left|\partial_\eta \chi_k\right|^2 + \tfrac{1}{2}\omega_k^2 \left|\chi_k\right|^2 \right) - \tfrac{1}{2} \right]$$

In terms of Bogoliubov coefficients:

$$\lim_{a\to\infty} n\, a^3 = \int \frac{dk}{k}\ \lim_{a\to\infty} n_k = \int \frac{dk}{k}\ \frac{k^3}{2\pi^2}\ \lim_{\eta\to\infty}\ \frac{\omega_k}{am}\ |\beta_k|^2 = \int \frac{dk}{k}\ \frac{k^3}{2\pi^2}\ \lim_{\eta\to\infty}\ |\beta_k|^2$$

Phew!

Finally, Calculation of Relic Abundance

Three (equivalent) ways to calculate relic abundance:

1. Integrate EOM: $\partial_\eta^2 \chi_k = -\omega_k^2 \chi_k$ (2nd-order equation for complex χ_k)

2. Integrate two 1st-order equations for complex α_k and β_k

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\Phi_k(\eta)/2} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{i\Phi_k(\eta)/2}$$

$$\partial_\eta \chi_k = -i\omega_k \left(\frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\Phi_k(\eta)/2} - \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{i\Phi_k(\eta)/2} \right)$$

$$\Phi_k(\eta) \equiv 2\int_{\eta_i}^{\eta} d\eta'\ \omega_k(\eta') \qquad \text{Phase}$$

$$\partial_\eta \alpha_k = \tfrac{1}{2} A_k\, \omega_k\, \beta_k\, e^{i\Phi_k}$$

$$\partial_\eta \beta_k = \tfrac{1}{2} A_k\, \omega_k\, \alpha_k\, e^{-i\Phi_k}$$

(for imaginary ω_k analytic continuation cumbersome)

Finally, Calculation of Relic Abundance

Three (equivalent) ways to calculate relic abundance:

3. Define a_k and b_k to avoid calculating Φ_k (two 1st-order equations for complex a_k and b_k):

$$\alpha_k(\eta) = a_k(\eta)\, e^{i\Phi_k(\eta)/2} \qquad \text{and} \qquad \beta_k(\eta) = b_k(\eta)\, e^{-i\Phi_k(\eta)/2}$$

$$\partial_\eta a_k(\eta) = -i\omega_k(\eta)\, a_k(\eta) + \frac{1}{2} A_k(\eta)\, \omega_k(\eta) b_k(\eta)$$
$$\partial_\eta b_k(\eta) = +i\omega_k(\eta)\, b_k(\eta) + \frac{1}{2} A_k(\eta)\, \omega_k(\eta) a_k(\eta)$$

Finally, Calculation of Relic Abundance

Initial Conditions: as $a \longrightarrow 0$, frequency $\omega_k^2 = k^2 + a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right] \longrightarrow k^2$ motivates "Bunch-Davies" initial conditions for χ_k and $\partial_\eta \chi_k$:

$$\chi_k(\eta) \xrightarrow{\eta \to -\infty} \chi_k^{\mathrm{BD}}(\eta) \equiv \frac{1}{\sqrt{2k}}\, e^{-ik\eta}$$

$$\partial_\eta \chi_k(\eta) \xrightarrow{\eta \to -\infty} -i\sqrt{\frac{k}{2}} e^{-ik\eta}$$

as $\eta \rightarrow \infty$ the physical momentum is much larger than H and the field should not "feel" the curvature of spacetime.

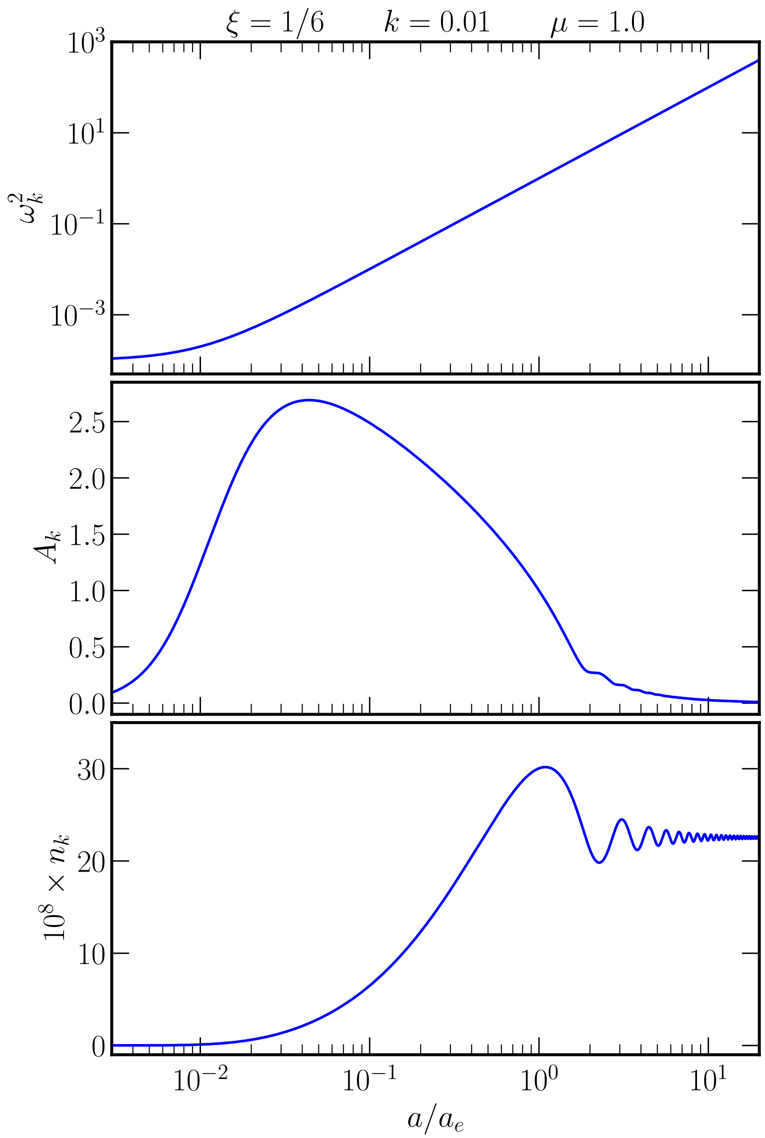
NO APOLOGIES!

Finally, Calculation of Relic Abundance

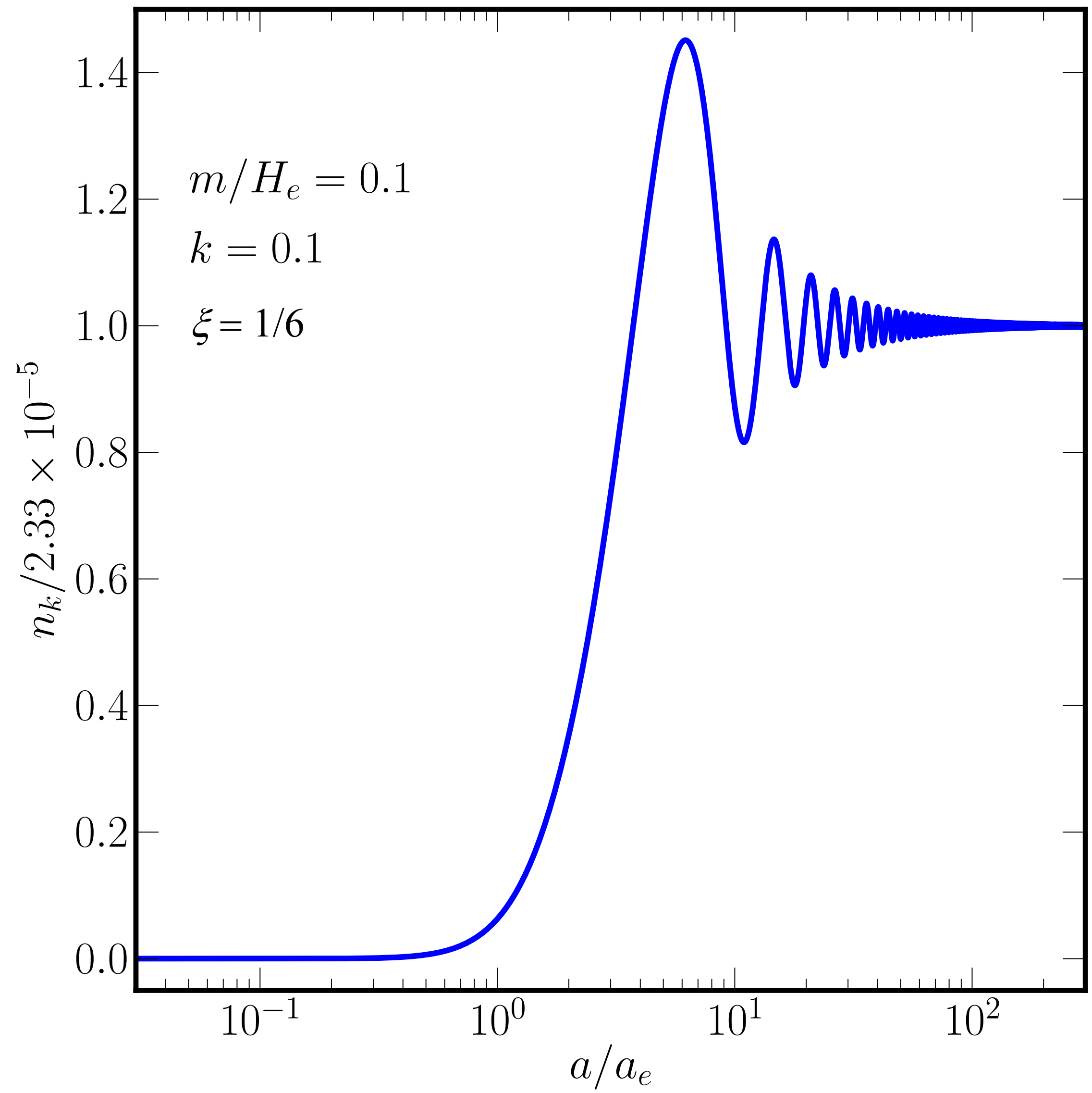
So just integrate EOM (either using 1, 2, or 3) with BD initial conditions and extract $|\beta_k|^2$ at late time and calculate spectral density and comoving number density.

$$n_k = \frac{k^3}{2\pi^2} \left[\frac{1}{am} \left(\frac{1}{2} \left| \partial_\eta \chi_k \right|^2 + \frac{1}{2} \omega_k^2 \left| \chi_k \right|^2 \right) - \frac{1}{2} \right] = \frac{k^3}{2\pi^2} \left| \beta_k \right|^2$$

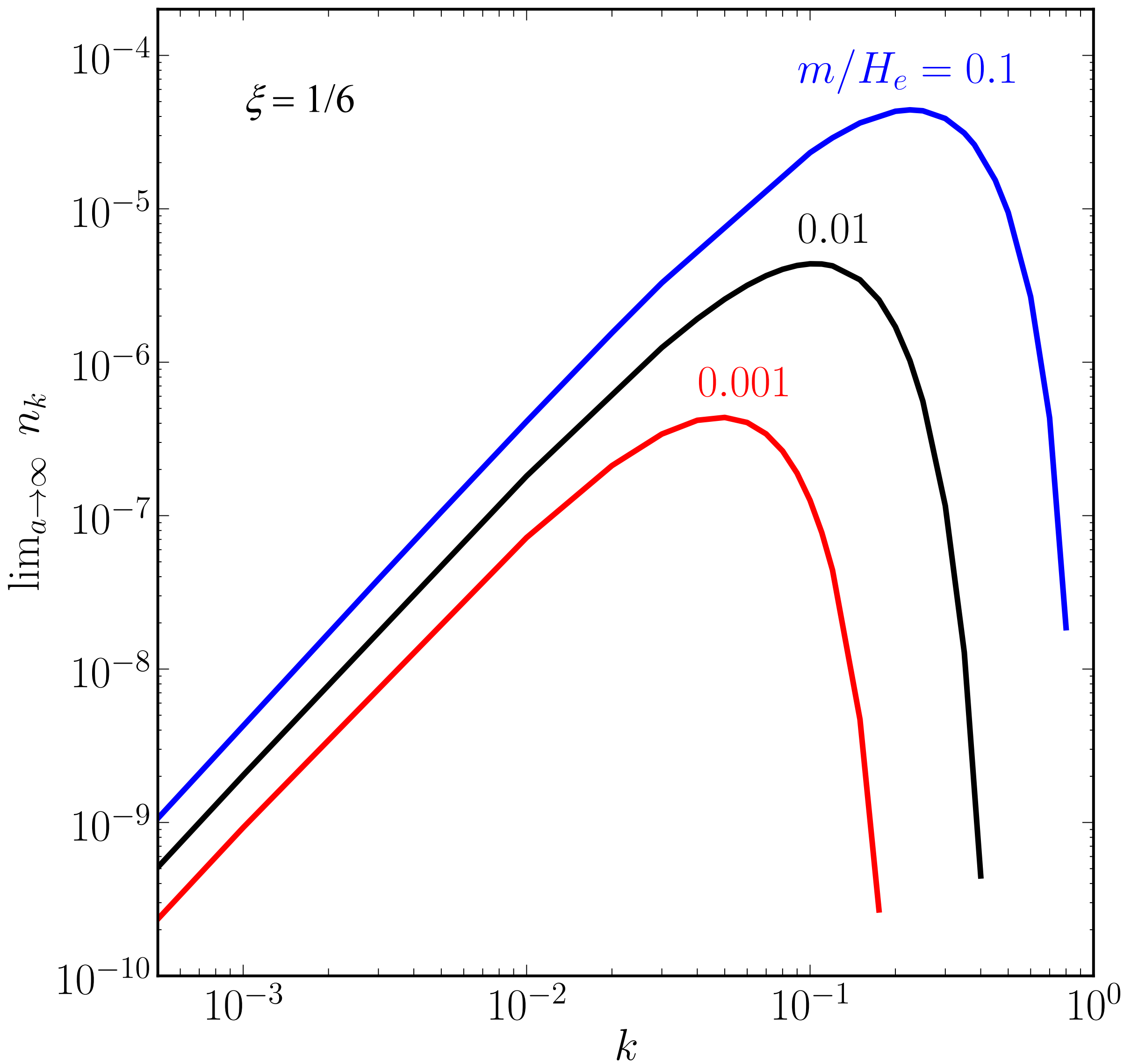
$$n\, a^3 = \int \frac{dk}{k}\, n_k$$



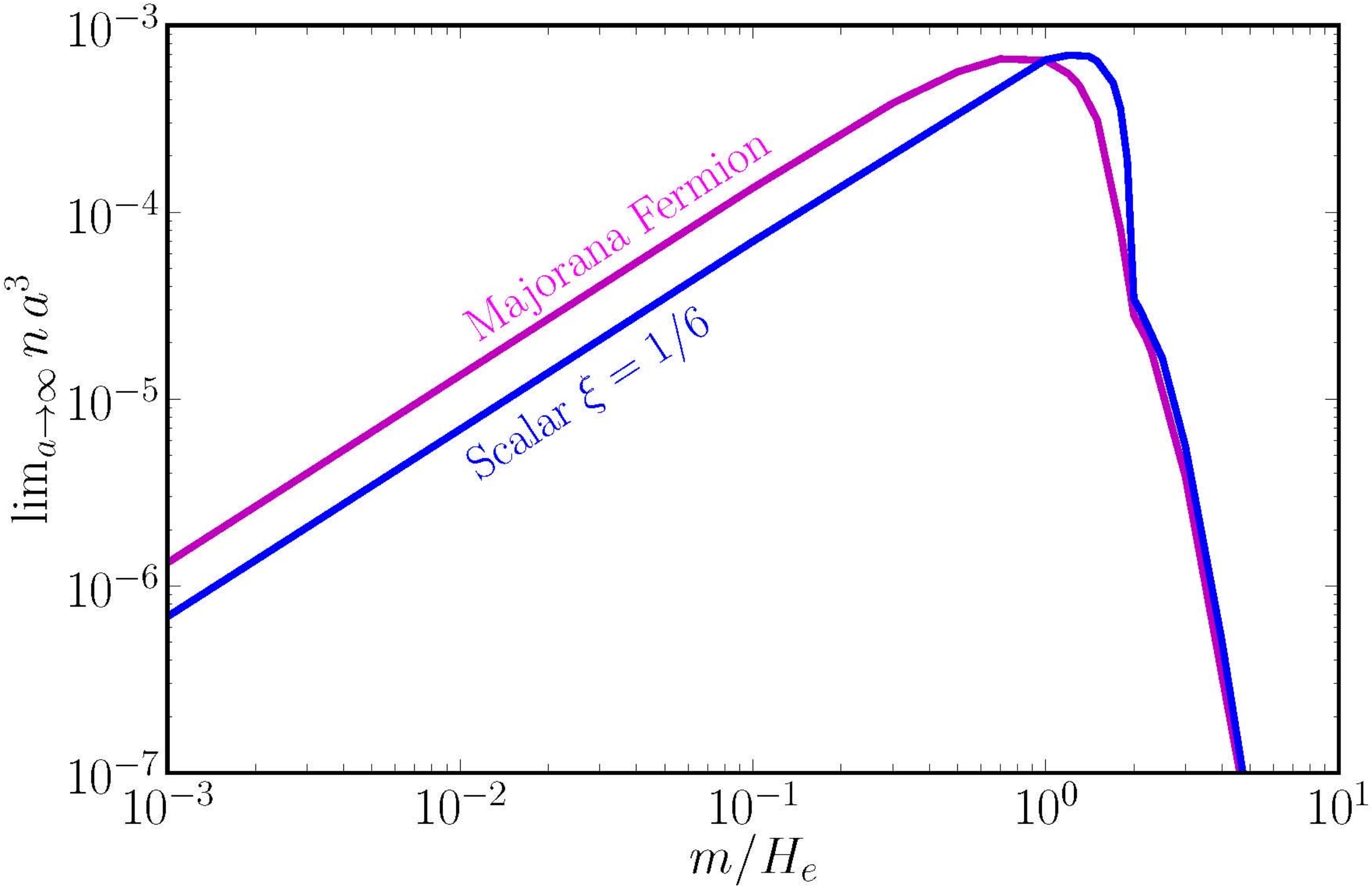
$\xi = 1/6$ $k = 0.01$ $\mu = 1.0$
ω_k^2
A_k
$10^8 \times n_k$
a/a_e



$m/H_e = 0.1$
$k = 0.1$
$\xi = 1/6$
$n_k/2.33 \times 10^{-5}$
a/a_e



Notice spectrum is BLUE, by which I mean spectrum vanishes as $k \longrightarrow 0$



$na^3 \rightarrow 0$ as $m \rightarrow 0$

For conformally-coupled scalar, conformal symmetry only broken by mass term.

Since metric is conformally Minkowski, massless, conformally-coupled scalar field does not feel expansion.

Conversion of na^3 to Ωh^2

After inflation universe dominated by coherent oscillations of inflaton. Energy density decreases as a matter-dominated universe. Eventually inflaton decays, “reheating” the universe to some “reheat” temperature $T_{\rm RH}$, after which the universe evolves as a radiation-dominated universe, eventually becoming matter dominated around $z = 30{,}000$, then dark-energy dominated at a redshift ≈ 1.

All the while na^3 remaining constant.

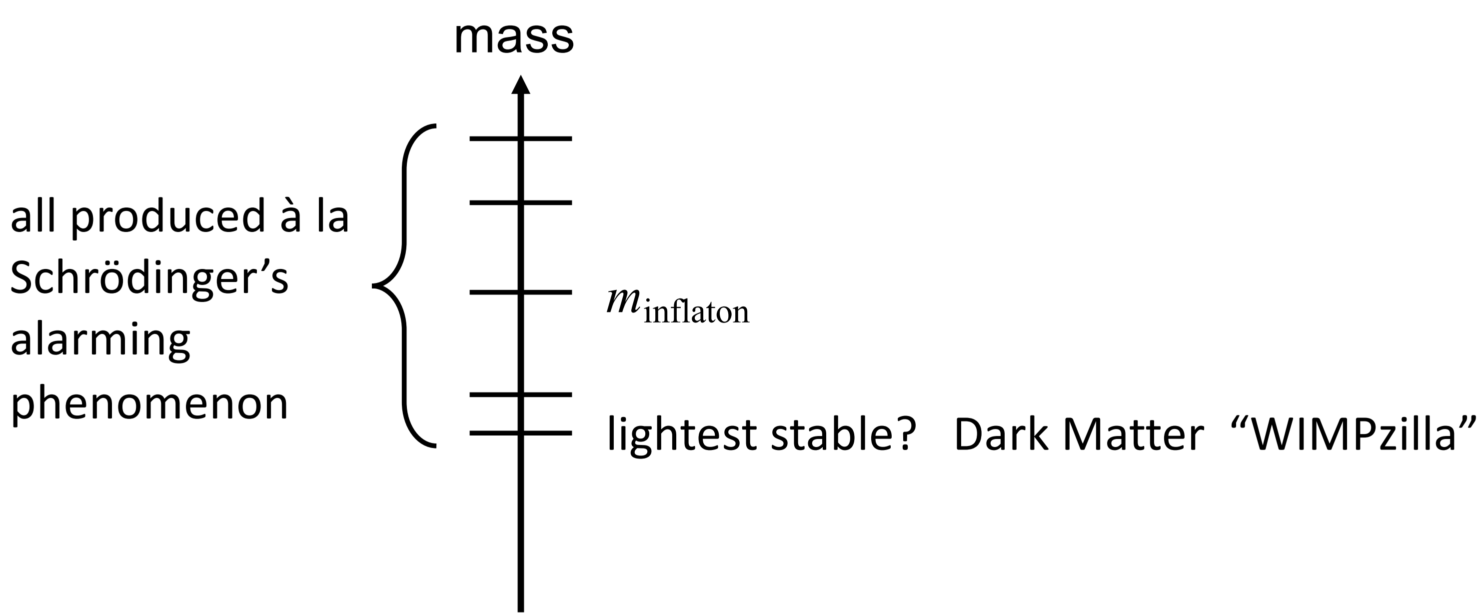
$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\,\mathrm{GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\,\mathrm{GeV}}\right)\frac{\lim_{a\to\infty} n\,a^3}{10^{-5}}$$

We don’t know H_e or $T_{\rm RH}$, but the above values are “representative” choices.

So $na^3 \approx 10^{-5}$ seems desirable.

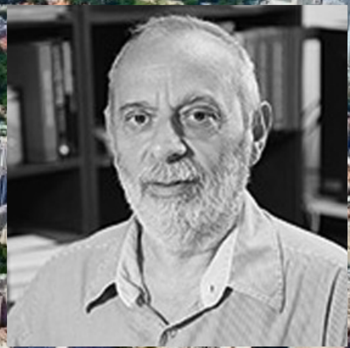
CGPP & Dark Matter

- Inflation indicates a new mass scale
- In most models, $m_{\text{inflaton}} \approx H_{\text{inflation}} \approx 10^{12} - 10^{14}$ GeV?
- $H_{\text{inflation}}$ detectable via primordial gravitational waves in CMB
- (I) expect other particles with mass $\approx m_{\text{inflaton}}$



Gravitational Particle Production and Dark Matter — Four Lectures

1. Dark Matter: Evidence and the Standard WIMP
2. Gravitational Particle Production (Schrödinger's Alarming Phenomenon)
3. GPP of Scalar Fields
4. Beyond Scalar Fields



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CBPF 9/2022