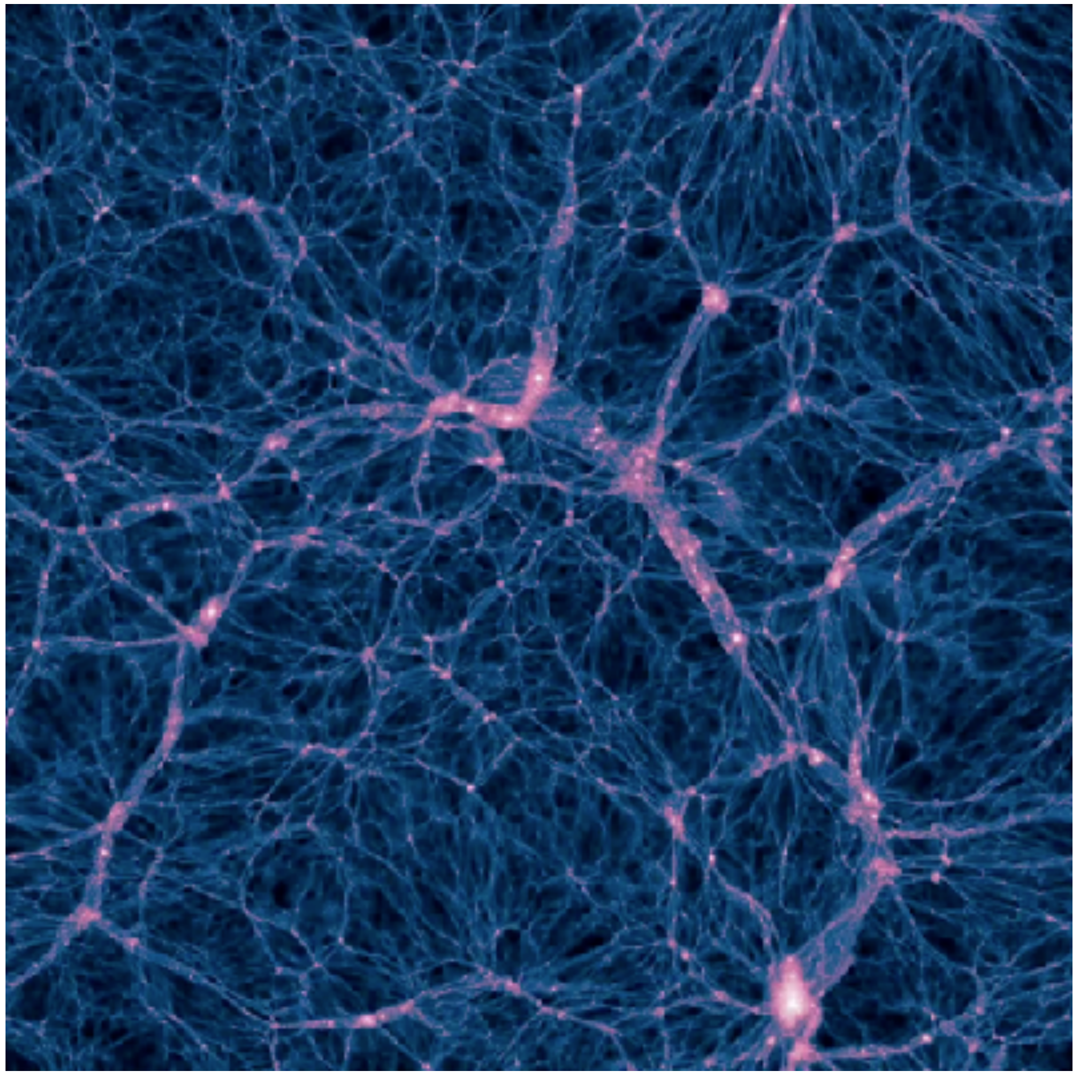
Relativistic cosmological large scale structures at one-loop

Radouane Gannouji

Pontificia Universidad Católica de Valparaíso

In Collaboration with Lina Castiblanco, Jorge Noreña, Clément Stahl



Illustris Simulation

XVIII Brazilian Cosmology and Gravitation School, CBPF

Алекса́ндр Алекса́ндрович Фри́дман
Alexander Alexandrovich Friedmann

1917

Cosmological considerations on the general theory of relativity

But if we are concerned with structure only on a large scale, we can represent matter as if it were uniformly distributed in huge spaces ....

Homogeneity

Pascal: a sphere whose centre is everywhere and circumference nowhere

Isotropy

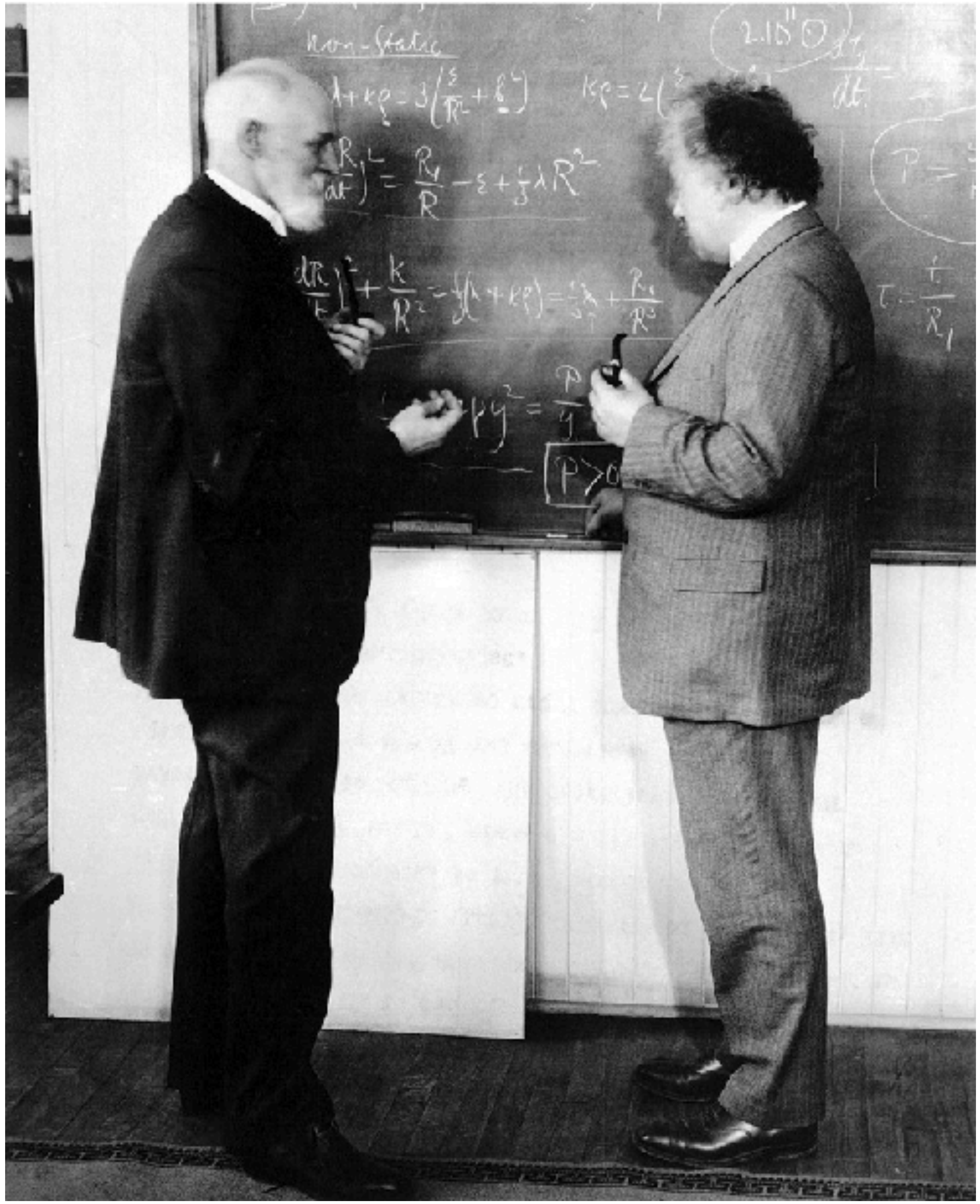
The world is spatially closed
Machian

The world is static

I will lead the reader along the road I have travelled myself, a rather bumpy and winding road, because otherwise I cannot expect him to be very interested in the result at the end of the journey.

The conclusion I will come to is that the field equations of gravitation that I have defended so far still need a slight modification. . .

Λ



1917

Einstein's theory of gravitation and its astronomical consequences. Third paper

Homogeneity

Isotropy

Λ

Vacuum

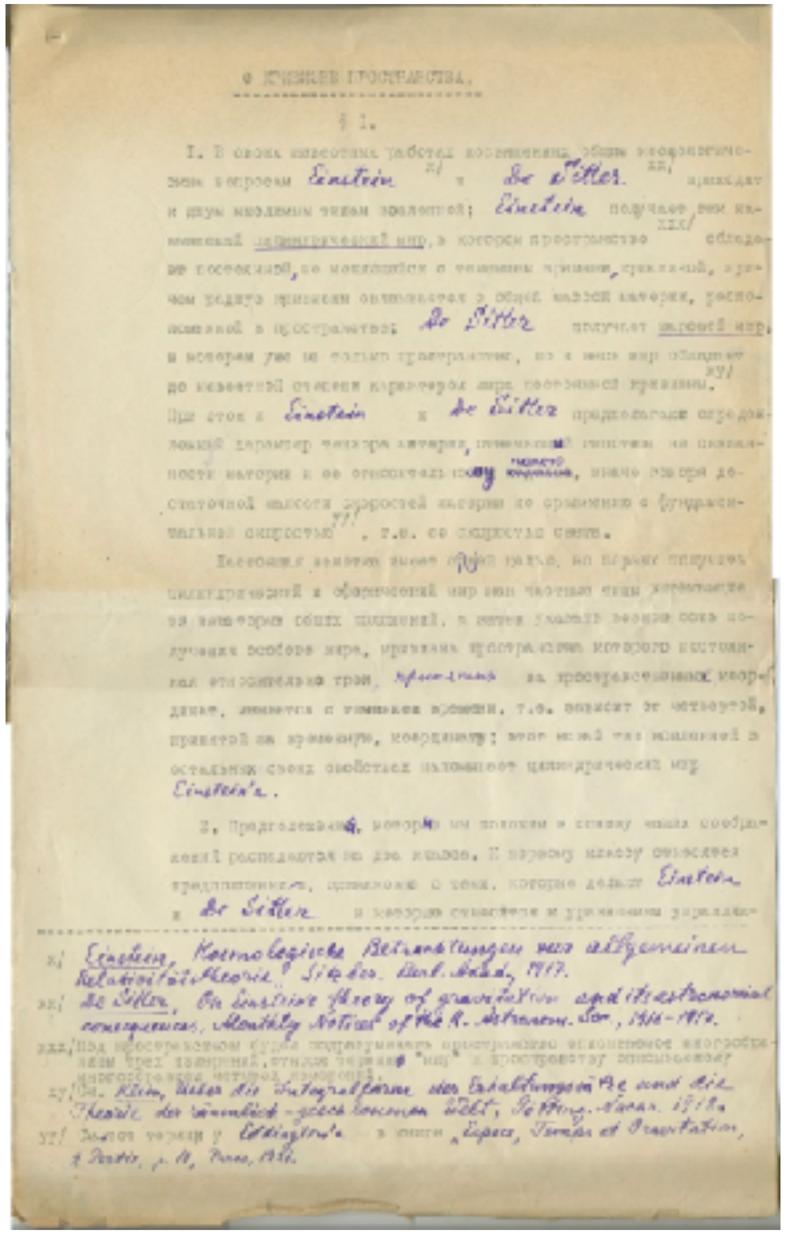
The de Sitter Universe

Anti-Machian

Felix Klein

"It would be unsatisfactory, in my opinion, if a world without matter were possible. Rather, the $g^{\mu\nu}$-field should be fully determined by matter and not be able to exist without the latter."

On the curvature of space



29-05-1922

29-06-1922

$$\kappa = \frac{8\pi G}{c^4}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Low velocity of matter compared to c

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

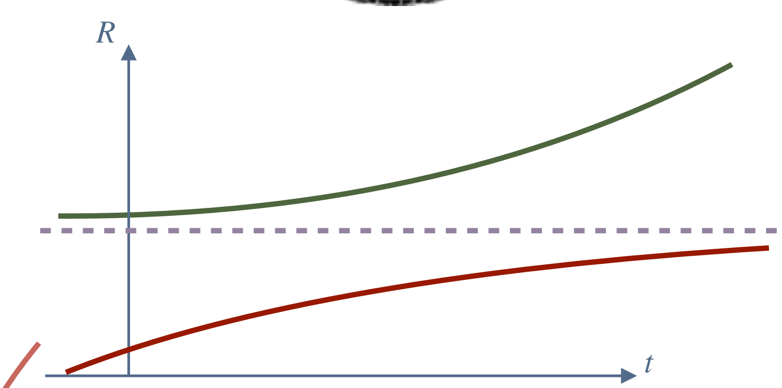
Time since the creation of the Universe

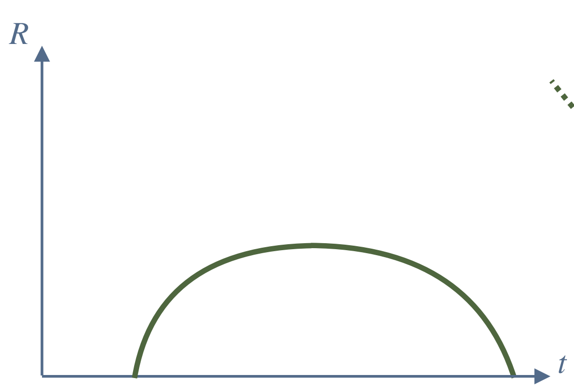
$$ds^2 = -dt^2 + \frac{R(t)^2}{c^2}\left(dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2\right)$$

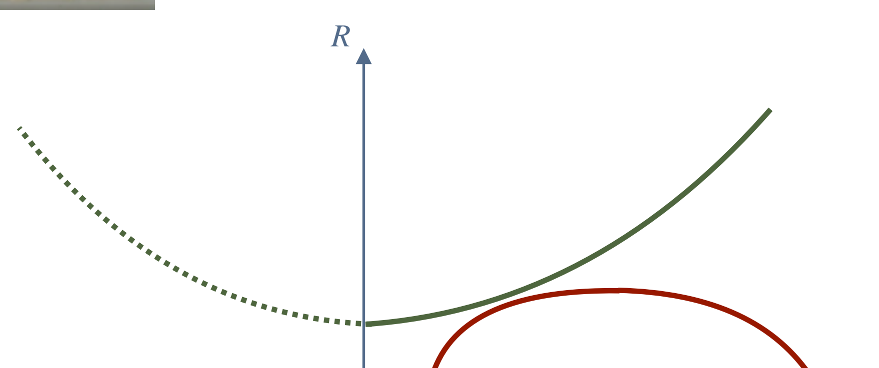
$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{c^2}{R^2} - \Lambda = 0$$

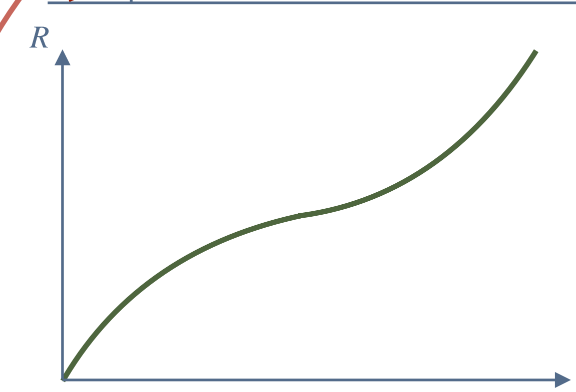
$$3\frac{\dot{R}^2}{R^2} + 3\frac{c^2}{R^2} - \Lambda = \kappa c^2 \rho$$

$\rho = \frac{\rho_0}{R^3}$ with $\rho_0 = \frac{M}{R^3}$









Einstein

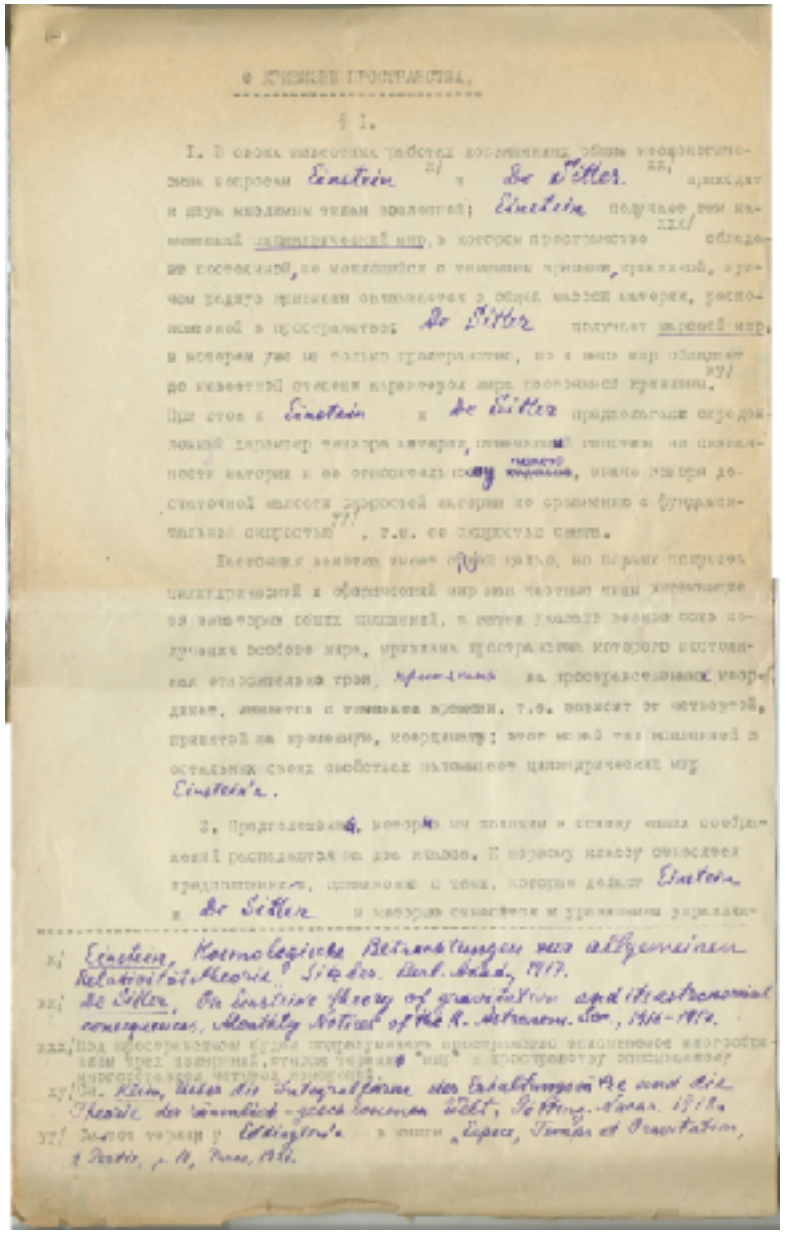
Λ

0

de Sitter

$$\Lambda_c = \frac{4c^2}{\kappa^2 \rho_0^2}$$

On the curvature of space



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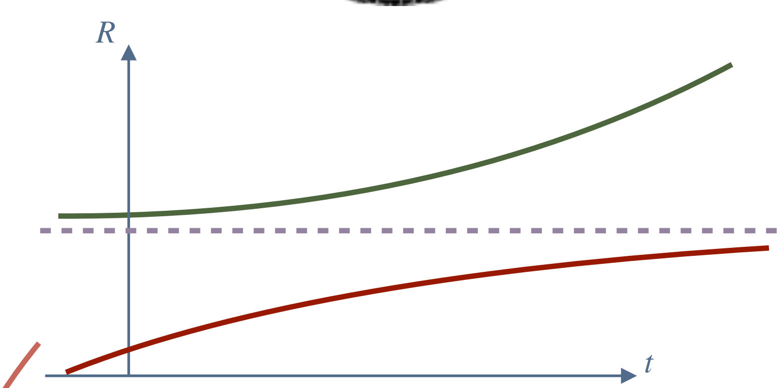
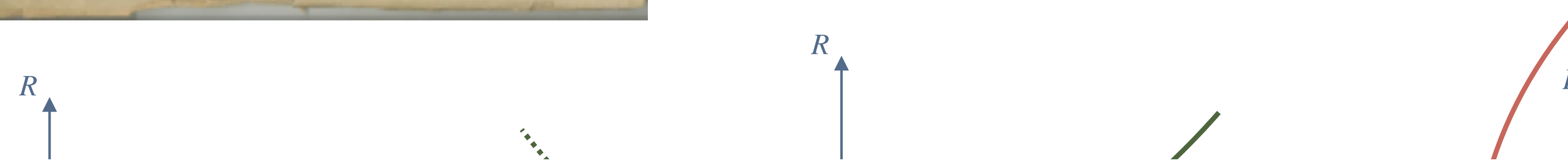
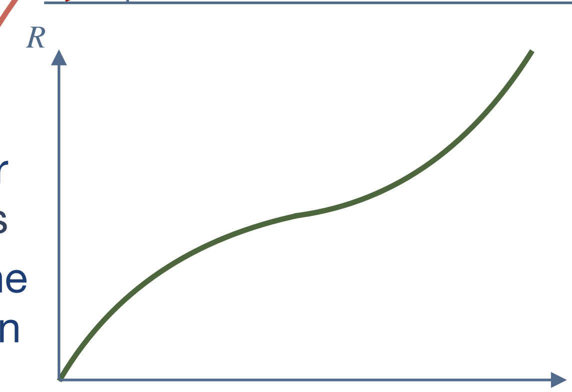
$$\rho = \frac{\rho_0}{R^3} \quad \text{with} \quad \rho_0 = \frac{M}{R^3}$$



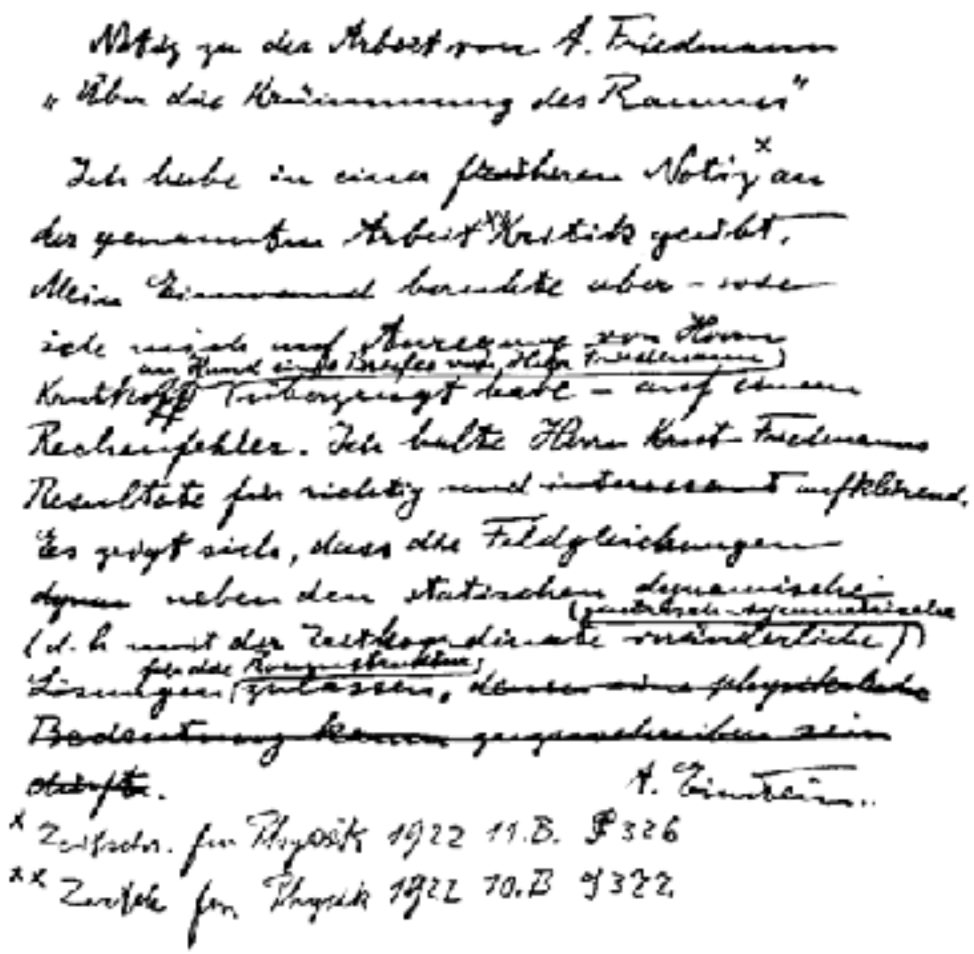




It remains to be pointed out that the cosmological quantity Λ remains undetermined in our formulae, as it is an extra constant in the problem; possibly electrodynamic considerations can lead to its evaluation. If we fix $\Lambda = 0$ y $M = 5.10^{21}$ solar masses, then the period of the world becomes of the order of 10 billion years. But these figures can surely only serve as an illustration for our calculations.

de Sitter

$$\Lambda_c = \frac{4c^2}{\kappa^2 \rho_0^2}$$



Notiz zu der Arbeit von A. Friedmann „Über die Krümmung des Raumes"

Ich habe in einer früheren Notiz[x] an der genannten Arbeit[xx] Kritik geübt. Mein Einwand beruhte aber – wie ich mich auf Anregung von Herrn Krutkoff an Hand eines Briefes von Herrn Friedmann überzeugt habe – auf einem Rechenfehler. Ich halte Herrn Friedmanns Resultate für richtig und ~~interessant~~ aufklärend. Es zeigt sich, dass die Feldgleichungen neben den statischen dynamische (d. h. mit der Zeitkoordinate veränderliche) zentrisch-symmetrische Lösungen für die Raumstruktur zulassen, ~~denen eine physikalische Bedeutung kaum zuzuschreiben sein dürfte~~.

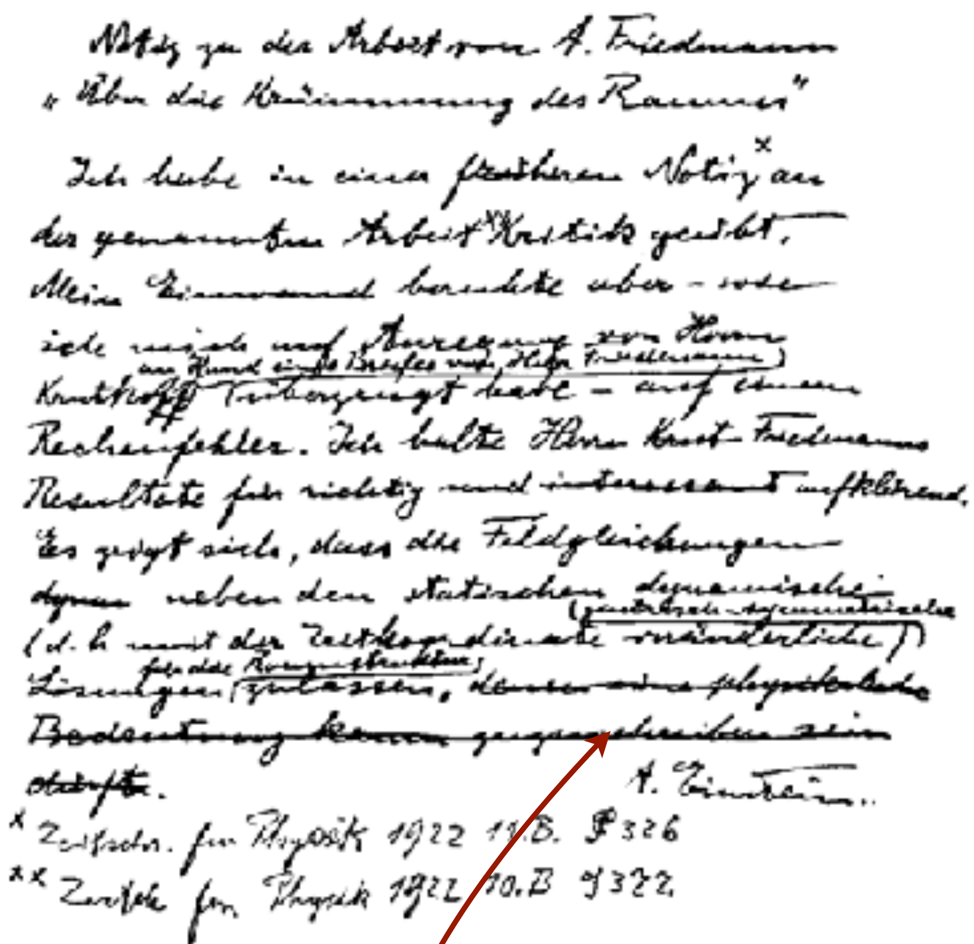
A. Einstein.

[x] Zeitschr. für Physik 1922 11.B. S 326

[xx] Zeitschr. für Physik 1922 10.B S 377

In an earlier note I exercised criticism on the mentioned paper. My objection, however, was based on a calculation error—as I have become persuaded, at the suggestion of Mr. Krutkoff, guided by a letter by Mr. Friedmann. I consider Mr. Friedmann's results correct and illuminating. It is demonstrated that the field equations permit, aside from the static solution, dynamic (i.e., variable with the time coordinate), centrally symmetrical solutions for the structure of space

I have won over Einstein in the argument about Friedmann.
The honor of Petrograd is saved!"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a physical signicance can hardly be ascribed to them

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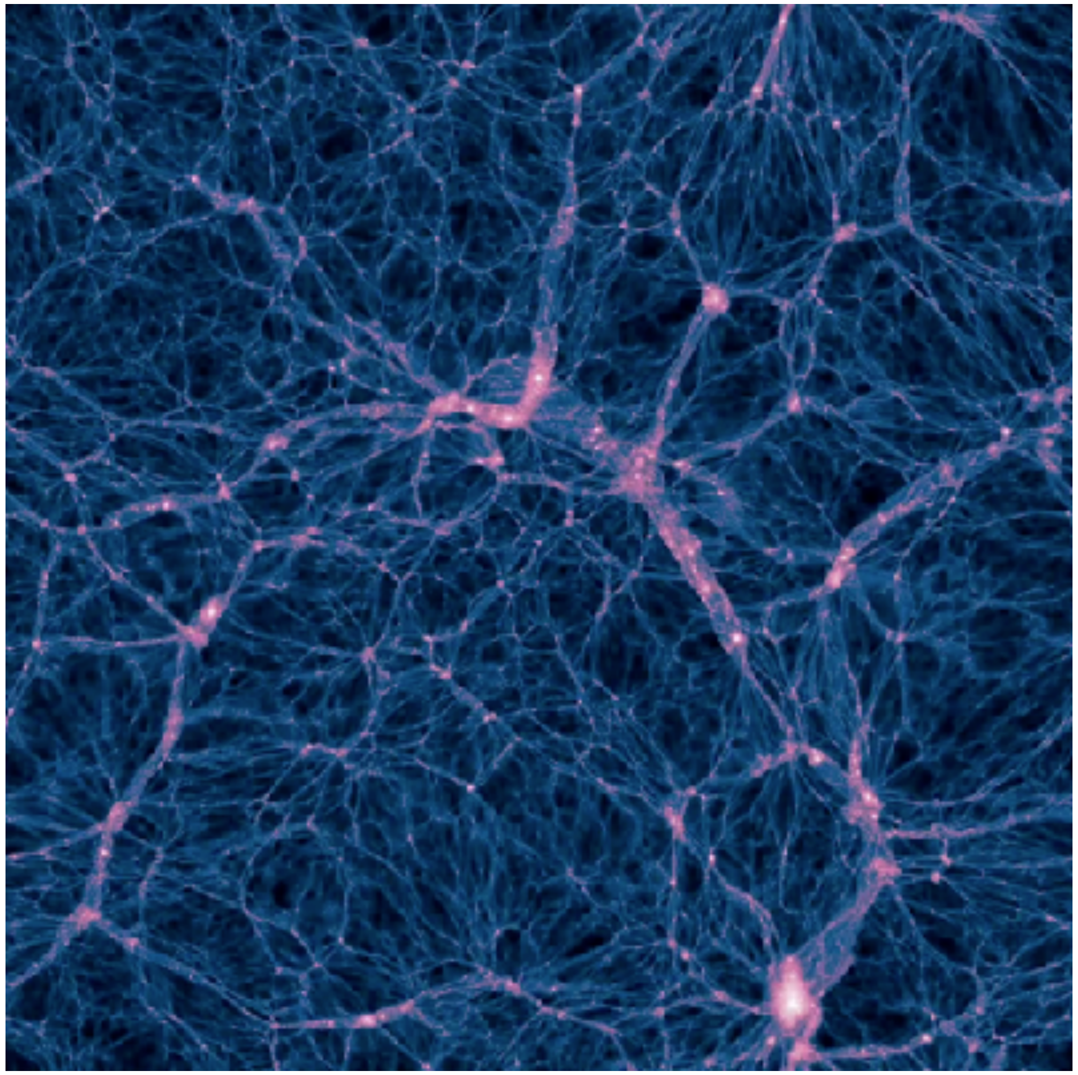
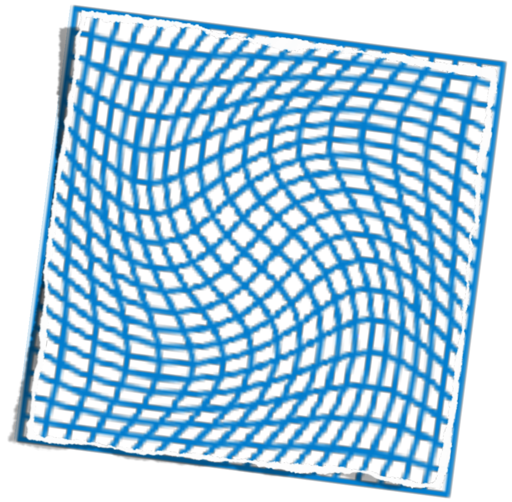


Illustris Simulation

XVIII Brazilian Cosmology and Gravitation School, CBPF

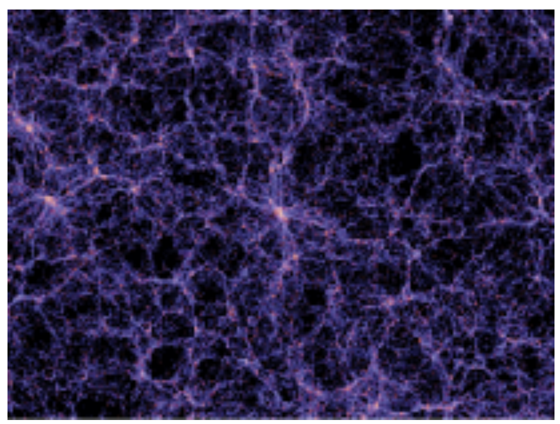
Summary

Large scales



Friedmann Universe
+
Linear Perturbations → Relativistic

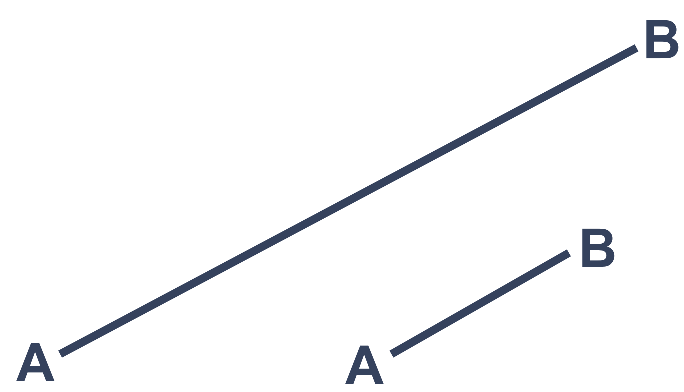
Small scales



Non-linear physics → Newtonian

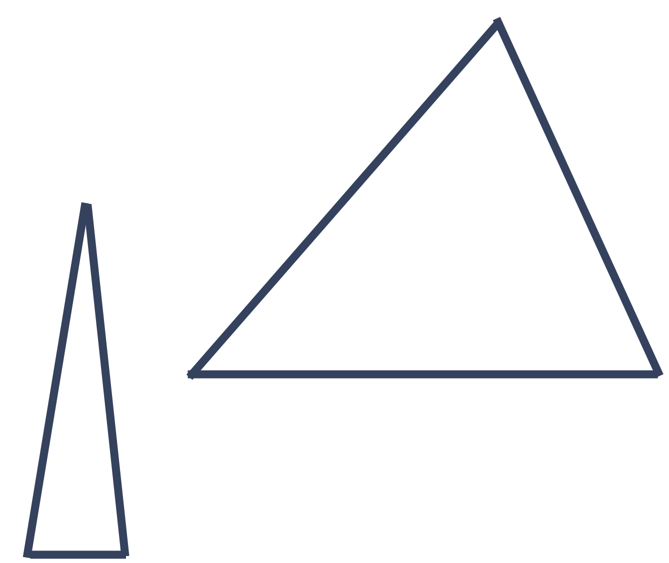
Power spectrum

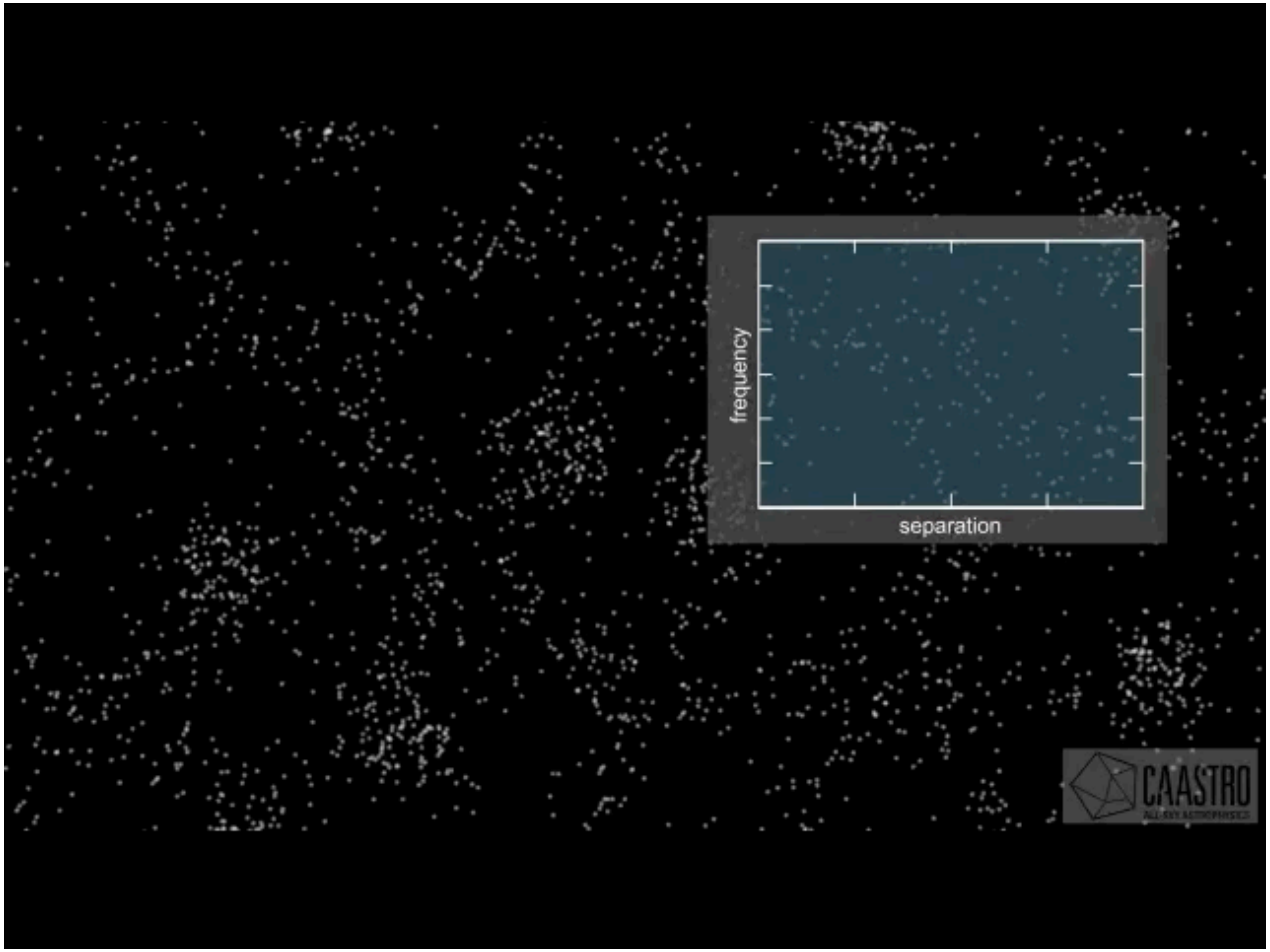
$$\langle \delta_m(\mathbf{k}_1, t)\delta_m(\mathbf{k}_2, t)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P(k_1, t)$$



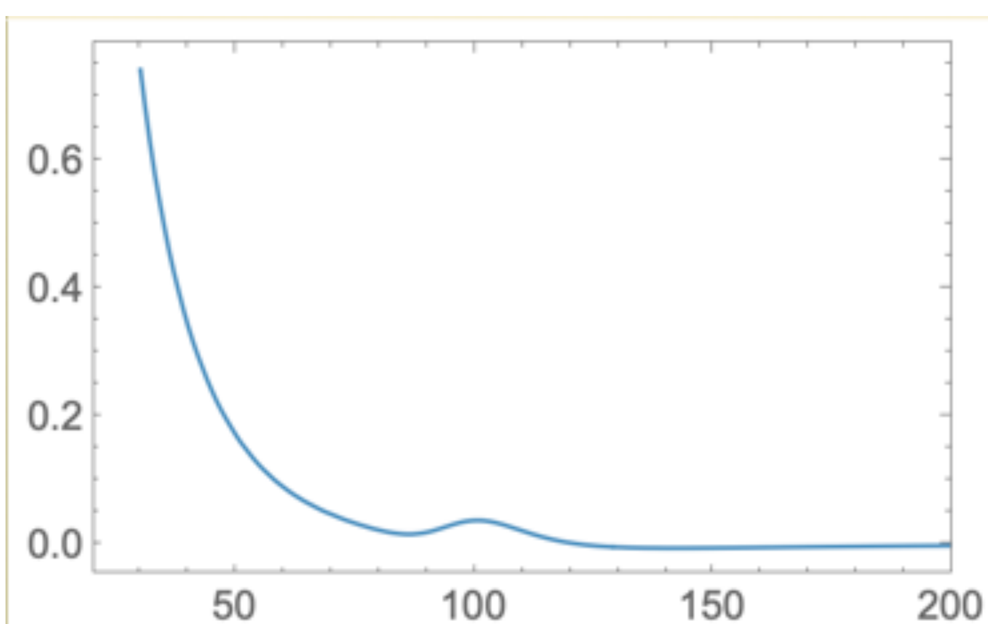
Bispectrum

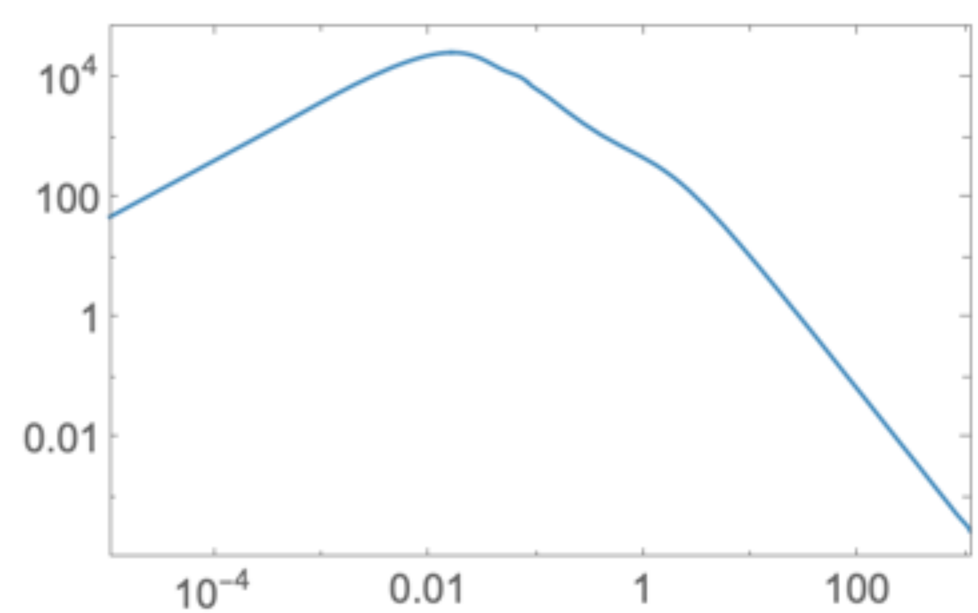
$$\langle \delta_m(\mathbf{k}_1, t)\delta_m(\mathbf{k}_2, t)\delta_m(\mathbf{k}_3, t)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3, t)$$





http://caastro.org/





Bispectrum

$$\langle \delta_m(\mathbf{k}_1, t)\delta_m(\mathbf{k}_2, t)\delta_m(\mathbf{k}_3, t)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B(k_1, k_2, k_3, t)$$

The squeezed limit contains model independent information about the physics during inflation

Single field inflation

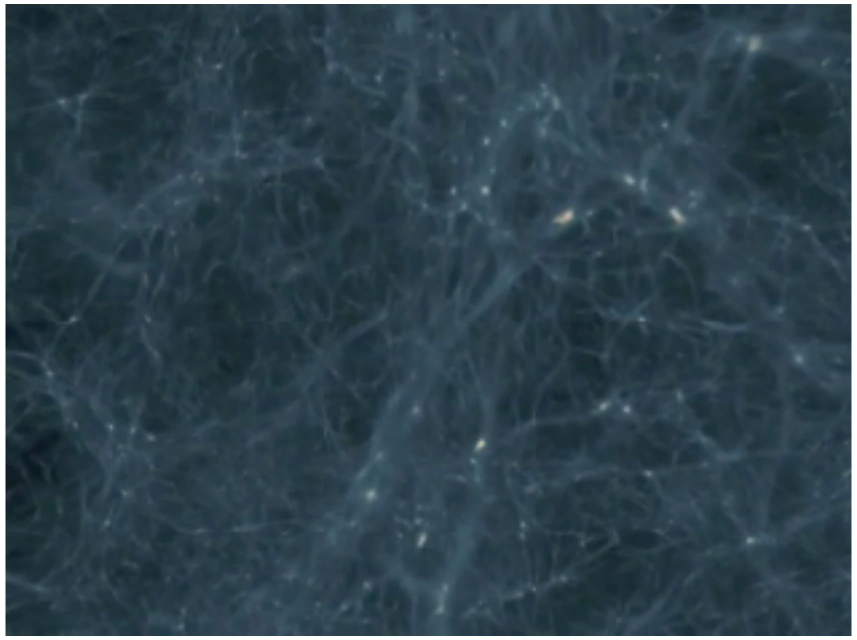
$$B(q, k_1, k_2) \xrightarrow{q\to 0} \frac{1}{q}$$

J. Maldacena astro-ph/0210603

P. Creminelli, M. Zaldarriaga astro-ph/0407059

with different behavior for multi-field inflation or higher spin

A self-gravitating expanding dust fluid



Particle number density in phase space $f(t, \mathbf{x}, \mathbf{p})$

Local mass density (zeroth order moment) $\rho_m(t, \mathbf{x}) = \int \mathrm{d}^3\mathbf{p} f(t, \mathbf{x}, \mathbf{p})$

Peculiar velocity flow (first order moment)

$$\rho_m(t, \mathbf{x})\mathbf{v}(t, \mathbf{x}) = \int \mathrm{d}^3\mathbf{p} \frac{\mathbf{p}}{am} f(t, \mathbf{x}, \mathbf{p})$$

Stress tensor (second order moment) $\rho_m(t, \mathbf{x})\mathbf{v}_i(t, \mathbf{x})\mathbf{v}_j(t, \mathbf{x}) + \sigma_{ij}(t, \mathbf{x}) = \int \mathrm{d}^3\mathbf{p} \frac{\mathbf{p}_i \mathbf{p}_j}{a^2 m^2} f(t, \mathbf{x}, \mathbf{p})$

$f(t, \mathbf{x}, \mathbf{p})$ is solution of the Vlasov equation (collision-less Boltzmann equation)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(t, \mathbf{x}, \mathbf{p}) + \frac{\mathbf{p}}{ma^2} \frac{\partial}{\partial \mathbf{x}} f(t, \mathbf{x}, \mathbf{p}) - m \frac{\partial}{\partial \mathbf{x}} \Phi(\mathbf{x}) \frac{\partial}{\partial \mathbf{p}} f(t, \mathbf{x}, \mathbf{p}) = 0$$

Coupled to the Poisson equation $\Delta \Phi(t, \mathbf{x}) = \frac{4\pi G m}{a} \left(\int \mathrm{d}^3\mathbf{p} f(t, \mathbf{x}, \mathbf{p}) - \bar{\rho}_m(t) \right)$

This is what (N-body mostly) codes aim at simulating

Single flow approximation

Matter density contrast

$$\delta_m(t,\mathbf{x}) = \frac{\rho_m(t,\mathbf{x}) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}$$

Taking moments of the Vlasov equation

$$\frac{\partial}{\partial t}\delta_m(t,\mathbf{x}) + \frac{1}{a}\partial_i\Big[(1+\delta_m(t,\mathbf{x}))\mathbf{v}^i(t,\mathbf{x})\Big] = 0$$

Continuity equation

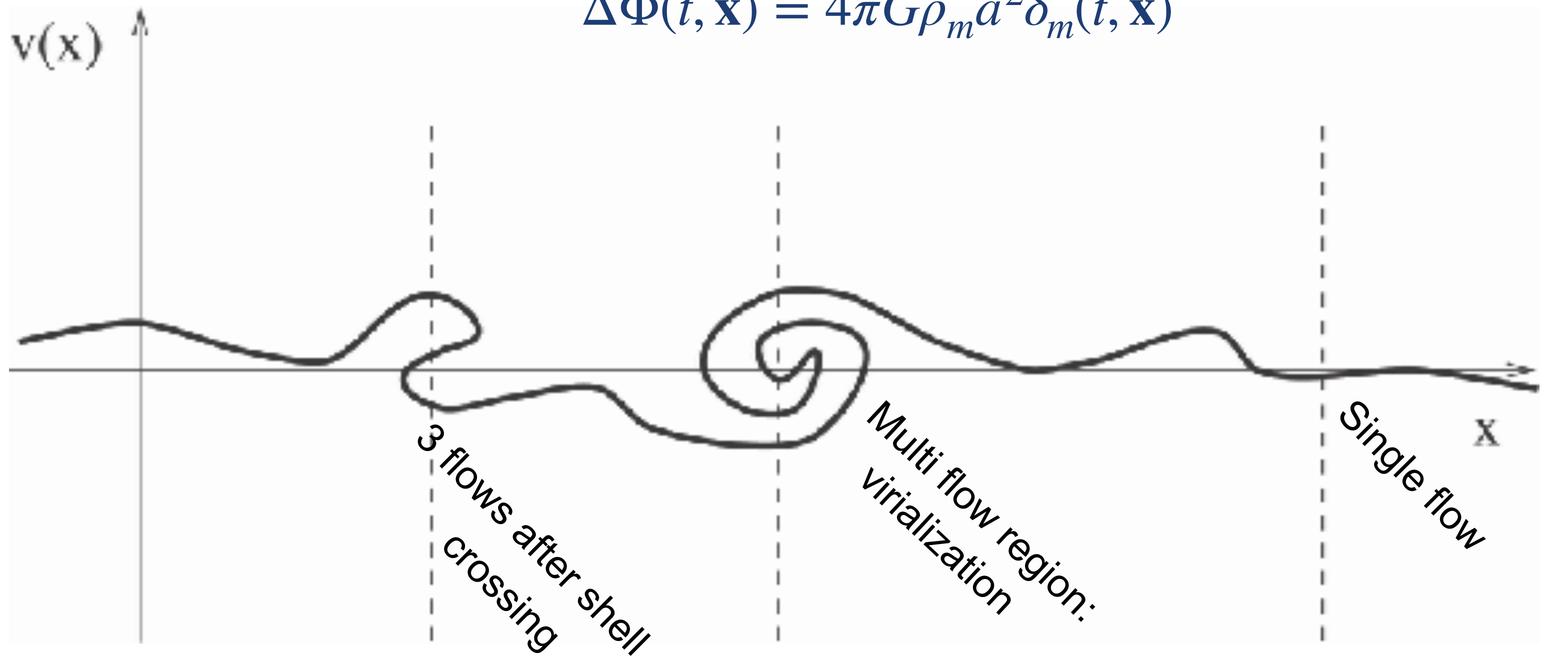
$$\frac{\partial}{\partial t}\mathbf{u}_i(t,\mathbf{x}) + H\mathbf{u}_i(t,\mathbf{x}) + \frac{1}{a}\mathbf{u}_j(t,\mathbf{x})\partial_j\mathbf{u}_i(t,\mathbf{x}) = -\frac{1}{a}\partial_i\Phi(t,\mathbf{x}) - \frac{1}{a\rho_m}\partial_j(\rho_m\sigma_{ij})$$

Euler equation

$$\frac{\partial}{\partial t}\sigma_{ij}(t,\mathbf{x}) + 2H\sigma_{ij} + \frac{1}{a}\mathbf{u}_k\nabla_k\sigma_{ij} + \frac{1}{a}\sigma_{jk}\nabla_k\mathbf{u}_i + \frac{1}{a}\sigma_{ik}\nabla_k\mathbf{u}_j = -\frac{1}{a\rho_m}\partial_k(\rho_m\Pi_{ijk})$$

$$\vdots$$

$$\Delta\Phi(t,\mathbf{x}) = 4\pi G\bar{\rho}_m a^2\delta_m(t,\mathbf{x})$$



Standard Perturbation Theory

$$\frac{\partial}{\partial t}\delta_m(t,\mathbf{x}) + \frac{1}{a}\partial_i\Big[(1+\delta_m(t,\mathbf{x}))\mathbf{v}^i(t,\mathbf{x})\Big] = 0$$

$$\frac{\partial}{\partial t}\mathbf{u}_i(t,\mathbf{x}) + H\mathbf{u}_i(t,\mathbf{x}) + \frac{1}{a}\mathbf{u}_j(t,\mathbf{x})\partial_j\mathbf{u}_i(t,\mathbf{x}) = -\frac{1}{a}\partial_i\Phi(t,\mathbf{x})$$

$$\Delta\Phi(t,\mathbf{x}) = 4\pi G\bar{\rho}_m a^2\delta_m(t,\mathbf{x})$$

In Fourier space:

F. Bernardeau, S. Colombi, E. Gaztanaga, R. Scoccimarro arXiv:astro-ph/0112551

$$\frac{1}{H}\dot{\delta}_m(t,\mathbf{k}) + \theta(t,\mathbf{k}) = -\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\alpha(\mathbf{k}_1,\mathbf{k}_2)\delta_m(t,\mathbf{k}_1)\theta(t,\mathbf{k}_2)$$

$$\frac{1}{H}\dot{\theta} + \left(2+\frac{\dot{H}}{H}\right)\theta + \frac{3}{2}\Omega_m\delta_m = -\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\theta(t,\mathbf{k}_1)\theta(t,\mathbf{k}_2)$$

Linear

Vertices (mode coupling)

$$\theta = \frac{\partial_i v_i}{aH}$$

$$\alpha(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_1+\mathbf{k}_2)\,.\,\mathbf{k}_1}{k_1^2}$$

$$\beta(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_1+\mathbf{k}_2)^2(\mathbf{k}_1\,.\,\mathbf{k}_2)}{2k_1^2k_2^2}$$

$$\delta_m(t,\mathbf{k}) = \sum_{n=1}^{\infty} f^n(t)\delta^{(n)}(\mathbf{k})$$

$$\theta(t,\mathbf{k}) = -\frac{\dot{f}}{H}\sum_{n=1}^{\infty} f^n(t)\theta^{(n)}(\mathbf{k})$$

$$\delta^{(n)}(\mathbf{k}) = \int \mathrm{d}^3\mathbf{k}_1\cdots\mathrm{d}^3\mathbf{k}_n\delta(\mathbf{k}-\mathbf{k}_{1\ldots n})\delta^{(1)}(\mathbf{k}_1)\cdots\delta^{(1)}(\mathbf{k}_n)F^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n)$$

$$\theta^{(n)}(\mathbf{k}) = \int \mathrm{d}^3\mathbf{k}_1\cdots\mathrm{d}^3\mathbf{k}_n\delta(\mathbf{k}-\mathbf{k}_{1\ldots n})\delta^{(1)}(\mathbf{k}_1)\cdots\delta^{(1)}(\mathbf{k}_n)G^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n)$$

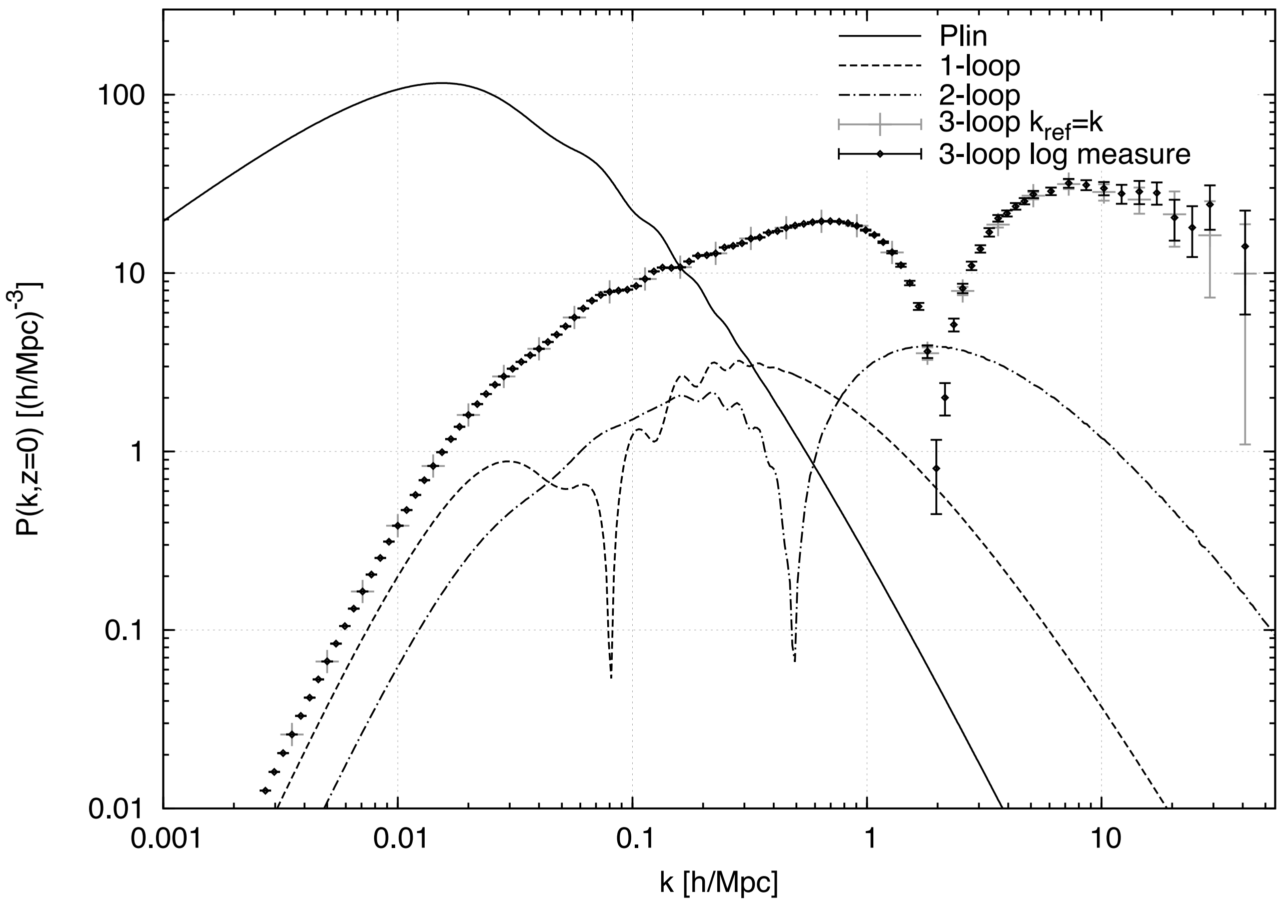
Standard Perturbation Theory

$$\delta_m = f\delta^{(1)} + f^2\delta^{(2)} + f^3\delta^{(3)} + \cdots$$

$$\langle\delta_m(t,\mathbf{k}_1)\delta_m(t,\mathbf{k}_2)\rangle = f^2\langle\delta^{(1)}(\mathbf{k}_1)\delta^{(1)}(\mathbf{k}_2)\rangle + f^3\langle\delta^{(1)}(\mathbf{k}_1)\delta^{(2)}(\mathbf{k}_2)\rangle + f^4\Big(\langle\delta^{(2)}(\mathbf{k}_1)\delta^{(2)}(\mathbf{k}_2)\rangle + \langle\delta^{(1)}(\mathbf{k}_1)\delta^{(3)}(\mathbf{k}_2)\rangle\Big)$$

$P(\mathbf{k})$ $P_L(\mathbf{k})$ $P^{(22)}(\mathbf{k})$ $P^{(13)}(\mathbf{k})$

1-loop correction



D. Blas, M. Garny, T. Konstandin arXiv:1309.3308

Weak field approximation: shortwave corrections

Metric perturbations remain small on all scales (except close to dense compact objects)

Universe can be described by a perturbed Friedmann metric

S. Green, R. Wald - arXiv:1011.4920, 1111.2997

Deviations of $T_{\mu\nu}$ can be large compared to average quantities $\langle T_{\mu\nu} \rangle$

But metric potentials remain small

Density of earth is 10^{29} times larger than density of the Universe but $\Phi \simeq 10^{-9}$

But space derivatives can be large

$$\Delta\Phi = 4\pi G\rho$$

Scales close to or beyond the horizon are in the linear regime at any time (observations)

Metric perturbations can contain fluctuations of short wavelengths with small amplitude but each spatial derivative is proportional to the inverse length scale

$\Rightarrow$ more important at small scales

Weak field approximation: shortwave corrections

Other directions exist

Cosmological linear theory of perturbations

1946: Lifshitz (SVT decomposition)

1980: Bardeen

1984: Kodama - Sasaki

Second order perturbations

2004: Noh - Hwang (using ADM formalism)

$$
\begin{aligned}
&\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]^{\cdot}+3H\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]-\frac{1}{a^{2}}A^{|\alpha}{}_{\beta}-\frac{1}{3}\delta^{\alpha}_{\beta}\left[\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)^{\cdot}+3H\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)-\frac{1}{a^{2}}A^{|\gamma}{}_{\gamma}\right]\\
&\quad+\frac{1}{a^{2}}\left[C^{\alpha\gamma}{}_{|\beta\gamma}+C^{\gamma}_{\beta}{}^{|\alpha}{}_{\gamma}-C^{\alpha|\gamma}_{\beta}{}_{\gamma}-C^{\gamma|\alpha}_{\gamma}{}_{\beta}-\frac{2}{3}R^{(3)}C^{\alpha}_{\beta}-\frac{1}{3}\delta^{\alpha}_{\beta}\left(2C^{\delta|\gamma}_{\gamma}{}_{\delta}-2C^{\gamma|\delta}_{\gamma}{}_{\delta}-\frac{2}{3}R^{(3)}C^{\gamma}_{\gamma}\right)\right]-8\pi G\Pi^{\alpha}_{\beta}\\
&=\left\{\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]A+2C^{\alpha\gamma}\left(\dot{C}_{\beta\gamma}+\frac{1}{a}B_{(\beta|\gamma)}\right)+\frac{1}{a}B_{\gamma}\left(C^{\alpha\gamma}{}_{|\beta}+C^{\gamma|\alpha}_{\beta}-C^{\alpha|\gamma}_{\beta}\right)\right\}^{\cdot}\\
&\quad+3H\left\{\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]A+2C^{\alpha\gamma}\left(\dot{C}_{\beta\gamma}+\frac{1}{a}B_{(\beta|\gamma)}\right)+\frac{1}{a}B_{\gamma}\left(C^{\alpha\gamma}{}_{|\beta}+C^{\gamma|\alpha}_{\beta}-C^{\alpha|\gamma}_{\beta}\right)\right\}\\
&\quad+\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]^{\cdot}A-\frac{1}{a}\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]_{|\gamma}B^{\gamma}+\delta K\left[\dot{C}^{\alpha}_{\beta}+\frac{1}{2a}\left(B^{\alpha}{}_{|\beta}+B_{\beta}{}^{|\alpha}\right)\right]\\
&\quad+\frac{1}{a^{2}}\left[-AA^{|\alpha}{}_{\beta}+\frac{1}{2}\left(-A^{2}+B^{\gamma}B_{\gamma}\right)^{|\alpha}{}_{\beta}-2C^{\alpha\gamma}A_{,\beta|\gamma}-\left(C^{\alpha\gamma}{}_{|\beta}+C^{\gamma|\alpha}_{\beta}-C^{\alpha|\gamma}_{\beta}\right)A_{,\gamma}\right]\\
&\quad-\frac{1}{3}\delta^{\alpha}_{\beta}\left\{\left[\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)A+2C^{\gamma\delta}\left(\dot{C}_{\gamma\delta}+\frac{1}{a}B_{\gamma|\delta}\right)+\frac{1}{a}B^{\delta}\left(2C^{\gamma}_{\delta|\gamma}-C^{\gamma}_{\gamma|\delta}\right)\right]^{\cdot}\right.\\
&\quad+3H\left[\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)A+2C^{\gamma\delta}\left(\dot{C}_{\gamma\delta}+\frac{1}{a}B_{\gamma|\delta}\right)+\frac{1}{a}B^{\delta}\left(2C^{\gamma}_{\delta|\gamma}-C^{\gamma}_{\gamma|\delta}\right)\right]+\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)^{\cdot}A-\frac{1}{a}\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)_{|\delta}B^{\delta}\\
&\quad\left.+\delta K\left(\dot{C}^{\gamma}_{\gamma}+\frac{1}{a}B^{\gamma}{}_{|\gamma}\right)+\frac{1}{a^{2}}\left[-AA^{|\gamma}{}_{\gamma}+\frac{1}{2}\left(-A^{2}+B^{\delta}B_{\delta}\right)^{|\gamma}{}_{\gamma}-2C^{\gamma\delta}A_{,\gamma|\delta}-\left(2C^{\gamma\delta}{}_{|\gamma}-C^{\gamma|\delta}_{\gamma}\right)A_{,\delta}\right]\right\}\\
&\quad+\frac{1}{a}B^{\alpha}{}_{|\gamma}\left[\dot{C}^{\gamma}_{\beta}+\frac{1}{2a}\left(B^{\gamma}{}_{|\beta}+B_{\beta}{}^{|\gamma}\right)\right]-\frac{1}{a}B^{\gamma}{}_{|\beta}\left[\dot{C}^{\alpha}_{\gamma}+\frac{1}{2a}\left(B^{\alpha}{}_{|\gamma}+B_{\gamma}{}^{|\alpha}\right)\right]\\
&\quad+\frac{1}{a^{2}}\left\{2C^{\gamma\delta}\left(C^{\alpha}_{\delta|\beta\gamma}+C_{\delta\beta}{}^{|\alpha}{}_{\gamma}-C^{\alpha}_{\beta|\delta\gamma}-C_{\delta\gamma}{}^{|\alpha}{}_{\beta}\right)+2C^{\alpha\gamma}\left(C^{\delta}_{\gamma|\beta\delta}+C^{\delta}_{\beta|\gamma\delta}-C_{\beta\gamma}{}^{|\delta}{}_{\delta}-C^{\delta}_{\delta|\gamma\beta}\right)\right.\\
&\quad-\frac{4}{3}R^{(3)}C^{\alpha}_{\gamma}C^{\gamma}_{\beta}+\left(2C^{\gamma}_{\delta|\gamma}-C^{\gamma}_{\gamma|\delta}\right)\left(C^{\alpha\delta}{}_{|\beta}+C^{\delta|\alpha}_{\beta}-C^{\alpha|\delta}_{\beta}\right)-C_{\gamma\delta|\beta}C^{\gamma\delta|\alpha}+2C^{\alpha\gamma|\delta}\left(C_{\beta\delta|\gamma}-C_{\beta\gamma|\delta}\right)-\frac{1}{3}\delta^{\alpha}_{\beta}\left[4C^{\gamma\delta}\left(C^{\epsilon}_{\gamma|\delta\epsilon}+C^{\epsilon}_{\gamma|\epsilon\delta}\right.\right.\\
&\quad\left.\left.\left.-C_{\gamma\delta}{}^{|\epsilon}{}_{\epsilon}-C^{\epsilon}_{\epsilon|\gamma\delta}\right)-\frac{4}{3}R^{(3)}C^{\delta}_{\gamma}C^{\gamma}_{\delta}+\left(2C^{\epsilon}_{\delta|\epsilon}-C^{\epsilon}_{\epsilon|\delta}\right)\left(2C^{\gamma\delta}{}_{|\gamma}-C^{\gamma|\delta}_{\gamma}\right)+C^{\gamma\delta|\epsilon}\left(2C_{\gamma\epsilon|\delta}-3C_{\gamma\delta|\epsilon}\right)\right]\right\}-16\pi GC^{\alpha\gamma}\Pi_{\beta\gamma}\\
&\equiv N_{4}{}^{\alpha}_{\beta}. \qquad (103)
\end{aligned}
$$

Weak field approximation: shortwave corrections

Other directions exist

- Cosmological linear theory of perturbations

  1946: Lifshitz (SVT decomposition)

  1980: Bardeen

  1984: Kodama - Sasaki

- Second order perturbations

  2004: Noh - Hwang (using ADM formalism)

- Third order perturbations

  2005: Noh - Hwang

Expansion is made without discrimination, like post-Newtonian formalism

We need to discriminate the elements of the metric

$$\nabla^2\phi = \frac{3}{2}a^2H^2\delta_m$$

Hubble scale: $k \sim aH \quad \Rightarrow \quad \phi \sim \delta_m \sim \mathcal{O}(10^{-5})$

Small scale: $aH/k \sim \mathcal{O}(10^{-3})$ and $\delta_m \sim \mathcal{O}(1) \quad \Rightarrow \quad \phi \sim \mathcal{O}(10^{-5})$

$$v^i = -\frac{\partial_i\phi}{aH}$$

Small scale: $v^i \sim \mathcal{O}(10^{-3})$

We perform a perturbative expansion in $\mathbf{aH/k}$ while keeping all orders in δ_m

Weak field approximation: shortwave corrections

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2\Big[(1-2\psi)\delta_{ij} + h_{ij}\Big]dx^i dx^j$$

$$\omega_i = \partial_i \omega + w_i\,, \quad \partial_i w_i = 0$$

$$T_{\mu\nu} = \bar{\rho}_m(1+\delta_m)u_\mu u_\nu$$

$$\delta^{ij}h_{ij} = \delta^{jk}\partial_k h_{ij} = 0$$

Poisson gauge
$\omega = 0$

C-gauge
$u^0 = 1$

Variable	Order in Poisson gauge	Order in C-gauge
∂_i/H	$\mathcal{O}(k/aH)$	$\mathcal{O}(k/aH)$
∂_t/H	$\mathcal{O}(1)$	$\mathcal{O}(1)$
ϕ	$\mathcal{O}(a^2H^2/k^2)$	$\mathcal{O}(a^2H^2/k^2)$
ψ	$\mathcal{O}(a^2H^2/k^2)$	$\mathcal{O}(a^2H^2/k^2)$
w_i	$\mathcal{O}(a^3H^3/k^3)$	$\mathcal{O}(a^3H^3/k^3)$
ω	-	$\mathcal{O}(aH/k)$
h_{ij}	$\mathcal{O}(a^4H^4/k^4)$	$\mathcal{O}(a^4H^4/k^4)$
δ_m	$\mathcal{O}(1)$	$\mathcal{O}(1)$
v^i	$\mathcal{O}(aH/k)$	$\mathcal{O}(aH/k)$

Poisson gauge

$$\dot{\delta}_m + \partial_i\left[(1+\delta_m)v^i\right] = \dot{\delta}_m\left(\phi - \frac{a^2v^2}{2}\right) - (1+\delta_m)\,\partial_t\left(\frac{a^2v^2}{2}\right) + 3(1+\delta_m)\dot{\phi} + 2\phi_{,i}(1+\delta_m)v^i$$

$$\dot{\theta} + 2H\theta + \partial_i(v^i\partial_j v^j) + \frac{\nabla^2\phi}{a^2} = \partial_i\left[\left(-\phi + \frac{a^2v^2}{2}\right)\dot{v}^i + 2\left(\frac{a^2v^2}{2}H - H\phi - \dot{\phi}\right)v^i + 2(\phi_{,i}v^2 - v^j\phi_{,j}v^i)\right]$$

$$\frac{2}{a^2}\nabla^2\phi(1-2\phi) - 6\frac{\ddot{a}}{a} + 6H(3\dot{\phi} - 2H\phi) + 6\ddot{\phi} - 4\phi_{,i}^2 = \bar{\rho}_m(1+\delta_m)(1-2\phi+2a^2v^2)$$

In the Newtonian limit, we recover

$$\dot{\delta}_N + \partial_i\left[(1+\delta_N)v_N^i\right] = 0$$

$$\dot{\theta}_N + 2H\theta_N + \partial_i(v_N^i\partial_j v_N^j) + \frac{3}{2}H^2\delta_N = 0$$

$$\delta_m = \delta_N + \delta_R$$

$$v^i = v_N^i + v_R^i$$

Relativistic

Newtonian

Poisson gauge

$$\dot{\delta}_R + \theta_R = -\int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} \delta_D(\mathbf{k} - \mathbf{k}_{12}) \alpha(\mathbf{k}_1, \mathbf{k}_2)(\theta_R(\mathbf{k}_1)\delta_N(\mathbf{k}_2) + \theta_N(\mathbf{k}_1)\delta_R(\mathbf{k}_2)) + \mathcal{S}_\delta[\phi_N, \delta_N, \theta_N]$$

$$\dot{\theta}_R + 2H\theta_R + \frac{3}{2}H^2\delta_R = -2\int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} \delta_D(\mathbf{k} - \mathbf{k}_{12}) \beta(\mathbf{k}_1, \mathbf{k}_2)\theta_N(\mathbf{k}_1)\theta_R(\mathbf{k}_2) + \mathcal{S}_\theta[\phi_N, \delta_N, \theta_N]$$

$$\alpha(\mathbf{k}_1, \mathbf{k}_2) = \frac{\mathbf{k}_{12} \cdot \mathbf{k}_1}{\mathbf{k}_1^2}$$

$$\beta(\mathbf{k}_1, \mathbf{k}_2) = \frac{\mathbf{k}_{12}^2(\mathbf{k}_1 \cdot \mathbf{k}_2)}{2\mathbf{k}_1^2\mathbf{k}_2^2}$$

$$\mathbf{k}_{12} = \mathbf{k}_1 + \mathbf{k}_2$$

$$\ddot{\delta}_R + 2H\dot{\delta}_R - \frac{3}{2}H^2\delta_R = S$$

newtonian

relativistic

Which can be solved using SPT

We need to consider initial conditions, up to second order, calculated in

A. Fitzpatrick, L. Senatore, M. Zaldarriaga arXiv: 0902.2814

$$\delta_m(\mathbf{k},t) = \sum_{n=1}^{\infty} a^n \int_{\mathbf{k}_1\ldots\mathbf{k}_n} (2\pi)^3 \delta_D(\mathbf{k}-\mathbf{k}_{1\ldots n}) \Big[F_n(\mathbf{k}_1,\ldots,\mathbf{k}_n) + a^2H^2F_n^R(\mathbf{k}_1,\ldots,\mathbf{k}_n)\Big]\delta_l(\mathbf{k}_1)\ldots\delta_l(\mathbf{k}_n)$$

$$\theta(\mathbf{k},t) = -H\sum_{n=1}^{\infty} a^n \int_{\mathbf{k}_1\ldots\mathbf{k}_n} (2\pi)^3 \delta_D(\mathbf{k}-\mathbf{k}_{1\ldots n}) \Big[G_n(\mathbf{k}_1,\ldots,\mathbf{k}_n) + a^2H^2G_n^R(\mathbf{k}_1,\ldots,\mathbf{k}_n)\Big]\delta_l(\mathbf{k}_1)\ldots\delta_l(\mathbf{k}_n)$$

$$F_1^R(\mathbf{k}) = \frac{3}{\mathbf{k}^2}$$

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7}\alpha(\mathbf{k}_1,\mathbf{k}_2) + \frac{2}{7}\beta(\mathbf{k}_1,\mathbf{k}_2)$$

$$F_2^R(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{4\mathbf{k}_1^2\mathbf{k}_2^2}\left[\left(\frac{59(\mathbf{k}_1+\mathbf{k}_2)^2}{14} - \frac{125}{14}(\mathbf{k}_1^2+\mathbf{k}_2^2) - \frac{9(\mathbf{k}_1^2-\mathbf{k}_2^2)^2}{7(\mathbf{k}_1+\mathbf{k}_2)^2}\right)\right]$$

$$G_2(\mathbf{k}_1,\mathbf{k}_2) = 2F_2(\mathbf{k}_1,\mathbf{k}_2) - \alpha(\mathbf{k}_1,\mathbf{k}_2) = \frac{3}{7}\alpha(\mathbf{k}_1,\mathbf{k}_2) + \frac{4}{7}\beta(\mathbf{k}_1,\mathbf{k}_2)$$

$$G_2^R(\mathbf{k}_1,\mathbf{k}_2) = F_2^R(\mathbf{k}_1,\mathbf{k}_2) + \frac{9}{2}\frac{F_2(\mathbf{k}_1,\mathbf{k}_2)}{(\mathbf{k}_1+\mathbf{k}_2)^2} - \frac{13}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2\mathbf{k}_2^2} - \frac{3}{4\mathbf{k}_1^2} - \frac{3}{4\mathbf{k}_2^2}$$

$$F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{1}{18}\left[7F_2(\mathbf{k}_1,\mathbf{k}_2)\alpha(\mathbf{k}_3,\mathbf{k}_{12}) + 7G_2(\mathbf{k}_1,\mathbf{k}_2)\alpha(\mathbf{k}_{12},\mathbf{k}_3) + 4G_2(\mathbf{k}_1,\mathbf{k}_2)\beta(\mathbf{k}_3,\mathbf{k}_{12})\right]$$

$$F_3^R(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{1}{14}\Bigg\{18\frac{F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)^2} + [52+3\alpha(\mathbf{k}_3,\mathbf{k}_{12})]\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2\mathbf{k}_2^2} + G_2^R(\mathbf{k}_1,\mathbf{k}_2)\left[10\alpha(\mathbf{k}_{12},\mathbf{k}_3) + 8\beta(\mathbf{k}_{12},\mathbf{k}_3)\right]$$
$$+10F_2^R(\mathbf{k}_1,\mathbf{k}_2)\alpha(\mathbf{k}_3,\mathbf{k}_{12}) + F_2(\mathbf{k}_1,\mathbf{k}_2)\left[-\frac{30}{\mathbf{k}_3^2} - \frac{81}{\mathbf{k}_{12}^2} - 75\frac{\mathbf{k}_3\cdot\mathbf{k}_{12}}{\mathbf{k}_3^2\mathbf{k}_{12}^2}\right] + G_2(\mathbf{k}_1,\mathbf{k}_2)\left[65\frac{\mathbf{k}_3\cdot\mathbf{k}_{12}}{\mathbf{k}_3^2\mathbf{k}_{12}^2} + \frac{15}{\mathbf{k}_3^2}\right]\Bigg\}$$

Kernels in Poisson gauge

$$
\begin{aligned}
F_4^R(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \frac{1}{36}\Bigg(& 36\frac{F_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)}{\mathbf{k}_{1234}^2} + F_2(\mathbf{k}_1,\mathbf{k}_2)G_2(\mathbf{k}_3,\mathbf{k}_4)\left[-33\frac{\mathbf{k}_{12}\cdot\mathbf{k}_{34}}{\mathbf{k}_{12}^2\mathbf{k}_{34}^2} - \frac{27}{\mathbf{k}_{12}^2}\right] \\
& + F_2(\mathbf{k}_1,\mathbf{k}_2)F_2(\mathbf{k}_3,\mathbf{k}_4)\left[-18\frac{\mathbf{k}_{12}\cdot\mathbf{k}_{34}}{\mathbf{k}_{12}^2\mathbf{k}_{34}^2} - \frac{105}{\mathbf{k}_{12}^2}\right] + 33G_2(\mathbf{k}_1,\mathbf{k}_2)G_2(\mathbf{k}_3,\mathbf{k}_4)\frac{\mathbf{k}_{12}\cdot\mathbf{k}_{34}}{\mathbf{k}_{12}^2\mathbf{k}_{34}^2} \\
& + F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\left[-99\frac{\mathbf{k}_{123}\cdot\mathbf{k}_4}{\mathbf{k}_{123}^2\mathbf{k}_4^2} - \frac{180}{\mathbf{k}_{123}^2} - \frac{63}{\mathbf{k}_4^2}\right] + G_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\left[105\frac{\mathbf{k}_{123}\cdot\mathbf{k}_4}{\mathbf{k}_{123}^2\mathbf{k}_4^2} + \frac{21}{\mathbf{k}_4^2}\right] \\
& + 14\alpha(\mathbf{k}_{12},\mathbf{k}_{34})\left[G_2^R(\mathbf{k}_1,\mathbf{k}_2)F_2(\mathbf{k}_3,\mathbf{k}_4) + G_2(\mathbf{k}_1,\mathbf{k}_2)F_2^R(\mathbf{k}_3,\mathbf{k}_4)\right] \\
& + G_3^R(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\left[14\alpha(\mathbf{k}_{123},\mathbf{k}_4) + 8\beta(\mathbf{k}_{123},\mathbf{k}_4)\right] \\
& + 8G_2^R(\mathbf{k}_1,\mathbf{k}_2)G_2(\mathbf{k}_3,\mathbf{k}_4)\beta(\mathbf{k}_{12},\mathbf{k}_{34}) + 14F_3^R(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\alpha(\mathbf{k}_4,\mathbf{k}_{123}) \\
& + F_2(\mathbf{k}_3,\mathbf{k}_4)\left\{\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2\mathbf{k}_2^2}\left[75 + 12\alpha(\mathbf{k}_{34},\mathbf{k}_{12})\right] + \frac{\mathbf{k}_1\cdot\mathbf{k}_{34}}{\mathbf{k}_1^2\mathbf{k}_{34}^2}\left[42 - 12\alpha(\mathbf{k}_2,\mathbf{k}_{134})\right]\right\} \\
& + G_2(\mathbf{k}_3,\mathbf{k}_4)\left\{\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2\mathbf{k}_2^2}\left[-7\alpha(\mathbf{k}_{34},\mathbf{k}_{12})\right] + \frac{\mathbf{k}_1\cdot\mathbf{k}_{34}}{\mathbf{k}_1^2\mathbf{k}_{34}^2}\left[33 + 15\alpha(\mathbf{k}_2,\mathbf{k}_{134})\right]\right. \\
& \left. + \frac{\mathbf{k}_2\cdot\mathbf{k}_{34}}{\mathbf{k}_2^2\mathbf{k}_{34}^2}\left[75 + 3\alpha(\mathbf{k}_1,\mathbf{k}_{234})\right]\right\}\Bigg) .
\end{aligned}
$$

$$\langle \delta(\mathbf{k}_1, t)\delta(\mathbf{k}_2, t)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P(k_1, t)$$

$$P_{\text{1-loop}}(\mathbf{k}, t) = a^4(t)\left(P_{13}(\mathbf{k}) + P_{22}(\mathbf{k})\right)$$

$$P^R_{\text{1-loop}}(\mathbf{k}, t) = H_0^2 a^3(t)\left(P^R_{13}(\mathbf{k}) + P^R_{22}(\mathbf{k})\right)$$

Newtonian contribution

$$P_{13}(\mathbf{k}) = 6 P_L(k) \int_{\mathbf{q}} P_L(q) F_3(\mathbf{q}, -\mathbf{q}, \mathbf{k})$$

$$P_{22}(\mathbf{k}) = 2 \int_{\mathbf{q}} F_2^2(\mathbf{q}, |\mathbf{k} - \mathbf{q}|) P_L(q) P_L(|\mathbf{k} - \mathbf{q}|)$$

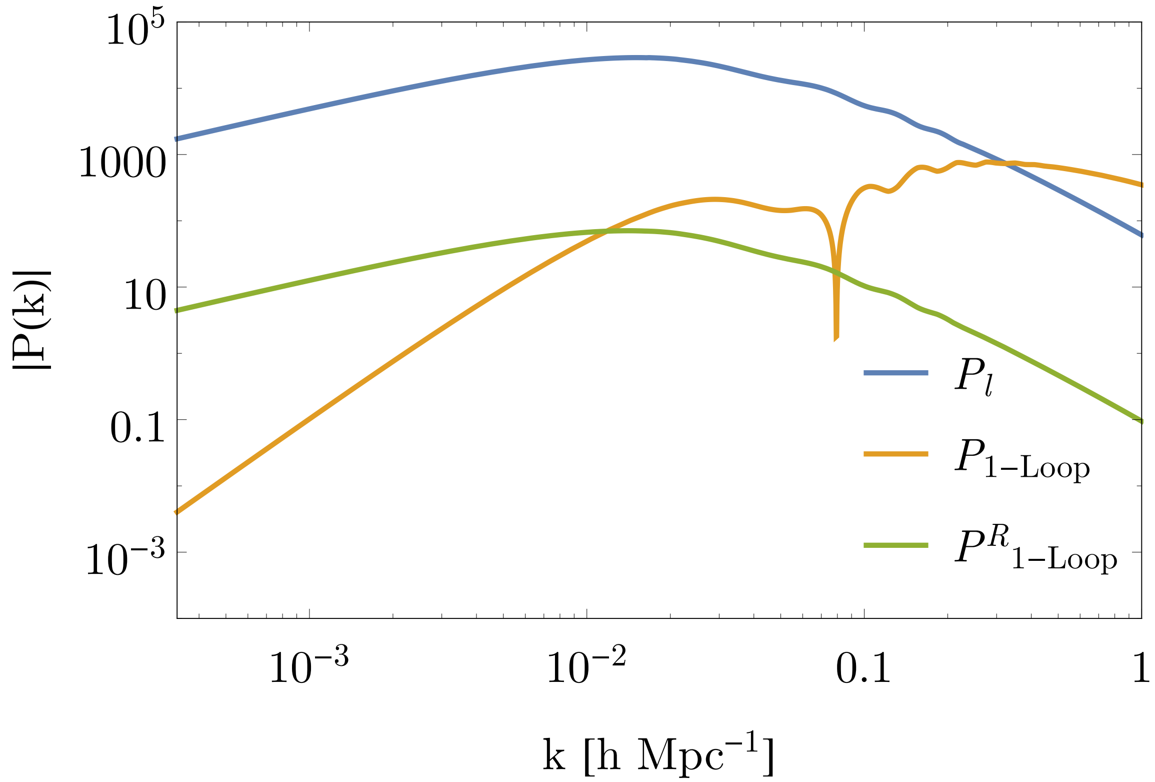
Relativistic contribution

$$P^R_{13}(\mathbf{k}) = 6 P_L(k) \int_{\mathbf{q}} P_L(q)\left[F_3^R(\mathbf{q}, -\mathbf{q}, \mathbf{k}) + F_1^R(k) F_3(\mathbf{q}, -\mathbf{q}, \mathbf{k})\right]$$

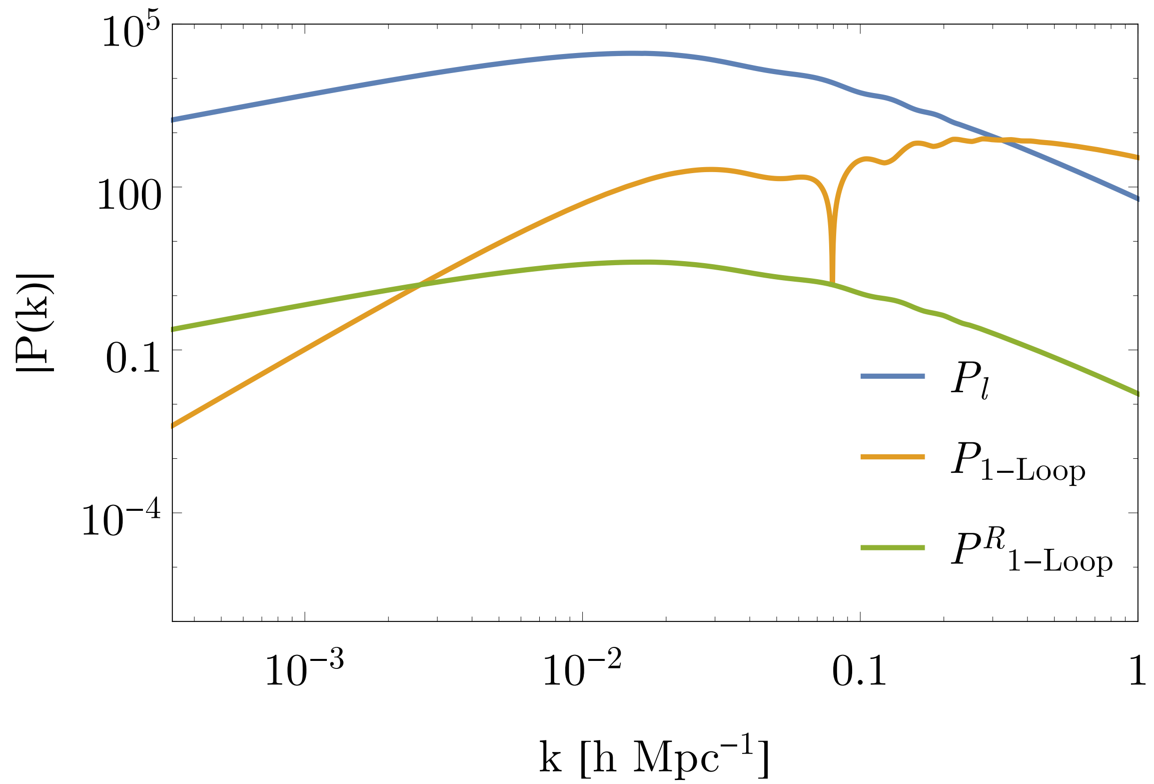
$$P^R_{22}(\mathbf{k}) = 4 \int_{\mathbf{q}} F_2(\mathbf{q}, |\mathbf{k} - \mathbf{q}|) F_2^R(\mathbf{q}, |\mathbf{k} - \mathbf{q}|) P_L(q) P_L(|\mathbf{k} - \mathbf{q}|)$$

One loop power spectrum

Poisson gauge



C-gauge



$$\langle \delta(\mathbf{k}_1, t)\delta(\mathbf{k}_2, t)\delta(\mathbf{k}_3, t)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3, t)$$

Newtonian tree level

$$B_{211}(k_1, k_2, k_3, t) = a^4(t) \left[F_2(\mathbf{k}_1, \mathbf{k}_2) P_L(k_1) P_L(k_2) + 2 \text{ cyclic permutations}\right]$$

Relativistic corrections to the tree level

$$B^R_{211}(k_1, k_2, k_3, t) = a^3(t) H_0^2 \left[2F_2^R(\mathbf{k}_1, \mathbf{k}_2) P_L(k_1) P_L(k_2) + 2F_2(\mathbf{k}_1, \mathbf{k}_2) F_1^R(k_1) P_L(k_1) P_L(k_2) \right.$$
$$\left. + 2F_2(\mathbf{k}_1, \mathbf{k}_2) F_1^R(k_2) P_L(k_1) P_L(k_2) + 2 \text{ cyclic permutations}\right]$$

Newtonian 1-loop bispectrum

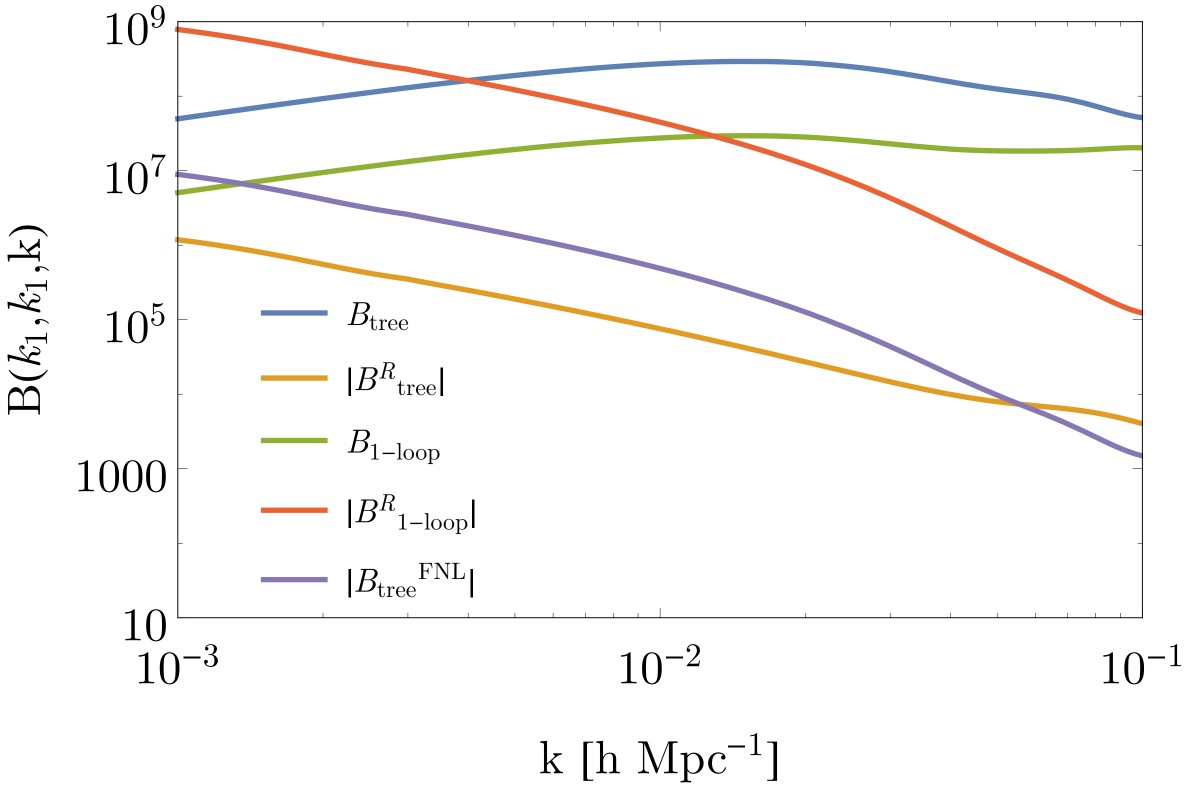
$$B_{\text{1-loop}}(k_1, k_2, k_3, t) = a^6(t) \left[B_{222}(k_1, k_2, k_3) + B^I_{321}(k_1, k_2, k_3) + B^{II}_{321}(k_1, k_2, k_3) \right.$$
$$\left. + B_{411}(k_1, k_2, k_3)\right]$$

Relativistic 1-loop bispectrum

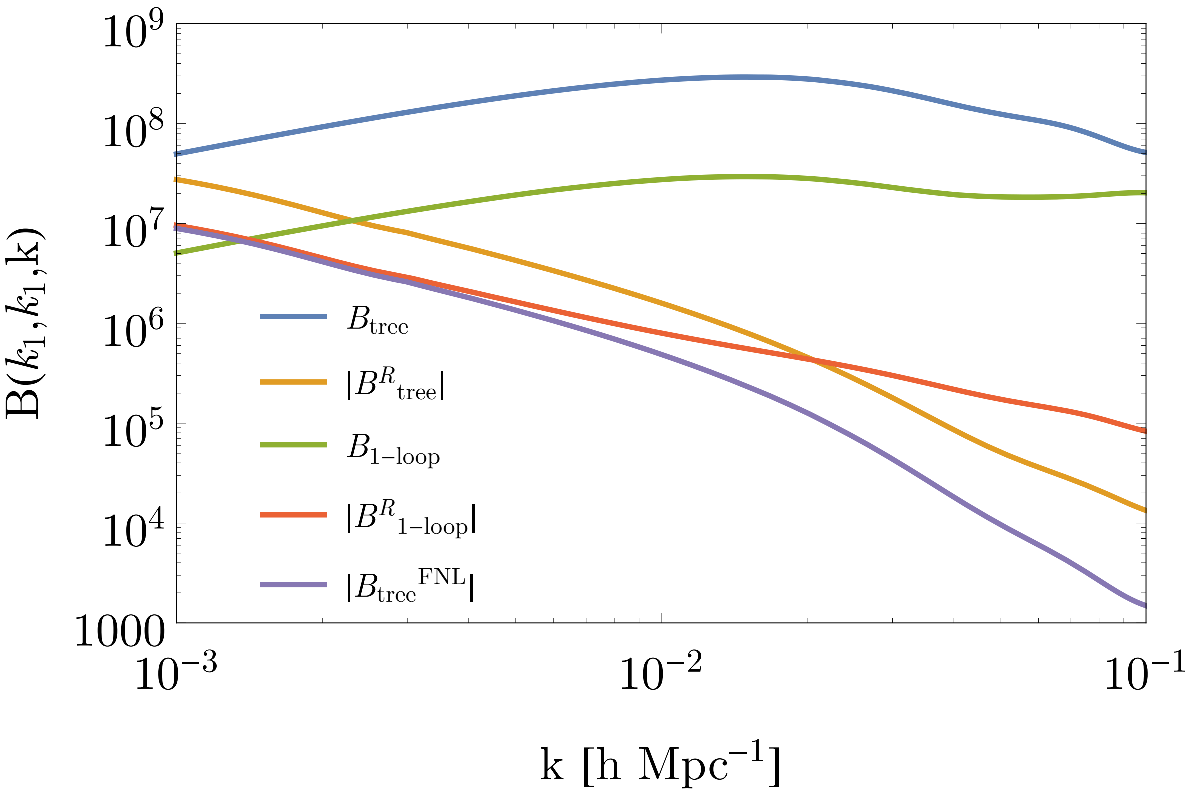
$$B^R_{\text{1-loop}}(k_1, k_2, k_3, t) = H_0^2 a^5(t) \left[B^R_{222}(k_1, k_2, k_3) + B^{I,R}_{321}(k_1, k_2, k_3) + B^{II,R}_{321}(k_1, k_2, k_3) \right.$$
$$\left. + B^R_{411}(k_1, k_2, k_3)\right]$$

One loop bispectrum

Poisson gauge

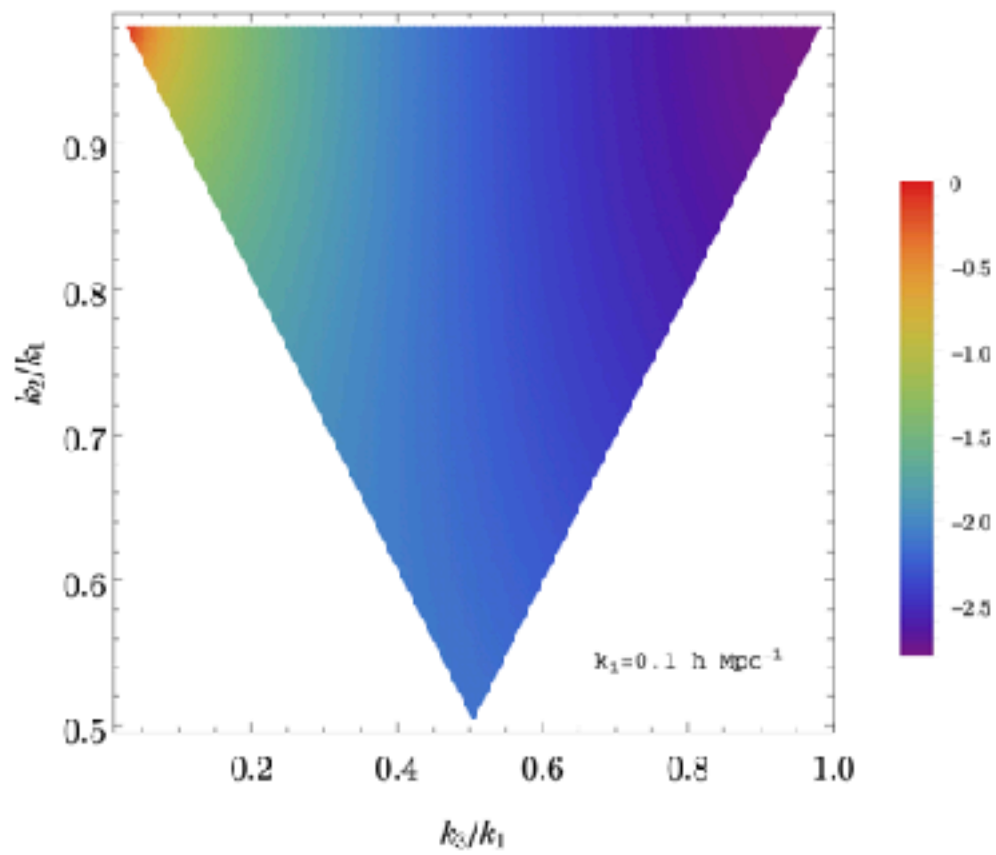


C-gauge

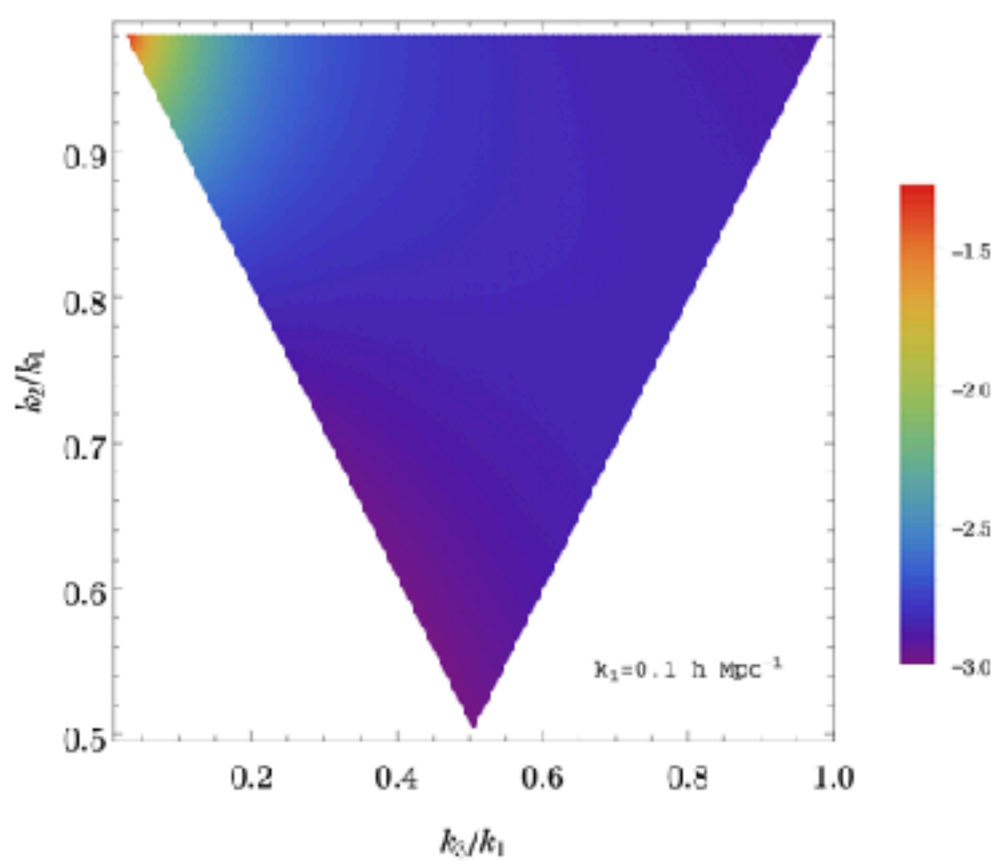


One loop bispectrum

Poisson gauge



C-gauge



Last minute comments

Renormalization of the background

$$\langle \delta_m \rangle = 0\,, \quad \langle \theta \rangle = 0\,, \quad \langle \phi \rangle = 0$$

L. Boubekeur, P. Creminelli, J. Norena, F. Vernizzi
arXiv:0806.1016

This is true in **newtonian** approach but not realized in the **relativistic** case

$$\rho_m \to \bar{\rho}_m(1 + \langle \delta_m \rangle)$$
$$P \to 0 + P$$

D. Baumann, A. Nicolis, L. Senatore, M. Zaldarriaga
arXiv:1004.2488

It modifies the Hubble parameter $H(t)$ by $\mathcal{O}(10^{-5})$

IR behavior

Loop integrals can depend on the IR cutoff chosen

In newtonian case, the divergences cancel each other, not for relativistic corrections

Actual observations have a limited resolution

$\Rightarrow$ All averages are taken with resolution of the largest scale measured

But the effect is weak

UV behavior

Fluid approach breaks at very small scales: shell-crossing

$\Rightarrow$ additional physics as an effective fluid which produce counterterms to renormalize this cutoff

Effective Field Theory of Large Scale Structure blah, blah, blah…

arXiv:1811.05452, 1912.13034

Thank you