# *Black Hole Remnants*

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## -Mimetic gravity

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{8\pi G}R + \lambda\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 1\right)\right),$$

- ***Constraint***

$$g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} = 1,$$

- ***Synchronous coordinate system***

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \gamma_{ik}\mathrm{d}x^i\mathrm{d}x^k$$

$$\phi = t$$

***Extrinsic curvature of hypersurfaces*** **$t = const$**

$$\kappa_{ik} = \frac{1}{2}\frac{\partial}{\partial t}\gamma_{ik} = -\phi_{;ik}$$

***and***

$$\Box\phi = g^{\alpha\beta}\phi_{;\alpha\beta} = \gamma^{ik}\kappa_{ik} = \kappa = \frac{\partial}{\partial t}\ln\sqrt{\gamma},$$

- **Action for modified mimetic gravity**

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(-f\left(\Box\phi\right)R - 2\Lambda\left(\Box\phi\right) + \lambda\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 1\right) + 2\mathcal{L}_m\right)$$

where

$$f\left(\Box\phi\right) = \frac{1}{8\pi G\left(\Box\phi\right)}$$

## *"Asymptotic freedom"*

$$G \, \alpha \frac{1}{f} \rightarrow 0 \quad \textit{when} \quad \Box\phi = k \rightarrow k_0$$

$$f = \frac{1}{1 - (\kappa^2/\kappa_0^2)},$$

- **Friedmann Universe**

$$ds^2 = dt^2 - a^2\,(t)\,\delta_{ik} dx^i dx^k$$

$$\frac{1}{3}\kappa^2 \boxed{\left(\frac{1 + 2\,(\kappa^2/\kappa_0^2)}{1 - (\kappa^2/\kappa_0^2)}\right)} = \varepsilon \quad \textit{where} \quad \kappa = 3\frac{\dot{a}}{a}$$

$$a \propto t^{\frac{2}{3(1+w)}}$$ ***for*** $\kappa^2/\kappa_0^2 \ll 1$

$$a \propto \exp\left(-\frac{\kappa_0 t}{3}\right)$$ ***when*** $\kappa^2/\kappa_0^2 \rightarrow \mathbf{1}$

- **Kasner Universe**

$$\gamma_{ik} = \gamma_{(i)}(t)\,\delta_{ik}$$ ***where*** $\gamma_{(i)} = \left(\frac{3}{2}\bar{\lambda}^2\right)^{1/3} t^{2p_i},$

***and*** $p_1 + p_2 + p_3 = 1,\qquad p_1^2 + p_2^2 + p_3^2 = 1$ ***for*** $\kappa^2/\kappa_0^2 \ll 1$

$$\gamma_{(1)} = \gamma_{(2)} = \gamma_{(3)} = \gamma^{1/3} \propto \exp\left(-\frac{2}{3}\kappa_0 t\right)$$ ***when*** $\kappa^2/\kappa_0^2 \rightarrow \mathbf{1}$

## Quantum fluctuations

$$\delta h\left(l\right) \simeq \frac{1}{\sqrt{f}l} \simeq \frac{\sqrt{G\left(\kappa\right)}}{l}$$

Hence, quantum fluctuations in a given physical scale $l \ll \kappa_0^{-1}$ vanish as $\kappa \rightarrow \kappa_0$ and correspondingly $G\left(\kappa\right) \rightarrow 0$.

# Black Hole Remnants

$$\mathrm{d}s^2 = \left(1 - a^2(r)\right)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{\left(1 - a^2(r)\right)} - r^2\mathrm{d}\Omega^2$$

***where*** $a^2(r) = r_g/r$ ***for*** ***BH***

***and*** $a^2 = (Hr)^2$ ***for*** ***de Sitter.***

*The Lemaître coordinates*

$$T = t + \int \frac{a}{1-a^2}\mathrm{d}r, \quad R = t + \int \frac{\mathrm{d}r}{a\left(1-a^2\right)}$$

$$\mathrm{d}s^2 = \mathrm{d}T^2 - a^2(x)\,\mathrm{d}R^2 - b^2(x)\mathrm{d}\Omega^2 \quad \textit{where} \quad x \equiv R - T$$

# *BH*

$$\mathrm{d}s^2 = \mathrm{d}T^2 - (x/x_+)^{-2/3}\,\mathrm{d}R^2 - (x/x_+)^{4/3}\,r_g^2\,\mathrm{d}\Omega^2,$$

and it is regular at the horizon $x = x_+ = 4M/3$.
region $x > 0$ covers both interior and exterior of the black

# *de Sitter*

$$\mathrm{d}s^2 = \mathrm{d}T^2 - \exp\left(2H(x - x_-)\right)\left(\mathrm{d}R^2 + H^{-2}\mathrm{d}\Omega^2\right)$$

where $x_-$ is a constant of integration in (4) and the de Sitter horizon occurs at $x = x_-$. The region $x < x_-$ corresponds to the patch of size $r = H^{-1}$ covered by static coordinates, which on larger scales do not exist.

## *Asymptotically free mimetic gravity with*

$$f(\tilde{\kappa}) = \frac{1+3\tilde{\kappa}^2}{\left(1+\tilde{\kappa}^2\right)\left(1-\tilde{\kappa}^2\right)^2}, \quad \text{where } \tilde{\kappa} \equiv \kappa/\kappa_0$$

***Exact solution***

$$\mathrm{d}s^2 = \mathrm{d}T^2 - a^2(x)\,\mathrm{d}R^2 - b^2(x)\mathrm{d}\Omega^2$$

***where***

$$a^3(\tilde{\kappa}) = \frac{4M\kappa_0}{3}\,|\tilde{\kappa}|\left(1-\tilde{\kappa}^4\right)\left(\frac{1+\tilde{\kappa}^2}{1+3\tilde{\kappa}^2}\right)^2,$$

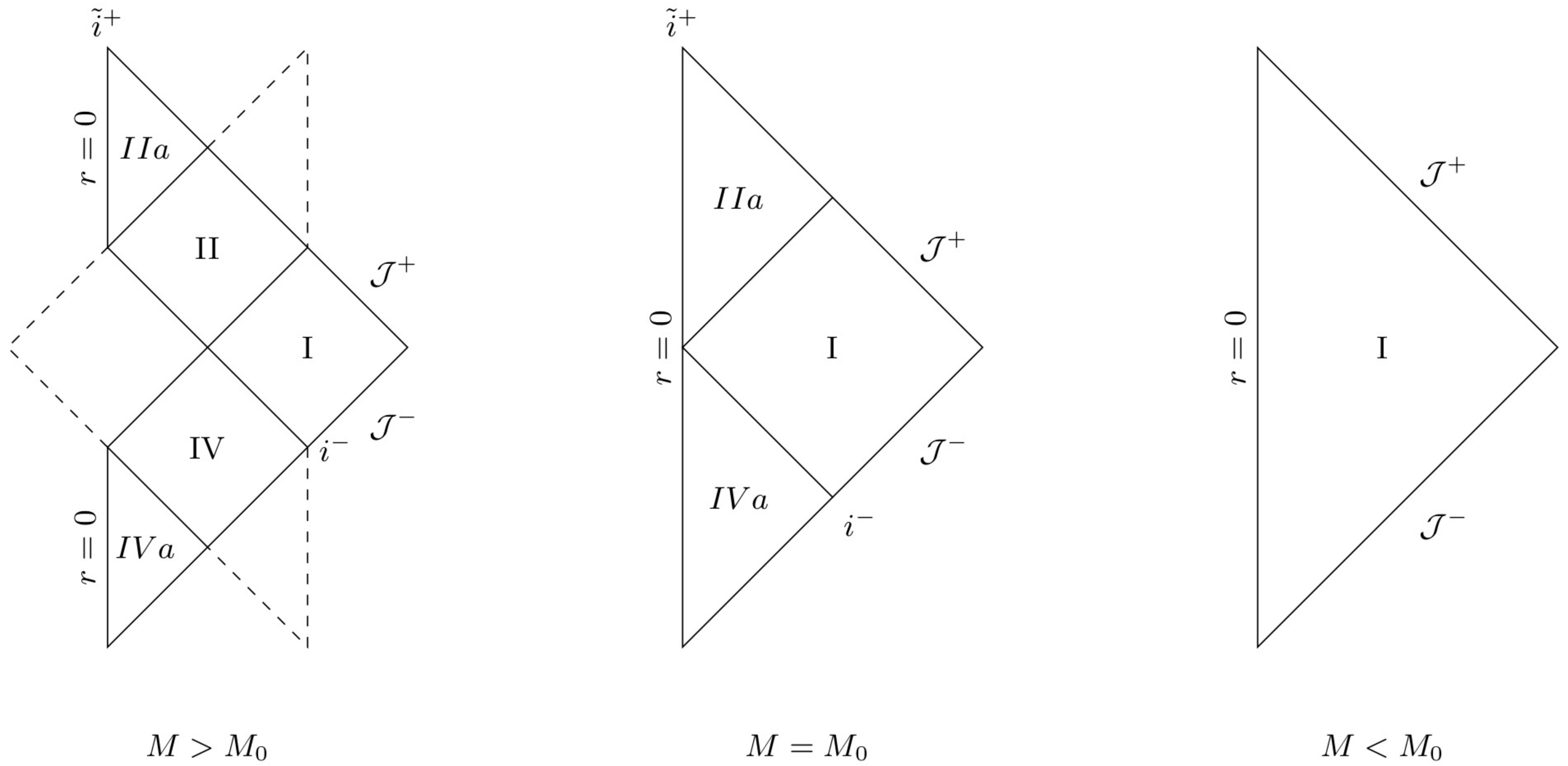
$$b^3(\tilde{\kappa}) = \frac{9M}{2\kappa_0^2\tilde{\kappa}^2}\left(1-\tilde{\kappa}^2\right)\left(1+3\tilde{\kappa}^2\right).$$

$$-\kappa_0\,x = \frac{1}{\tilde{\kappa}} + 2\left(\arctan\tilde{\kappa} - \tanh^{-1}\tilde{\kappa}\right).$$

***For*** $x \rightarrow +\infty$ ***we have BH***

***When*** $x \rightarrow -\infty$ ***we obtain static de Sitter patch***

***Horizons exist only if*** $$M \geq M_{\min} = \frac{5^{5/2}}{18\kappa_0}.$$



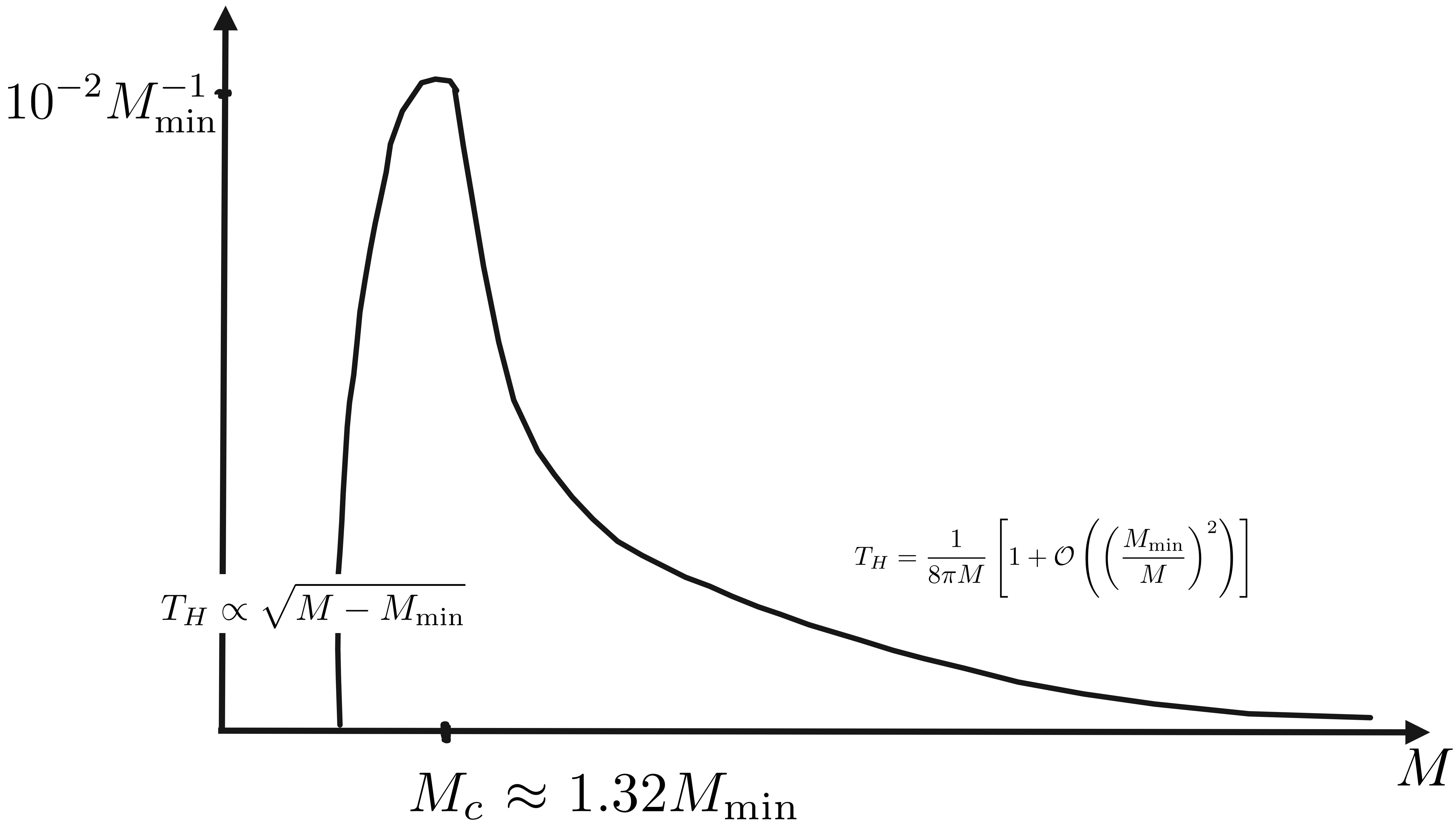
***Near horizon metric for*** $M = M_{\min}$

$$ds^2 \approx \frac{10}{7}\left(1-\frac{r}{r_*}\right)^2 dt^2 - \frac{\mathrm{d}r^2}{\frac{10}{7}\left(1-\frac{r}{r_*}\right)^2} - r^2 \mathrm{d}\Omega^2$$

***where*** $r_* = \left(\frac{6}{5}\right)^2 M_{\min}$

# *Black hole thermodynamics*

$$T_H = \frac{g_s}{2\pi} = \frac{\kappa_0}{6\pi}\,|\tilde{\kappa}_+|\,\frac{1-5\tilde{\kappa}_+^2}{1+3\tilde{\kappa}_+^2}, \qquad M = \frac{3}{4\kappa_0\,|\tilde{\kappa}_+|\left(1-\tilde{\kappa}_+^4\right)}\left(\frac{1+3\tilde{\kappa}_+^2}{1+\tilde{\kappa}_+^2}\right)^2 .$$




$$M(t) - M_{\min} \propto t^{-1}$$