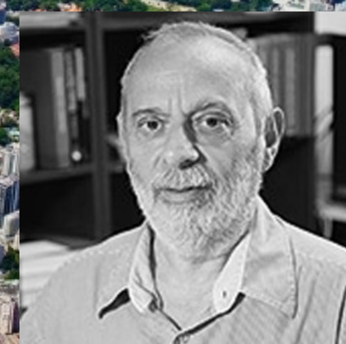
Gravitational Particle Production and Dark Matter — Four Lectures

1. Dark Matter: Evidence and the Standard WIMP
2. Gravitational Particle Production (Schrödinger's Alarming Phenomenon)
3. GPP of Scalar Fields
4. Beyond Scalar Fields



Rocky Kolb, University of Chicago

CBPF 9/2022

For 40 Years, Leading DM Candidate
"Weak"-Scale Cold Thermal Relic

- Mass: GeV – TeV
- "Weak-scale" interaction strength with SM
- No self-interactions
- Produced by "freeze-out" from primordial plasma. COLD dark matter.
- "Detectable" by direct detection, indirect detection, decay products, production at colliders
- Just BSM

But not (convincingly) seen

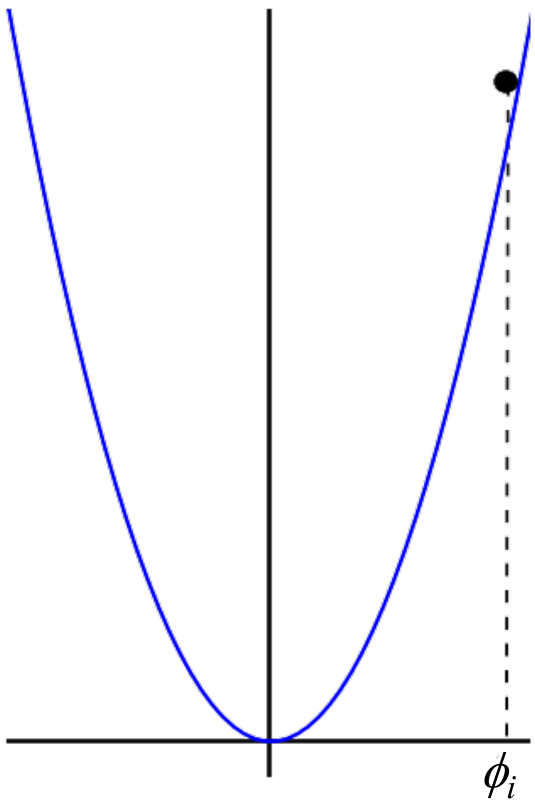
- In Direct detection (but DAMA/LIBRA)
- In Indirect Detection (but galactic-center excess)
- In Decay (but $3.5\ \mathrm{keV}$ γ-ray line)
- In Colliders no BSM signal (but μ_{g-2}, m_W)

What if DM interacts only gravitationally?

- Gravity must play a prole in its cosmological production
- But gravity weak!

Ideas for gravitational particle production

Produce particles through misalignment mechanism



- Scalar field has quantum fluctuations during inflation

$$\Delta\phi = \frac{H}{2\pi}$$

- After inflation field frozen by "Hubble drag" until

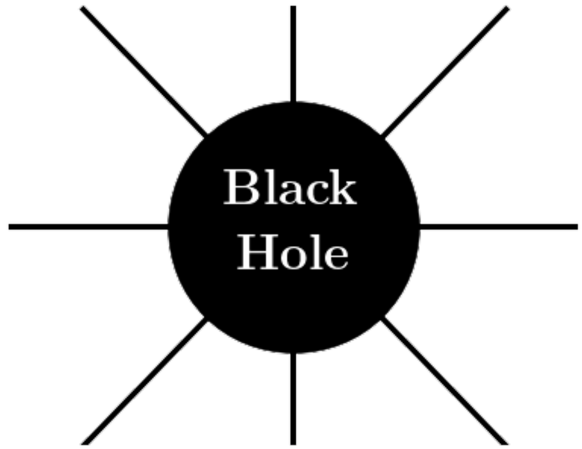
$$H \simeq m_\phi$$

- After which it oscillates with energy density in oscillating field

- cf., axion

Ideas for gravitational particle production

Produce particles via Hawking radiation from primordial black holes
(Hooper, Krnjaic, & McDermott)



$$\frac{\Omega h^2}{0.12} \approx \left(\frac{10^{11}\mathrm{GeV}}{m}\right)\left(\frac{10^{12}\mathrm{GeV}}{T_i}\right)^3\left(\frac{\epsilon_{\mathrm{BH}}}{10^{-16}}\right)$$

- PBHs of current interest (after first LIGO event)
- Seeds for PBHs from inflation
- Assumes DM mass about 10^{11} GeV (WIMPzilla)

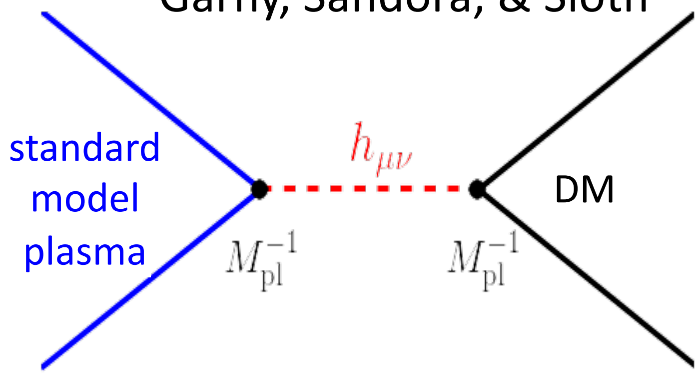
Ideas for gravitational particle production

$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from SM plasma via

graviton exchange

Garny, Sandora, & Sloth



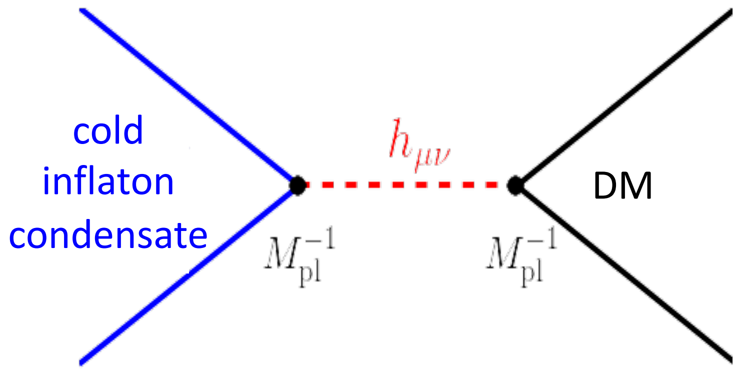
$$\frac{\Omega h^2}{0.12} \approx \left(\frac{\langle\sigma v\rangle}{T^2/M_{\rm Pl}^4}\right)\left(\frac{m}{10^{13}{\rm GeV}}\right)\left(\frac{T_{\rm RH}}{10^{14}{\rm GeV}}\right)^3$$

- Freeze-in
- For DM mass about $10^{13}\,{\rm GeV}$ (WIMPzilla)
- Assumes $m < T_{\rm RH}$

Ideas for gravitational particle production

$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from inflaton field after quasi-de Sitter era via graviton exchange
Ema, Nakayama, Tang; Mambrini & Olive



- Only works for DM mass < inflaton mass
- DM mass for correct Ωh^2 involved function of several parameters
- "Boltzmann" approach not complete treatment (Kaneta, Lee, Oda)

Boltzmann  $\subset$  Schrödinger $+$  Bolgoliubov

Cosmological Gravitational Particle Production (CGPP)

- A space-dependent (e.g., black holes) or time-dependent (e.g., big bang) gravitational field can create particles from vacuum

  Black Holes – Hawking 1974
  Big Bang – Schrödinger 1939

- Interest here on <u>time-dependent</u> gravitational fields, in particular, the big bang

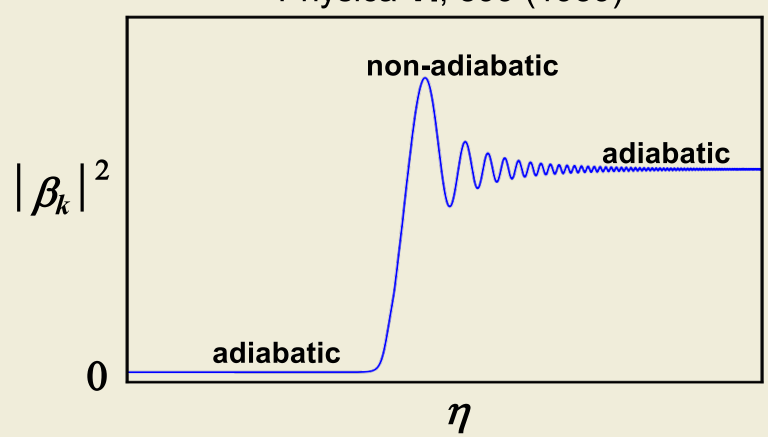
  Inflation: quasi deSitter phase followed by transition to matter-dominated then radiation-dominated phase

- CGPP is an example of QFT in classical gravitation background. Many interesting facets, but …

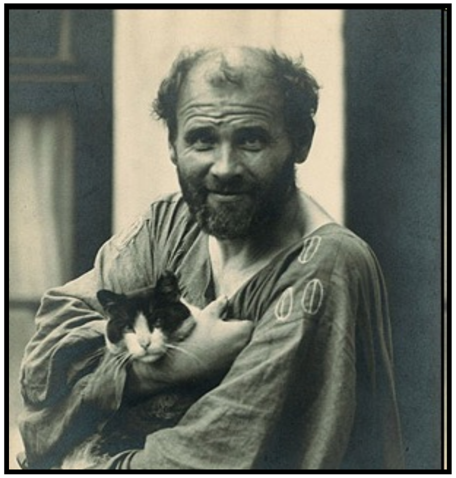
- … my motivation is whether
  - CGPP can be the origin of DARK MATTER (DM), and
  - CGPP can result in cosmological constraints on particle properties

The Proper Vibrations of the Expanding Universe

Physica **VI**, 899 (1939)

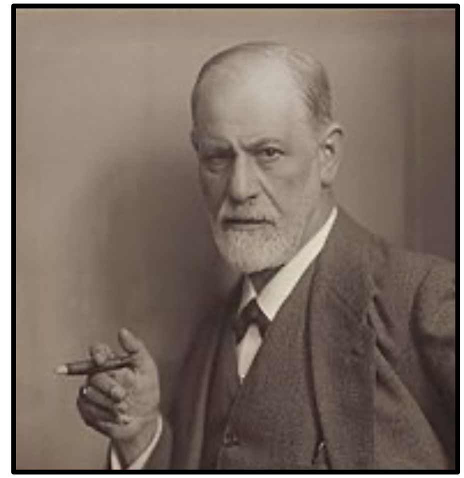


Erwin Schrödinger (1887 — 1961)

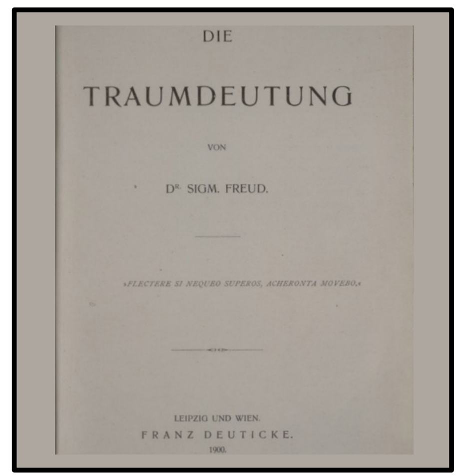


Gustav Klimt 1862–1918

Gustav Mahler 1860–1911



Sigmund Freud 1856–1939



DIE

TRAUMDEUTUNG

VON

Dr. SIGM. FREUD.

»FLECTERE SI NEQUEO SUPEROS, ACHERONTA MOVEBO.«

LEIPZIG UND WIEN.

FRANZ DEUTICKE.

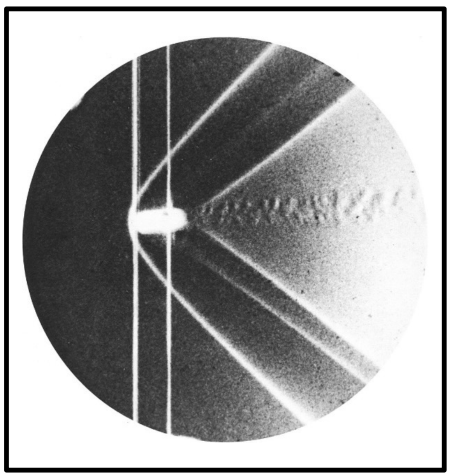
1900.

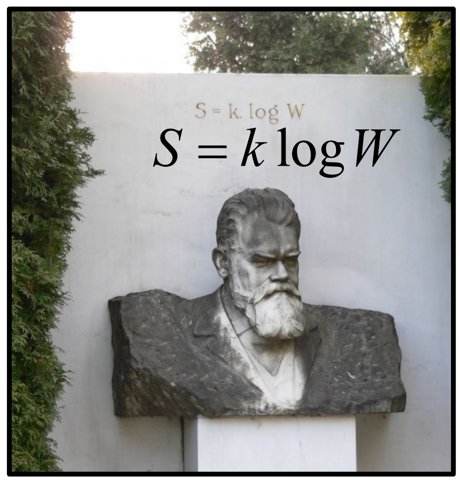
Erwin Schrödinger (1887 — 1961)

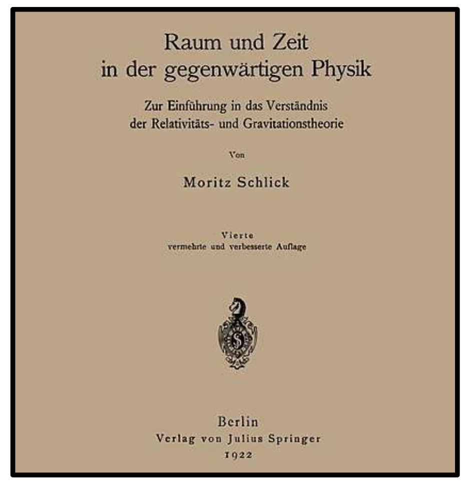
Ernst Mach 1838–1916

Ludwig Boltzmann 1844–1906

Moritz Schlitz 1882–1936







Raum und Zeit
in der gegenwärtigen Physik

Zur Einführung in das Verständnis
der Relativitäts- und Gravitationstheorie

Von

Moritz Schlick

Vierte
vermehrte und verbesserte Auflage

Berlin
Verlag von Julius Springer
1922

Erwin Schrödinger (1887 — 1961)

Erwin Schrödinger 1887–1961

Schrödinger Wave Equation

$$i\hbar \frac{\partial}{\partial t}\Psi(r,t) =$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(r,t) + V(r)\Psi(r,t)$$

Erwin Schrödinger (1887 — 1961)

Schoolboy ca. 1900

With spouse Annemarie (Anny) Bertel
Wedding day 1920, and in 1956

Max Born on Schrödinger's “private life”

His private life seemed strange to bourgeois people like ourselves. But all this does not matter. He was a most lovable person, independent, amusing, temperamental, kind and generous, and he had a most perfect and efficient brain.

Schrödinger's Alarming Times

1926: "Quantisierung als Eigenwertproblem," *Annalen der Physik* **384**, 273

1927: Schrödinger visited U.S.

Found noise and dirt of New York "shattering"

Found Chicago worse, feared "bandits who spring with loaded guns from speeding autos." (Anny liked Chicago.)

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http://www.shardcore.org/

Schrödinger – Fermi

http://www.theflorentine.net/

Dublin
February 10, 1951

Dear Fermi,

….. I beg you to help me remove once and for all, a remorse that I cannot help associating with my memory of you at our last meeting, namely that I still owe you Lire 400 val. Sept 1938. To re-calculate this sum to date, now that all money-value has gone down is very difficult, but I think something like 200 Swedish Crowns would be a modest estimate for re-payment. If you agree and if you still have an account at Stockholm, this would be very simple. If the later is not the case, please indicate me your bankers' account at Chicago, and I hope to manage even so.
…..

Yours very sincerely,
E. Schrödinger

http://www.shardcore.org/

Schrödinger – Fermi

http://www.theflorentine.net/

Chicago
February 27, 1951

Dear Shrodinger [sic],

….. As to the old debt that you mention, I believe that you are estimating the value of 400 lire too high. At that time the lire was worth about one twentieth of one dollar and it seems therefore a $20.00 settlement would be correct. I no longer have an account in Sweden. My bank here in Chicago is the University National Bank, 1354 East 55th Street, Chicago 15. Please however, be sure if there are any difficulties whatsoever about transferring this amount not to worry about it because it is certainly not worth it.

…..

Yours very sincerely,
Enrico Fermi

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Schrödinger the Cosmologist

No. 3648, Sept. 30, 1939 NATURE 593

LETTERS TO THE EDITORS

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 600.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Nature of the Nebular Red-Shift

FROM an investigation (to be published in *Physica*) of the proper vibrations of expanding spherical space, it follows that—in extremely good approximation—light is propagated with respect to co-moving co-ordinates irrespective of the expansion, except that (*a*) the time-rate of events is slowed down and (*b*) all energy portions decrease, both inversely proportional to the radius of curvature.

The slowing down secures the constancy of the velocity of light and entails the nebular red-shift, which from this point of view takes place *during the passage*. The attempt[1] to decide by observation, whether it is actually due to expansion, rests on two important formulæ, which follow from the new view with great ease. Let l be the linear diameter of a nebula *at the moment of emission* and χ its angular distance from the observer (linear distance divided by the circumference of space), then the angle $d\theta$ between two geodesics of space, pointing at the moment of emission from the observer to the ends of the diameter, is from pure geometry:

$$d\theta = \frac{l}{R \sin \chi}, \quad (1)$$

R being the radius of curvature at the moment of emission. By the theorem quoted above, $d\theta$ is also the observed angular diameter of the nebula (Hubble and Tolman, equation 3).

Again, let the energy emitted by the nebula within an appropriately chosen unit of time be E_0. It will soon assume the shape of a spherical shell of thickness c (say). Let $R_{obs.}$ be the radius of space, when this shell reaches the observer. Its surface at this moment is, by pure geometry, $4\pi R^2_{obs.} \sin^2\chi$. By the theorem quoted above, its thickness then is $c\, R_{obs.}/R$ and its energy is $E_0\, R/R_{obs.}$. Hence its energy density ρ is

$$\rho = \frac{E_0}{4\pi c R^4_{obs.}} \cdot \frac{R^2}{\sin^2\chi}. \quad (2)$$

ρ is a measure of the bolometric luminosity, observed outside the earth's atmosphere (Hubble and Tolman, equation 4).

My purpose in re-stating here these two important formulæ due to Tolman is to make the following remarks. Both l and E_0 refer to the moment of emission, which is different for two nebulæ observed simultaneously. Should l and E_0 exhibit a general dependence on R, then it would no longer be reasonable to regard them as *constants*, when equations (1) and (2) are combined (as they actually are) with the hypothesis of uniform spatial distribution of the nebulæ. For the latter, if admitted at all, has to apply to nebulæ which are intrinsically similar at the *same* moment of time—not at such moments as depend on the accidental position of our galaxy.

As regards l, the question is, whether we are inclined to assume (*a*) that the distances between the stars within a nebula behave, on the average, like the distances between two points of a rigid body—say, the ends of the Paris metre rod; or (*b*) like the distance between two distant nebulæ. Clearly the case of the stars is intermediate. To regard l as a constant means to decide for the first alternative. The second one would make l/R constant, giving formula (1) the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 3′).

As regards E_0, the possible general decline of the nebular candle-powers has already been mentioned by Hubble and Tolman (see their concluding remarks). To the assumption that the same amount of energy is emitted during every second, there is a peculiarly simple alternative, namely, that the amounts of energy, which *have* been emitted during a second, *remain* equal. On account of the decay of travelling energy, this assumption would mean $E_0 \sim 1/R$, which reduces equation 2 to the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 4′). I do not mean to suggest $E \sim 1/R$ particularly. I mention it in the way of an example.

These remarks detract nothing from the importance of deciding by observation how $d\theta$ and ρ actually behave, if the photographs are interpreted as assuming uniform spatial distribution. I understand that present evidence points to observed luminosities (ρ) decreasing with distance *not even quite as rapidly* as we should expect (with E_0 = const.) from the non-recessional explanation. If that is so, I should say they rather support the recessional explanation, in spite of its predicting a still more rapid decrease of the ρ's. The discrepancy, though greater, can here be removed by assuming the E_0's to decrease with time; an assumption which is very plausible in an expanding universe, which, on the whole, cools down; but not at all plausible in a static one.

E. SCHRÖDINGER.

7 Sentier des Lapins,
La Panne, Belgium.
July 31.

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The Forbidden $^3P_0$–$^1D_2$ Line of O III in the Nebular Spectrum of Nova Herculis 1934

ALTHOUGH the two well-known lines of [O III] λ = 5007 A. ($^3P_2$—$^1D_2$) and λ = 4959 A. ($^3P_1$—$^1D_2$) are the most prominent features in the spectra of planetary nebulæ and novæ at the nebular stage, the third line of the triplet, corresponding to the $^3P_0$—$^1D_2$

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Previously several interactions with Sir Arthur Stanley Eddington.

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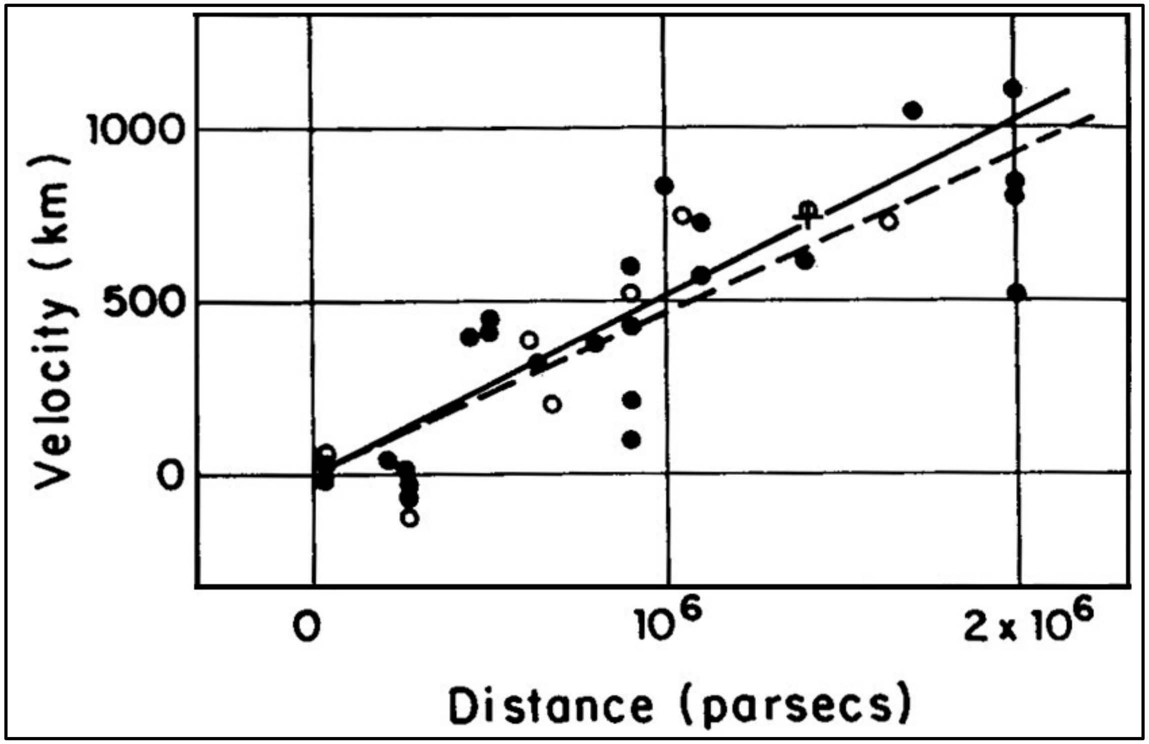
Expansion of the Universe



Hubble's discovery paper

Proc. Natl. Acad. Sci. **15**, 168 (1929)

University of Chicago **1908-1909 National Champions**

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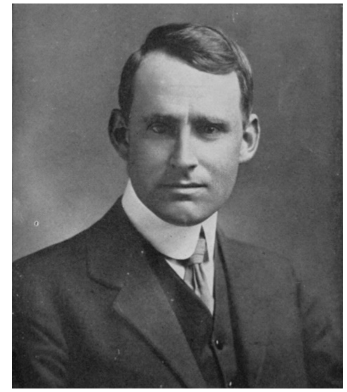

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Physica VI, no 9 October 1939

THE PROPER VIBRATIONS
OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einsteins theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (Einstein- and De Sitter-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

For *light*: when referred to the customary *co-moving* coordinates, an *arbitrary* wave process exhibits essentially the same succession of states as without expansion. Briefly, the wave function shares the general dilatation. Hence all *wave lengths* increase proportionally to the radius of curvature. — The *time rate* of events is slowed down. It is, in every moment, proportional to R^{-1}. Moreover all *intensities* are affected by a common factor such as to make the total energy of an arbitrary wave process proportional to R^{-1}.

For the *material particle* the broad results are these: a strictly monochromatic process (i.e. a proper vibration) again shares the

THE PROPER VIBRATIONS
OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

Physica **VI**, 899 (1939)

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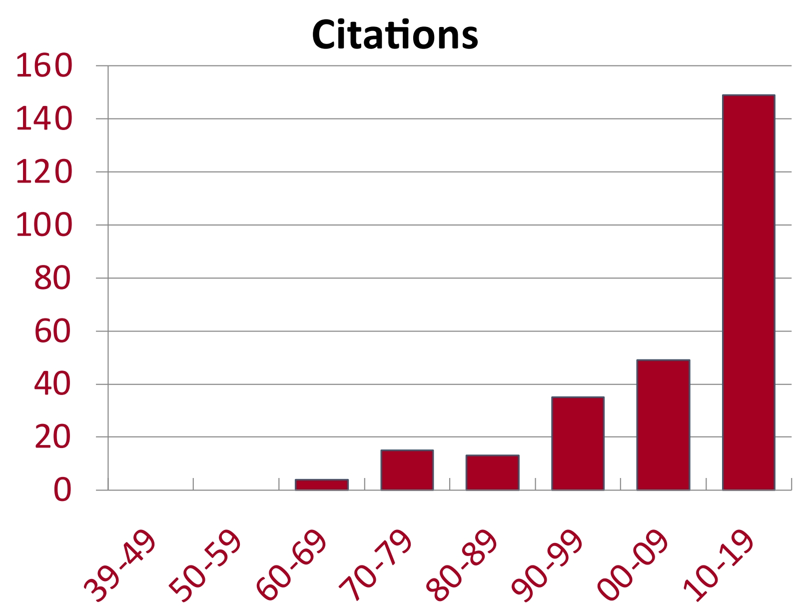
Published October 1939
No author affiliation listed
Cited 268 times (Google Scholar)



$e^{2\pi i\nu t}$ will re-assume (or approximately re-assume) the form $Ae^{2\pi i\nu' t}$ — and *not* $Ae^{2\pi i\nu' t} + Be^{-2\pi i\nu' t}$ — whenever $R(t)$, after an intermediate period of arbitrary variation, returns to constancy (or to approximate constancy). I can see no reason whatsoever for $f(t)$ to behave rigorously in this way, and indeed I do not think it does. There will thus be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called „the alarming phenomena". They are certainly very slight, though, in two cases, viz. 1) when R varies slowly 2) when it is a linear function of time (see the following sections).

A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitters universe I know three of them. Besides the static one, for which P. O. Müller (l.c.) has redently given the proper vibrations, there is an expanding form with infinite R and an expanding form with finite R *). A proper vibration of one frame will not transform into a proper vibration of the other frame, for the separation of variables is destroyed by the transformation.

Schrödinger's two favorite phrases:

1. alarming phenomenon
2. adulteration

Schrödinger was alarmed by creation of a *single* particle

per Hubble time ($H_0^{-1} \sim 10^{10}$ yr)

per Hubble volume ($H_0^{-3} \sim 10^{57}$ km^3)

with Hubble energy ($H_0 \sim 10^{-33}$ eV)

Of all the circumstances faced by Schrödinger in 1939, why did this alarm him?

common dilatation, so that its wave length λ is proportional to R, as before. From the changing λ the changing *frequency* is calculated by de Broglies formula. This implies different frequencies to be affected by different factors. Therefore an arbitrary wave function can no longer be said to simply share the common dilatation. But since de Broglie's dispersion formula persists, the familiar connection (momentum $= h/\lambda$) between linear group velocity ($=$ particle velocity) and wave length is also preserved, which causes the former or more precisely the momentum, to decrease proportional to R^{-1}. As regards the amplitudes, the most reliable information about them, valid for any particle wave function whatsoever, is this, that the *normalisation* is rigorously conserved during the expansion.

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time. Waves travelling in one direction can be rigorously separated from those travelling in the opposite direction. The results for D'Alemberts equation (light) and Gordons equation (material particles), which have been used throughout in this paper for the sake of simplicity, are given in sect. 5 and 6 respectively. I have confirmed the results with Diracs equation, but reserve it to a subsequent paper.

Even in an expanding universe, a particle's wavefunction can be decomposed into "proper vibrations" (positive & negative frequency modes):

$$\Psi(t) = \frac{\alpha}{\sqrt{2\omega}}\, e^{-i\omega t} + \frac{\beta}{\sqrt{2\omega}}\, e^{+i\omega t}$$

Particle occupancy number $\propto |\beta|^2$

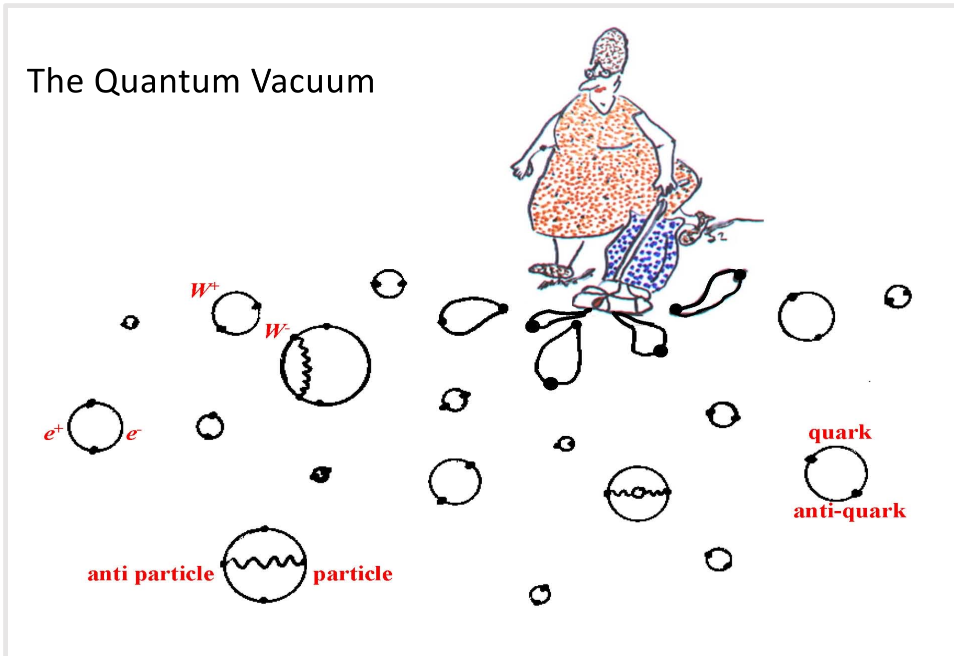
If start with pure incoming or outgoing waves, in and out will become mixed.

The expansion of the universe creates particles!

This alarms me [ed. why?], so I wrote a paper.

Schrödinger's Alarming Phenomenon

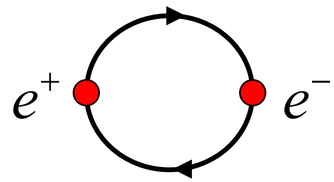
How to understand mutual adulteration (particle creation)



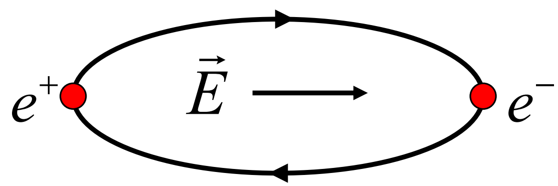
External Fields Can Disturb The Quantum Vacuum

Disturbing the Quantum Vacuum

Electric Field → Particle creation

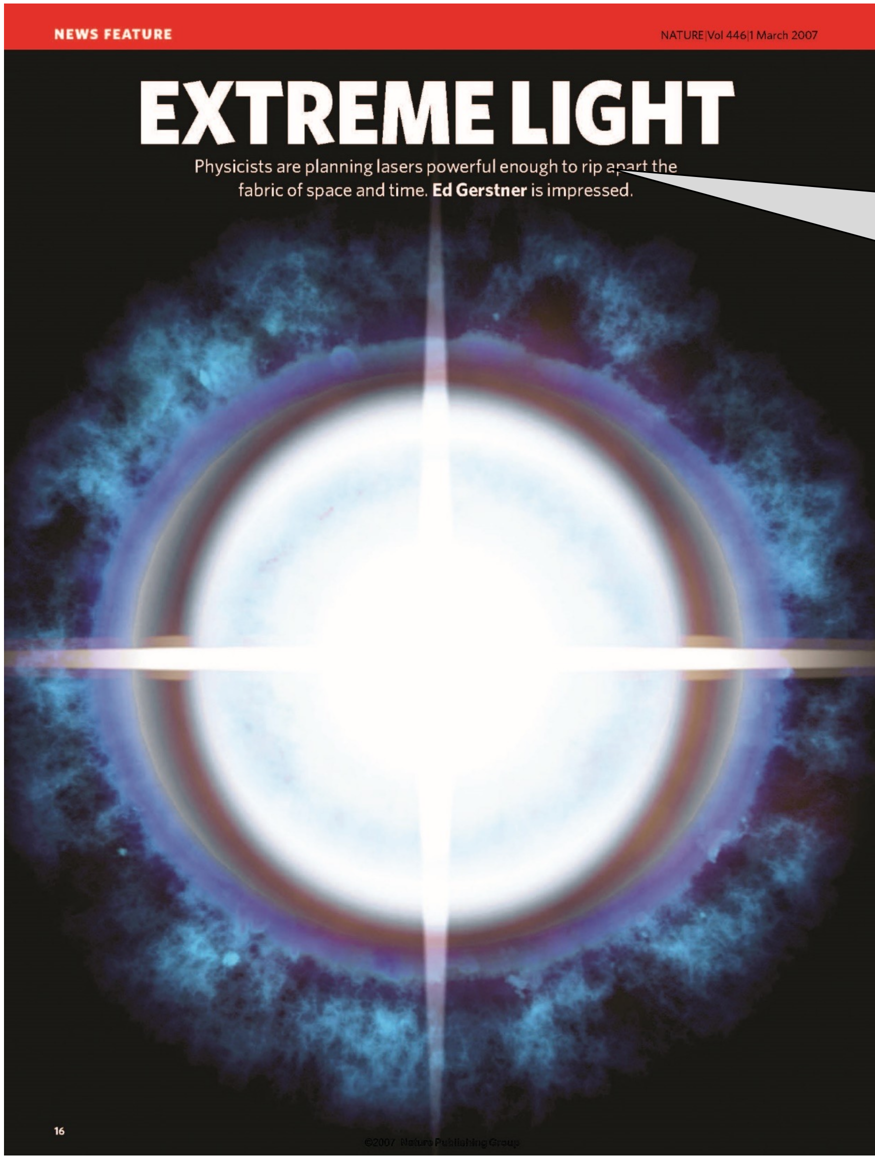


Particle creation if energy gained in acceleration from E-field over a Compton wavelength exceeds the particle's rest mass.



$$\left|\vec{E}_{\text{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16} \text{ V cm}^{-1} \qquad \Gamma \propto e^{-\pi E_{\text{crit}}/\left|\vec{E}\right|}$$

Sauter (1931); Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)



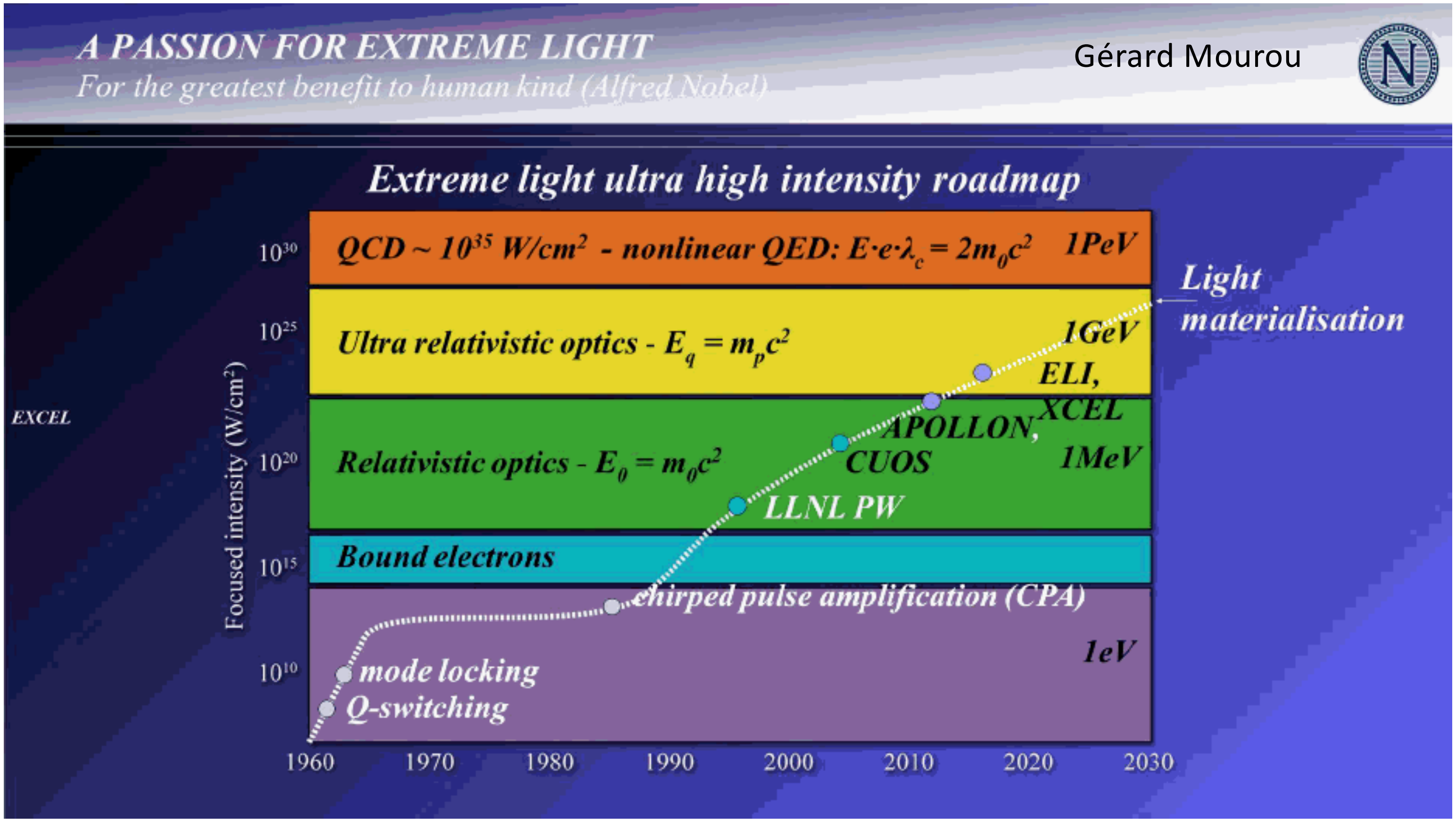
NATURE, Vol 446/1 March 2007

Physicists are planning lasers powerful enough to rip apart the fabric of space and time.

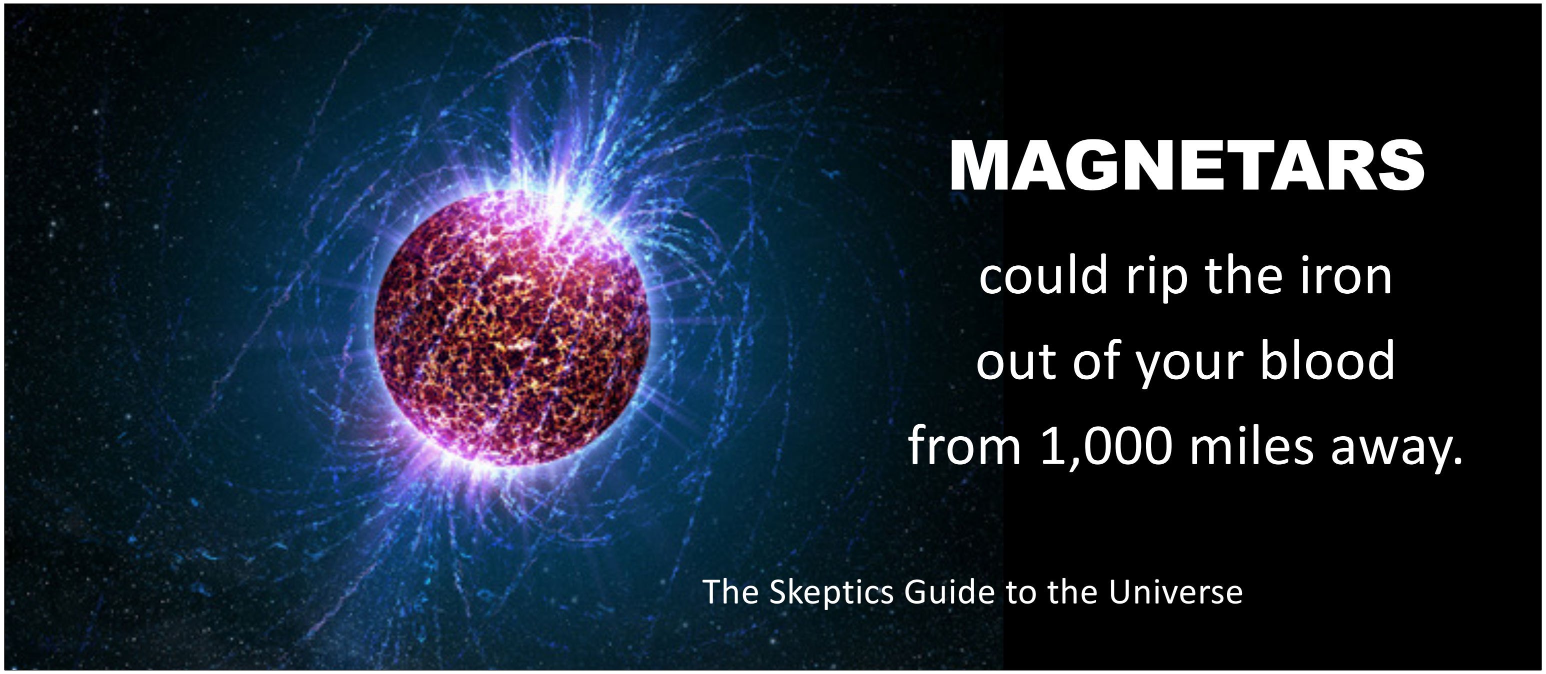
"We're going to change the index of refraction of the vacuum and produce new particles."

Gérard Mourou

Disturbing the Quantum Vacuum



$$I_C \approx \frac{c}{8\pi}\left|\vec{E}_{\text{crit}}\right|^2 \approx 10^{30}\ \text{W cm}^2$$



$$\left|\vec{B}_{\mathrm{crit}}\right| = \frac{m_e^2}{e} \approx 5 \times 10^{13}\,\mathrm{G}$$

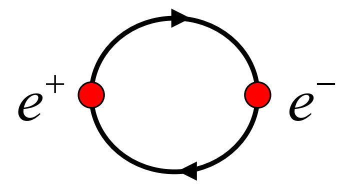
Crab pulsar	3×10^{13} G
Magnetars	$10^{14} - 10^{15}$ G

Strong magnetic fields imply existence of strong electric fields.

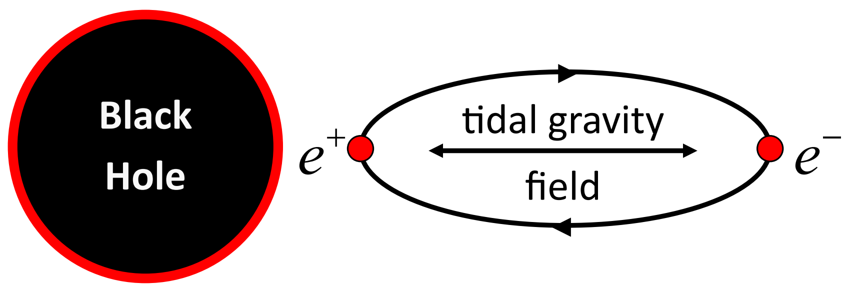
Many strange phenomena associated with pulsars, magnetars, etc.

Disturbing the Quantum Vacuum

Tidal gravitational field → Particle creation



Particle creation if energy gained in acceleration from gravity over a Compton wavelength exceeds the particle's rest mass.

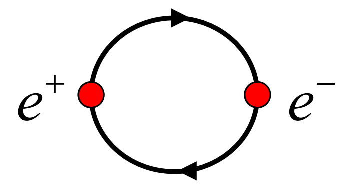


$\mathrm{v} = c$ at Black Hole horizon

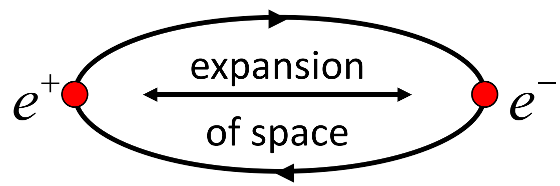
Hawking (1974); Bekenstein (1972)

Disturbing the Quantum Vacuum

Expanding universe → Particle creation



Particle creation if energy gained in acceleration from expansion over a Compton wavelength exceeds the particle's rest mass.



$\mathrm{v} = c$ at Hubble radius

$$H_{\text{crit}} = m \qquad\qquad \Gamma \propto e^{-\pi H_{\text{crit}}/H}$$

Schrödinger's Alarming Phenomenon (1939)

Schrödinger's Alarming Phenomenon

Why was Schrödinger alarmed?

- Appearance of particles from the vacuum sounds crazy.
- Technical issues with calculation:
  Quantum mechanical calculation (requires quantum field theory).
  Only create particles with mass less than expansion rate H
  (today $H_0 \sim 10^{-33}$ eV).
  Only create particles if violate Weyl Conformal Invariance
  (don't create photons).
  Would Schrödinger still have been alarmed?
- Schrödinger looked for (and found) a cosmological solution without mutual adulteration (not a very physical solution).
- Perhaps he thought it was conceptual challenge to Quantum Mechanics or General Relativity.
- Infinite particle creation in standard big-bang at $t = 0$.
- (Sometimes should just follow the equations).

Schrödinger's Alarming Phenomenon

"Outstanding" importance?

Schrödinger 1939: "Generally speaking this is a phenomenon of outstanding importance. With particles it would mean the production or annihilation of matter, merely by the expansion." [why would that be of outstanding importance?]

Forgotten in 40s, 50s, 60s (by Schrödinger also).

Leonard Parker Thesis 1966. In 1968 paper: "…for the early stages of a Friedmann expansion it [particle creation] may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity." [no speculation as to the "great cosmological significance"]

Zel'dovich 1970s proposed an application: explaining why the universe is homogeneous and isotropic.

Schrödinger's Alarming Phenomenon

Other interest in CGPP in the 1970s (mostly regarded as a curiosity).

US: Parker, Ford, Fulling, Allen, Friedman, Wald, …
Soviet Union: Zel'dovich, Starobinski, Grishchuk, Grib, Mostepanenko, Lukash, … (CGPP in the CCCP)
UK: Bunch, Davies, Birrell, Hawking, …

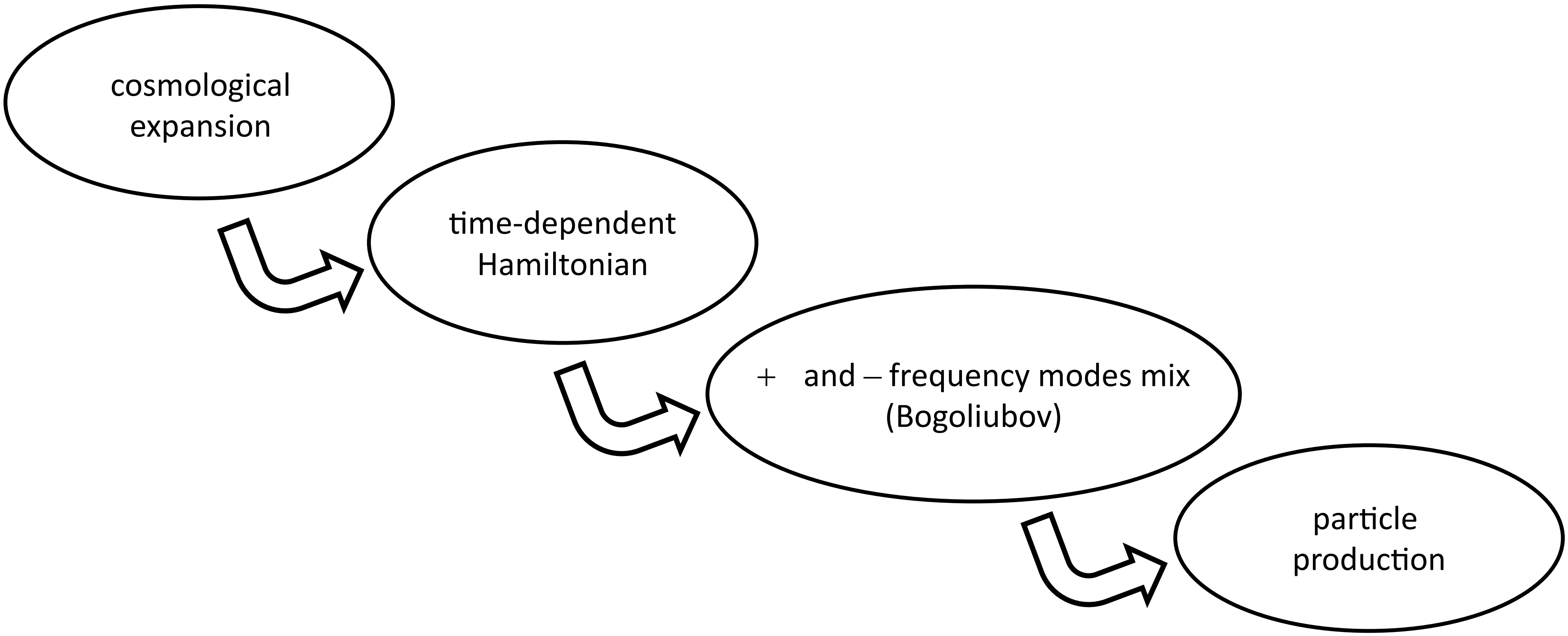
Great cosmological significance in the 1980s (inflation):

Sasaki, Kodama, Mukhanov, Vilenkin, Linde, Abbott, Wise, Lyth, Salopek, Bond, …

Cosmological Gravitational Particle Production (CGPP)

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a “particle” is approximate.

Schrodinger (1939); Parker (1965, 68); Fulling, Ford, & Hu; Zel’dovich; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



Scalar field ϕ in Minkowski space

Action:

$$S[\phi(x)] = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\right]$$

Fourier mode decomposition:

$$\widehat{\phi}(t,\boldsymbol{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\widehat{a}_{\mathbf{k}}\chi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + \widehat{a}^\dagger_{\mathbf{k}}\chi^*_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

Mode functions satisfy wave equation and normalization condition:

$$\partial_t^2\chi_k(t) + \omega_k^2\chi_k(t) = 0 \qquad \chi_k\partial_t\chi_k^* - \chi_k^*\partial_t\chi = 0$$

Dispersion relation: $\omega_k^2 = k^2 + m^2$

Vacuum state $|0\rangle$ defined as state where $\widehat{a}_{\mathbf{k}}|0\rangle = 0$

Scalar field ϕ in Minkowski space

Solution to wave equation:

$$\chi_k(t) = \frac{\alpha_k}{\sqrt{2\omega_k}}\, e^{-i\omega_k t} + \frac{\beta_k}{\sqrt{2\omega_k}}\, e^{+i\omega_k t}$$

Bogoliubov coefficients satisfy: $|\alpha_k|^2 - |\beta_k|^2 = 1$

If choose $|\alpha_k| = 0$, will have pure outgoing wave … no mode mixing

Some Notation

FLRW model $$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

My convention: scale factor a has dimension of length. An observer "at rest" has constant (r, θ, ϕ) "comoving coordinates"

In early universe $k = 0$

Define conformal time $$a d\eta = dt$$

Expansion rate $$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$ prime indicates derivative wrt conformal time

Assume Standard Inflationary Picture

Quasi-de Sitter Phase driven by vacuum energy of inflaton displaced from potential minimum, expansion rate H_I roughly constant

Matter-dominated phase due to inflaton oscillations about minimum of potential

Inflaton decays and leads to radiation-dominated phase characterized by a reheat temperature $T_{\rm RH}$

Conformal time $-\infty < \eta < 0$ de Sitter and $0 < \eta < \infty$ matter-dominated → radiation-dominated

Roughly inflation matter-dominated (a_e , H_e are values at end of inflation)

inflation:

$$a = \frac{a_e}{1-\eta}$$

$$H = H_e$$

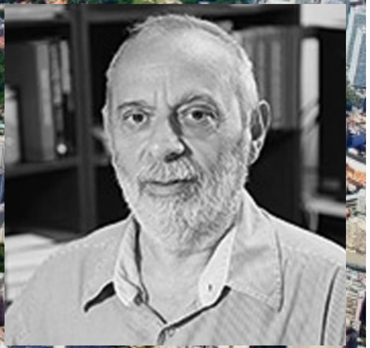
matter-dominated:

$$a = a_e(1+\tfrac{1}{2}\eta)^2$$

$$H = \frac{H_e}{(1+\frac{1}{2}\eta)^3}$$

Gravitational Particle Production and Dark Matter — Four Lectures

1. Dark Matter: Evidence and the Standard WIMP
2. Gravitational Particle Production (Schrödinger's Alarming Phenomenon)
3. GPP of Scalar Fields
4. Beyond Scalar Fields



Rocky Kolb, University of Chicago

CBPF 9/2022