



# *Andrei Sakharov* *and Cosmological Baryogenesis*

*A critical view after more than half century of the seminal paper*

COSMOS & CONTEXTO

· SAKHAROV: FÍSICO TEÓRICO OU INVENTOR? ·

Ignacio Alfonso de Bediaga e Hickman / 26 de abril de 2019

***Ignacio Bediaga: Centro Brasileiro de Pesquisas Físicas.***

***Mário Novello's 80th Anniversary Symposium***

***September 12 Centro Brasileiro de Pesquisas Físicas.***

# *Sakharov conditions*

It is traditional to start any discussion of baryogenesis with the list of three necessary ingredients needed to create a baryon asymmetry :

- 1- Baryonic number violation.**
- 2- Loss of thermal equilibrium**
- 3- C and CP violation.**



Although these principles have come to be attributed to Sakharov, he did not enunciate them as clearly in his three-page paper as one might have been led to think, especially the second point.

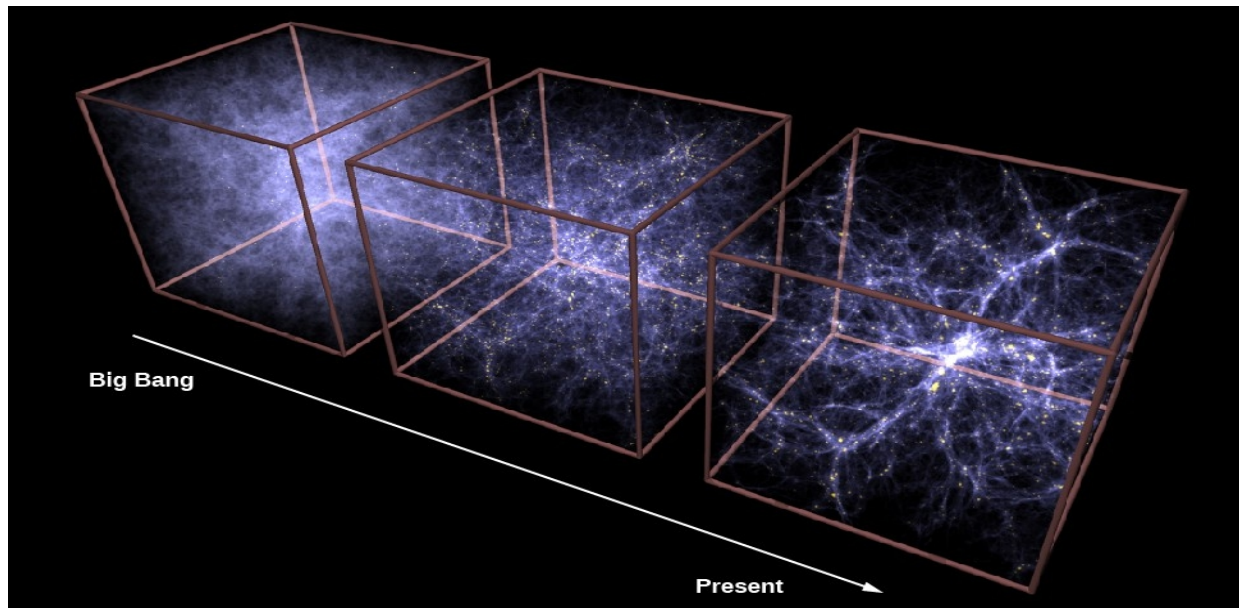
James M. Cline

# *Cold Universe Expansion*

Based in the Zeldovich Cold Universe:

“The initial stage of an expanding Universe and the appearance of a nonuniform distribution of matter” Soviet Physics JETP, Vol. 22, p.241.

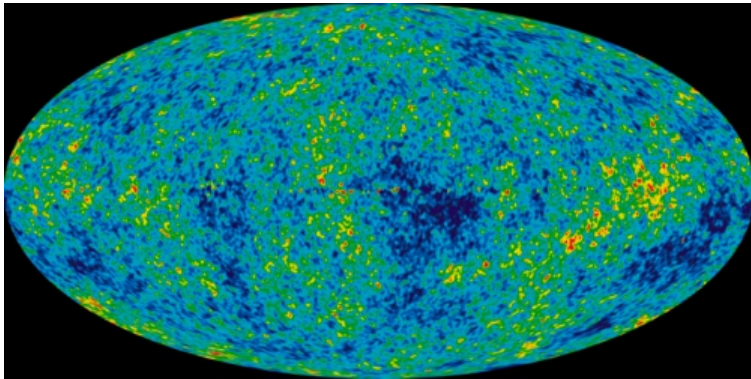
Initial state super-dense matter at  $-273^{\circ}\text{C}$  with a non uniform matter distribution, with a state equations arrive to a non-uniform mass distribution.



# *Cosmic Radiation Background*



Arno Penzias and Robert Wilson



Uniform distribution of the radiation  
2.7248 to 2.7252 Kelvin variation of 0.01%

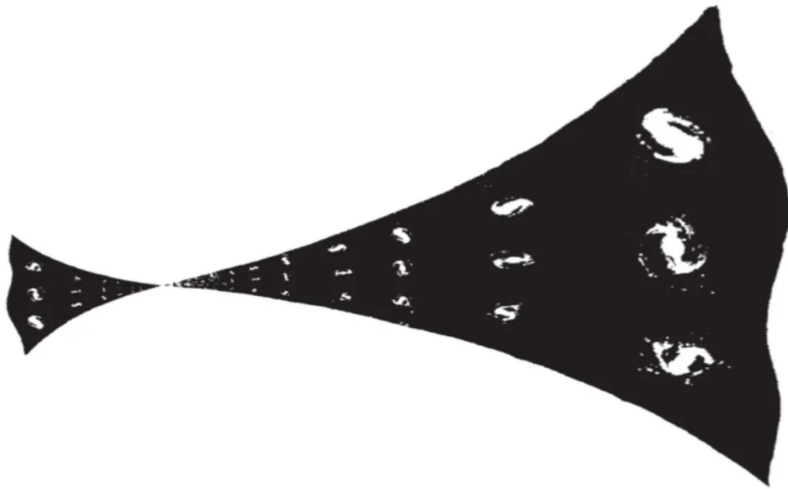
# Hot Universe Expansion

Based in the Gamow/Zeldovich Hot Universe:

‘Violation of CP invariance, C asymmetry, and baryon asymmetry of the Universe’,  
Pisma Zh. Eksp. Teor. Fiz. 5, 32-35, 1967)

Sakharov describes a scenario where a universe which was initially contracting and with equal and opposite baryon asymmetry to that existing today goes through a bounce at the singularity and reverses the magnitude of its baryon asymmetry

**This scenario kept the CPT Invariance**



The world of Andrei Sakharov:  
a Russian physicist's path to  
freedom, de Gennady Gorelik.  
Oxford University Press (2005).

$T < 0$  anti-matter Universe

$T = 0$  Bounce with “Maximum” particles X

Massa  $2 \times 10^{-5} \text{g}$  or  $10^{19}$  proton mass

Possible only out of the thermodynamical  
equilibrium

Maximum: Double baryon number violation

$T > 0$  matter Universe

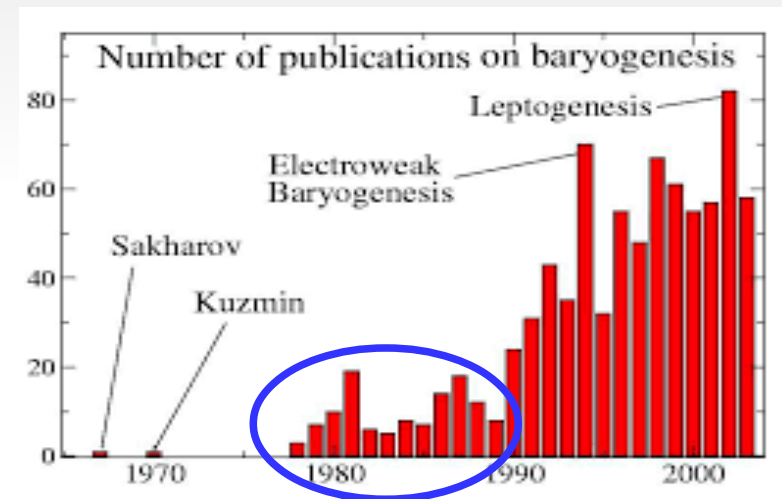
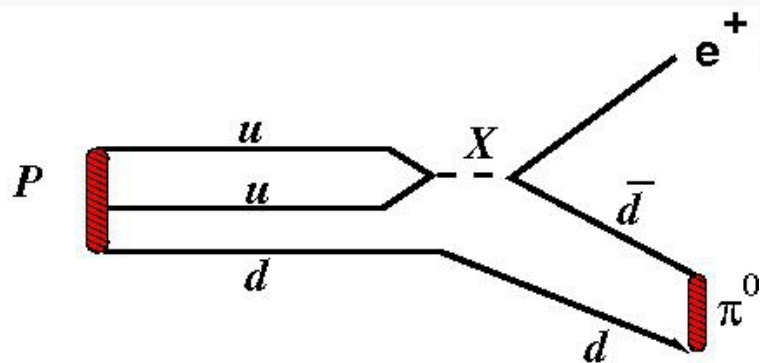


# Baryonic violation problem

In 1955, Lee and Yang discussed a new massless gauge field based on the established conservation of baryon number.

They predicted the existence of a repulsive force between baryonic matter.

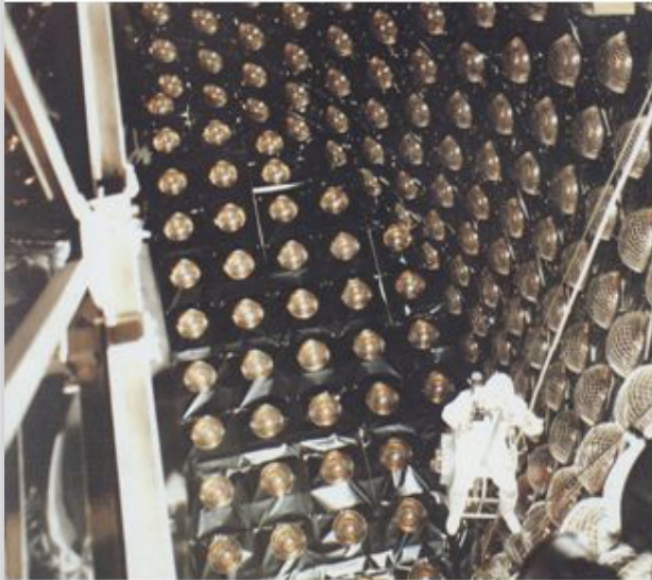
## Grand Unify Theory SU(5)



**Big experimental effort to looking for  
protons decays**

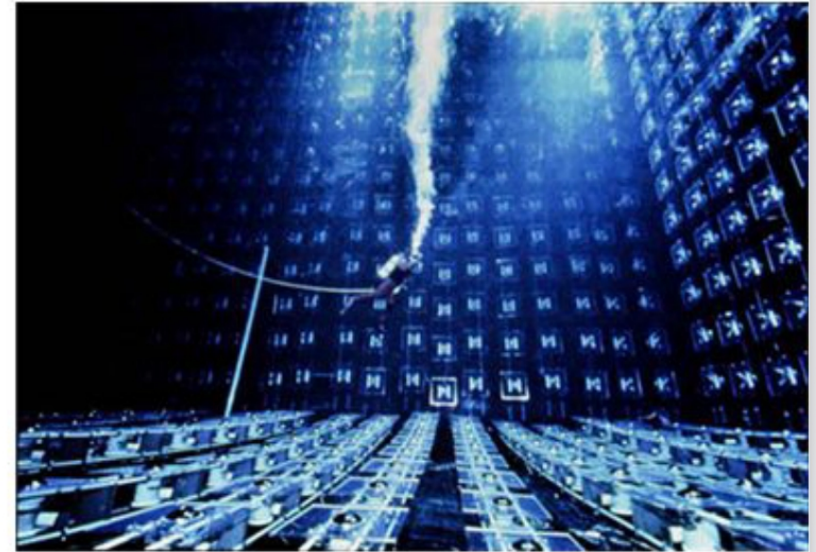
# Proton decay experiments

Grand Unified Theories (in the late 1970's)  $\rightarrow \tau_p = 10^{30 \pm 2}$  years



Kamiokande  
(1000ton)

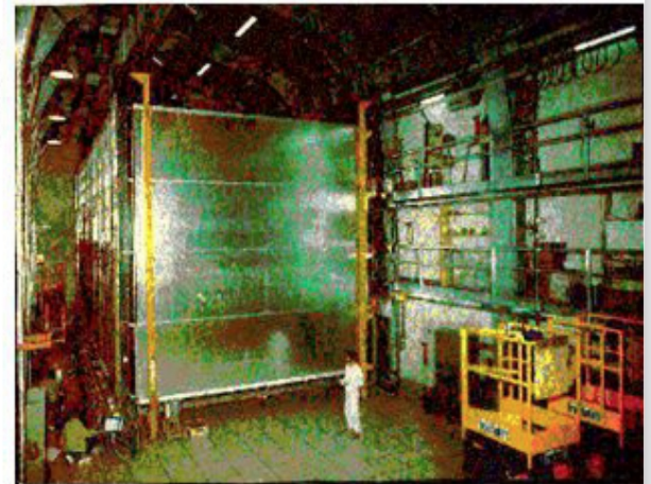
IMB  
(3300ton)



NUSEX  
(130ton)

Frejus  
(700ton)

These experiments  
observed many contained  
atmospheric neutrino  
events (background for  
proton decay).



# Negative experimental results: Particle Data Group

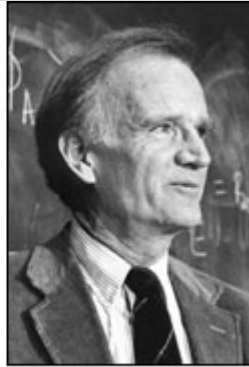
<b>p DECAY MODES</b>	Partial mean life ( $10^{30}$ years)	Co
<b>Antilepton + meson</b>		
$N \rightarrow e^+ \pi$	$> 158 (n), > 1600 (p)$	
$N \rightarrow \mu^+ \pi$	$> 100 (n), > 473 (p)$	
$N \rightarrow \nu \pi$	$> 112 (n), > 25 (p)$	
$p \rightarrow e^+ \eta$	$> 313$	
$p \rightarrow \mu^+ \eta$	$> 126$	
$n \rightarrow \nu \eta$	$> 158$	
$N \rightarrow e^+ \rho$	$> 217 (n), > 75 (p)$	
$N \rightarrow \mu^+ \rho$	$> 228 (n), > 110 (p)$	
$N \rightarrow \nu \rho$	$> 19 (n), > 162 (p)$	
$p \rightarrow e^+ \omega$	$> 107$	
$p \rightarrow \mu^+ \omega$	$> 117$	
$n \rightarrow \nu \omega$	$> 108$	
$N \rightarrow e^+ K$	$> 17 (n)$	
$p \rightarrow e^+ K_S^0$	$> 120$	
$p \rightarrow e^+ K_L^0$	$> 51$	
$N \rightarrow \mu^+ K$	$> 26 (n)$	
$p \rightarrow \mu^+ K_S^0$	$> 150$	
$p \rightarrow \mu^+ K_L^0$	$> 83$	
$N \rightarrow \nu K$	$> 86 (n)$	
$n \rightarrow \nu K_S^0$	$> 51$	
$p \rightarrow e^+ K^*(892)^0$	$> 84$	
$N \rightarrow \nu K^*(892)$	$> 78 (n)$	
<b>Antilepton + mesons</b>		
$p \rightarrow e^+ \pi^+ \pi^-$	$> 82$	
$p \rightarrow e^+ \pi^0 \pi^0$	$> 147$	
$n \rightarrow e^+ \pi^- \pi^0$	$> 52$	
$p \rightarrow \mu^+ \pi^+ \pi^-$	$> 133$	
$p \rightarrow \mu^+ \pi^0 \pi^0$	$> 101$	
$n \rightarrow \mu^+ \pi^- \pi^0$	$> 74$	
$n \rightarrow e^+ K^0 \pi^-$	$> 18$	
<b>Lepton + meson</b>		
$n \rightarrow e^- \pi^+$	$> 65$	
$n \rightarrow \mu^- \pi^+$	$> 49$	
$n \rightarrow e^- \rho^+$	$> 62$	
$n \rightarrow \mu^- \rho^+$	$> 7$	
$n \rightarrow e^- K^+$	$> 32$	
$n \rightarrow \mu^- K^+$	$> 57$	
<b>Lepton + mesons</b>		
$p \rightarrow e^- \pi^+ \pi^+$	$> 30$	
$n \rightarrow e^- \pi^+ \pi^0$	$> 29$	
$p \rightarrow \mu^- \pi^+ \pi^+$	$> 17$	
$n \rightarrow \mu^- \pi^+ \pi^0$	$> 34$	
$p \rightarrow e^- \pi^+ K^+$	$> 75$	
$p \rightarrow \mu^- \pi^+ K^+$	$> 245$	
<b>Antilepton + photon(s)</b>		
$p \rightarrow e^+ \gamma$	$> 670$	
$p \rightarrow \mu^+ \gamma$	$> 478$	
$n \rightarrow \nu \gamma$	$> 28$	
$p \rightarrow e^+ \gamma \gamma$	$> 100$	
$n \rightarrow \nu \gamma \gamma$	$> 219$	



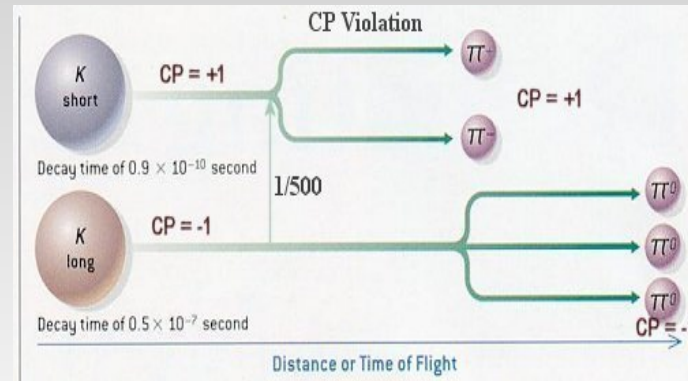
# 1964 CP violation observed



James Cronin



Val Fitch



Epigraph in a copy of his paper to his friend  
Evgeny Feinberg

“*Making use of the effect*

*1. Okubo has proposed,*

*While the temperature is high*

*The universe is richly clothed*

*In a coat made to fit*

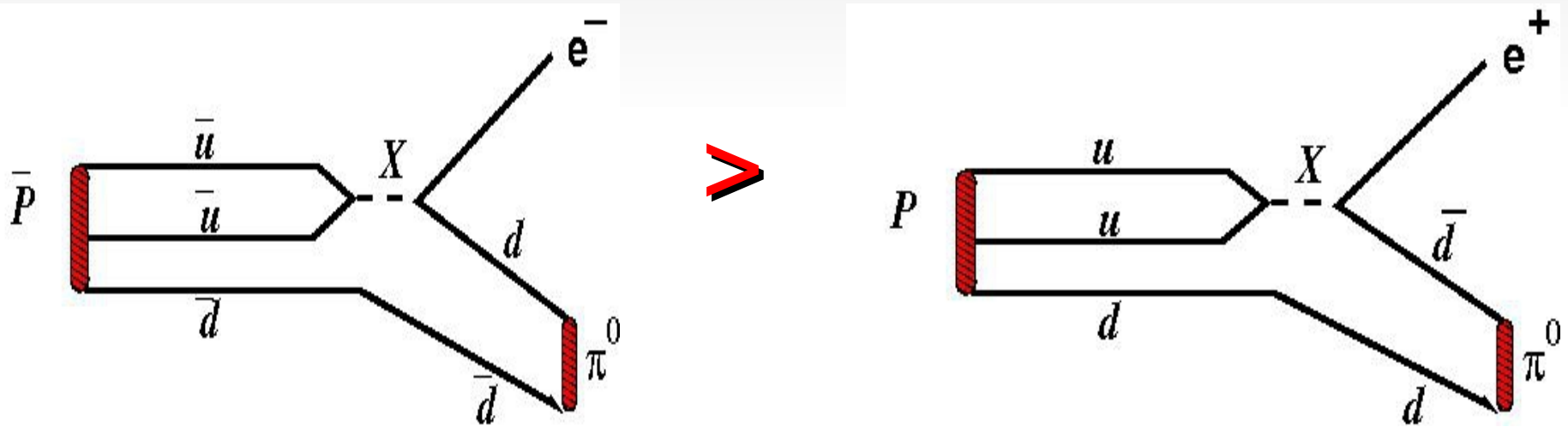
*Its crooked figure-head to foot.”*

In 1957 Okubo proposed that CP violation imply in a  
different rate between particles and anti-particles decays

Direct CP violation

# Direct CP violation

**Anti-proton decay rate bigger than proton decay.**





## Real life: directly ~~CP~~ violation:

Phases  $\phi_i$  change signal with charge conjugate operation: weak phase.

Phases  $\delta_i$  no change signal with charge conjugate operation: strong phase.

$$\langle f|T|i\rangle = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)},$$

$$\langle \bar{f}|T|\bar{i}\rangle = A_1 e^{i(\delta_1 - \phi_1 + \theta)} + A_2 e^{i(\delta_2 - \phi_2)} .$$

CP Violation  
Branco, Lavoura and Silva  
Oxford U. Press

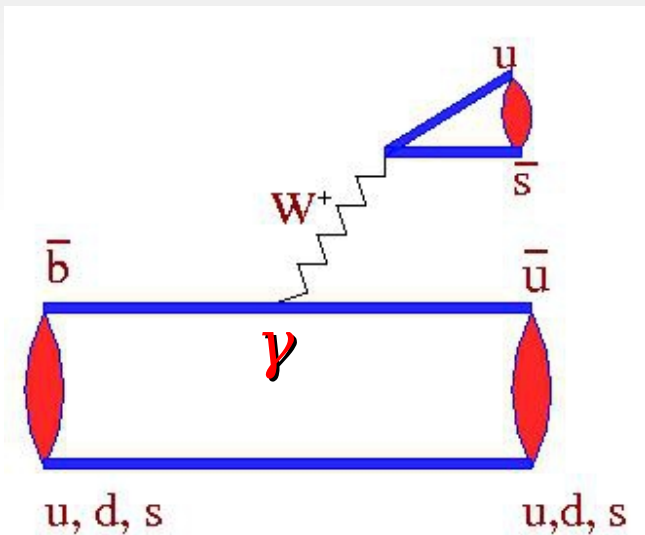
$$|\langle f|T|i\rangle|^2 - |\langle \bar{f}|T|\bar{i}\rangle|^2 = -4A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) .$$

Directly CP violation: two amplitudes with different strong and weak phase.

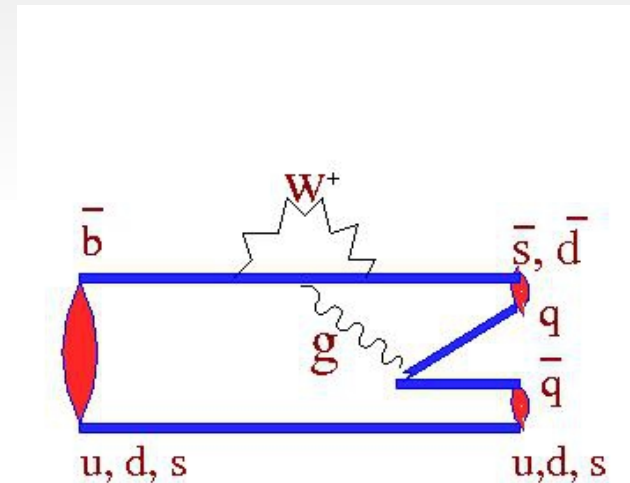
# ~~CP~~ in charge meson decays: quark graphics contributions to the charmless B decays

*Myron Bander, D. Silverman, A. Soni : Phys.Rev.Lett. 43 (1979) 242*

Tree



Penguin



Interference=>~~CP~~

Weak CKM phase  $\gamma$  change signal with charge conjugate operation  
**Strong phase coming from the loop in the Penguin contribution.**

~~CP~~ in charge meson decays:  
hadronic FSI contributions to the charmless B decays

Wolfenstein ( Phys.Rev. D43 (1991) 151-156)

Simplified formulation: P particle decay in a family of only two final states  $\alpha$  e  $\beta$



Weak CKM phase  $\gamma$  change sign with charge conjugate operation

**Strong phase coming from hadronic final state interaction.**



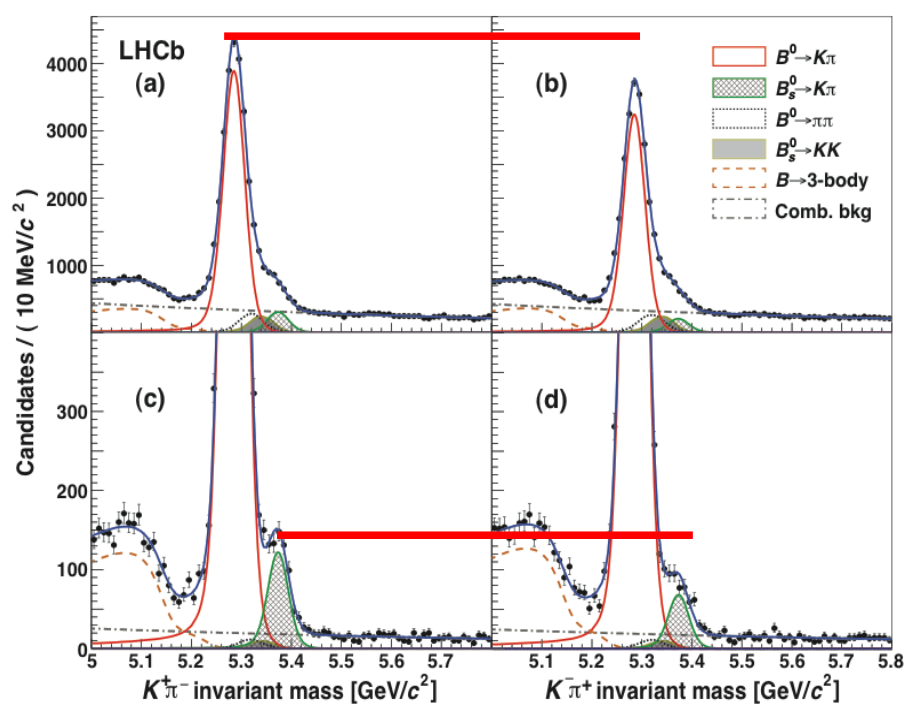
# Directly ~~CP~~ violation for :



LHCb: *Phys. Rev. Lett.* **110**, 221601 (2013)

Directly CP violation:

simple counting of events between charge conjugates final states.



$$A_{CP}(B^0 \rightarrow K^+ \pi^-) = \frac{N(B^0 \rightarrow K^+ \pi^-) - N(B^0 \rightarrow K^- \pi^+)}{N(B^0 \rightarrow K^+ \pi^-) + N(\bar{B}^0 \rightarrow K^- \pi^+)}$$

$$A_{CP}(B^0 \rightarrow K^+ \pi^-) = -0.080 \pm 0.007 \text{ (stat)} \pm 0.003 \text{ (syst)},$$

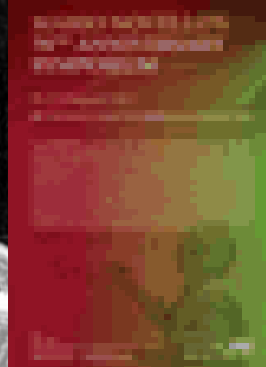
$$A_{CP}(B^0_s \rightarrow K^- \pi^+) = 0.27 \pm 0.04 \text{ (stat)} \pm 0.01 \text{ (syst)}.$$

CP violation

in heavy meson three body decay



# Search of $CP$ violation in three body Beauty decays.



*General questions about  $CP$  Violation.*

*$CP$  Violation in heavy meson three body decays.*

*Experimental evidence of  $CP$  Violation in  
 $B^{\pm} \rightarrow K^{\pm} \pi^{+} \pi^{-}$   $B^{\pm} \rightarrow K^{\pm} K^{+} K^{-}$  decays.*

*$CPT$  constraint for direct  $CP$  asymmetry.*

*Perspectives*

**Ignacio Bediaga: Centro Brasileiro de Pesquisas Físicas.**

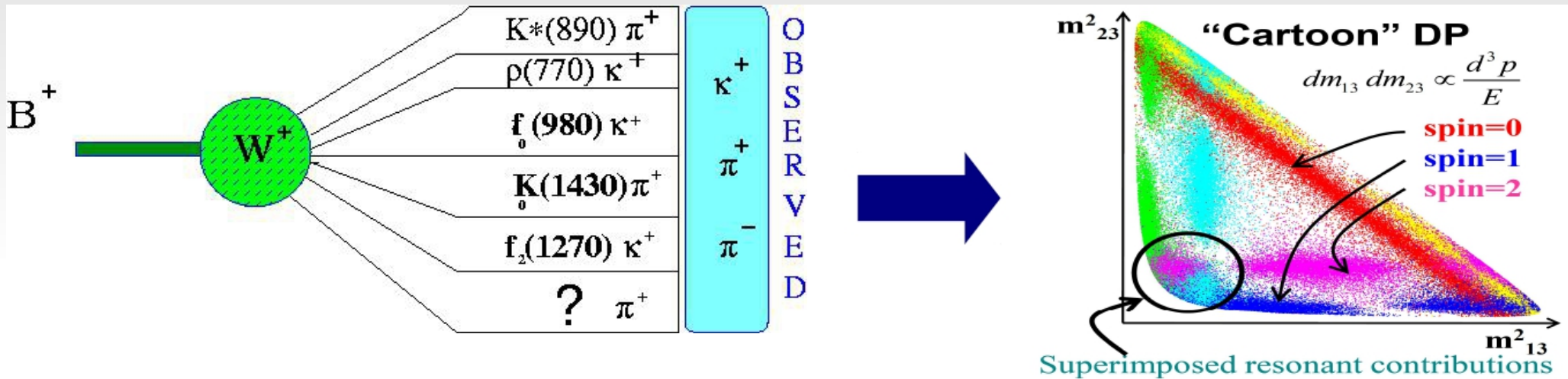
**Mário Novello's 70th Anniversary Symposium**

**August 15 to 17 Centro Brasileiro de Pesquisas Físicas**



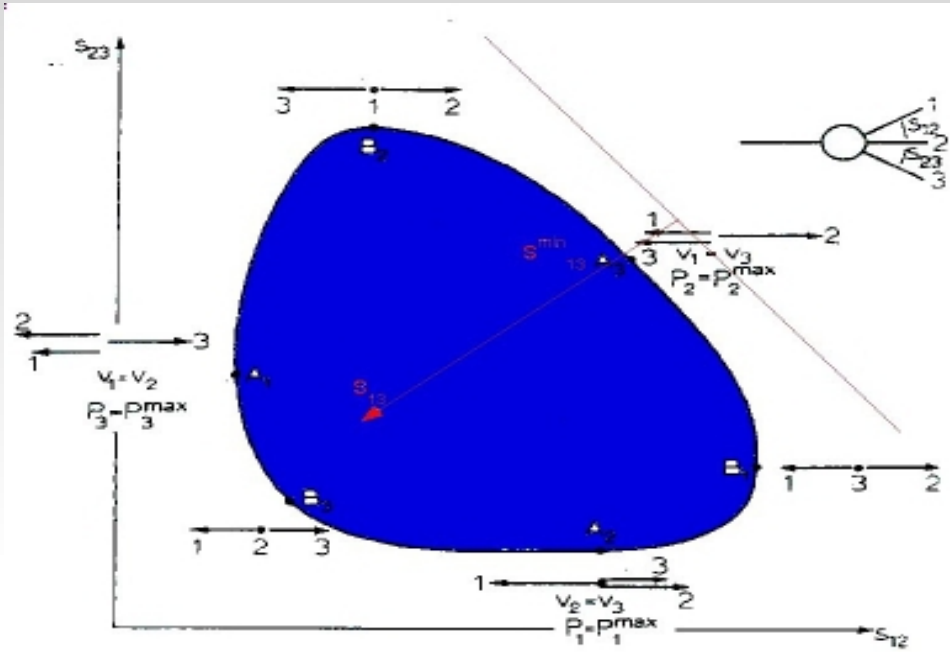
# Charmless three body B charge decays

Study the B decays and their intermediary states:



- $B^\pm \rightarrow K^\pm \pi^+ \pi^-$
- $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$
- $B^\pm \rightarrow \pi^\pm K^+ K^-$
- $B^\pm \rightarrow K^\pm K^+ K^-$

# Dalitz Plot



$$s_{12} = M_{12}^2 = (p_1^\nu + p_2^\nu)^2$$

$$s_{13} = M_{13}^2 = (p_1^\nu + p_3^\nu)^2$$

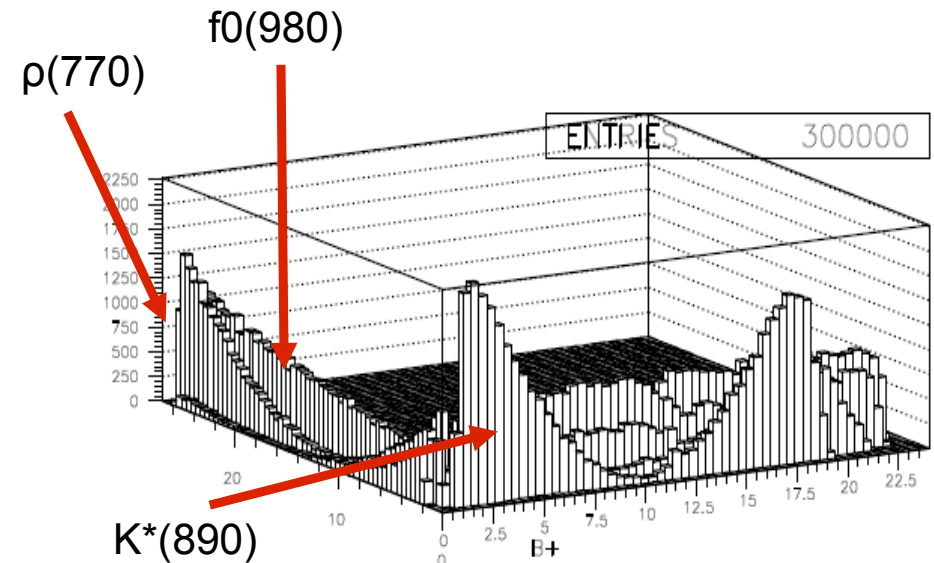
$$s_{23} = M_{23}^2 = (p_2^\nu + p_3^\nu)^2$$

**With one constraint**

Flat phase space where it is write the dynamics.

$$d\Gamma(s_{12}, s_{23}) = \frac{1}{(2\pi)^3 32 M_B^3} |\mathcal{M}|^2 ds_{12} ds_{23}$$

**$|M|^2 \Rightarrow$  resonances**



# Phases in amplitude analysis

Signature of the phase difference between two interfering resonances

$$|\mathcal{M}|^2 = |a_{\pi^+\pi^-}|^2 + |a_{\pi^+\pi^0}|^2 + 2|a_{\pi^+\pi^-} a_{\pi^+\pi^0}|$$

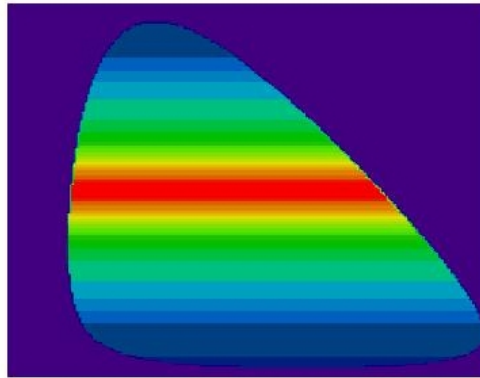


Figure 1:  $|a_{\pi^+\pi^-}| = 1, |a_{\pi^+\pi^0}| = 0$

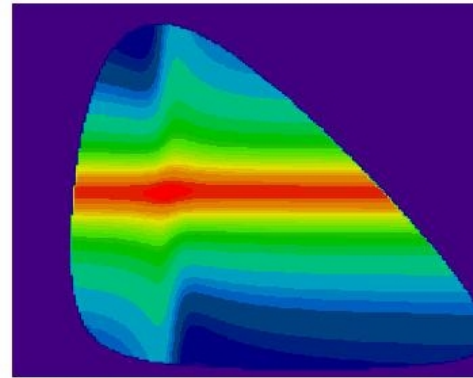


Figure 2:  $|a_{\pi^+\pi^-}| = 1, |a_{\pi^+\pi^0}| > 0$

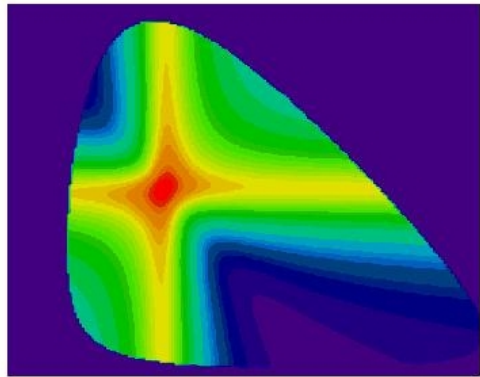


Figure 3: \*  
 $|a_{\pi^+\pi^-}| = |a_{\pi^+\pi^0}| = 1, \Delta\Phi = 0^\circ$

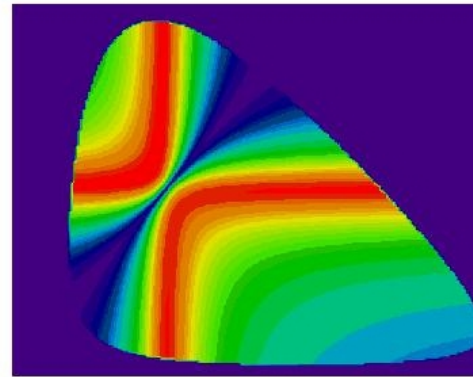


Figure 4: \*  
 $|a_{\pi^+\pi^-}| = |a_{\pi^+\pi^0}| = 1, \Delta\Phi = 90^\circ$



arXiv:2206.07622v1 [hep-ex] 15 Jun 2022

# Direct $CP$ violation in charmless three-body decays of $B^\pm$ mesons

LHCb collaboration

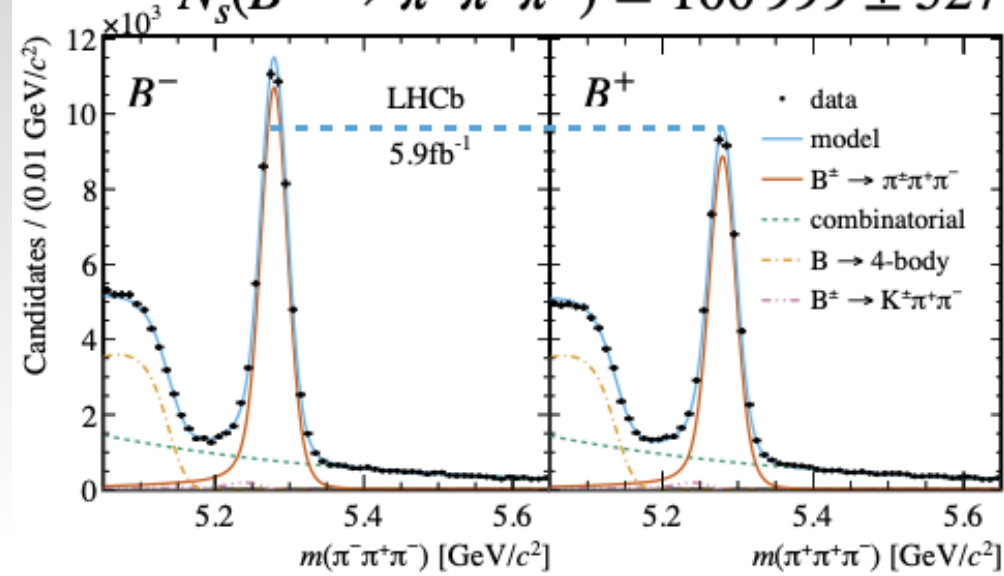
## Abstract

Measurements of  $CP$  asymmetries in charmless three-body decays of  $B^\pm$  mesons are reported using proton-proton collision data collected by the LHCb detector, corresponding to an integrated luminosity of  $5.9 \text{ fb}^{-1}$ . The previously observed  $CP$  asymmetry in  $B^\pm \rightarrow \pi^\pm K^+ K^-$  decays is confirmed, and  $CP$  asymmetries are observed with a significance of more than five standard deviations in the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  and  $B^\pm \rightarrow K^\pm K^+ K^-$  decays, while the  $CP$  asymmetry of  $B^\pm \rightarrow K^\pm \pi^+ \pi^-$  decays is confirmed to be compatible with zero. The distributions of these asymmetries are also studied as a function of the three-body phase space and suggest contributions from rescattering and resonance interference processes. An indication of the presence of the decays  $B^\pm \rightarrow \pi^\pm \chi_{c0}(1P)$  in both  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  and  $B^\pm \rightarrow \pi^\pm K^+ K^-$  decays is observed, as is  $CP$  violation involving these amplitudes.

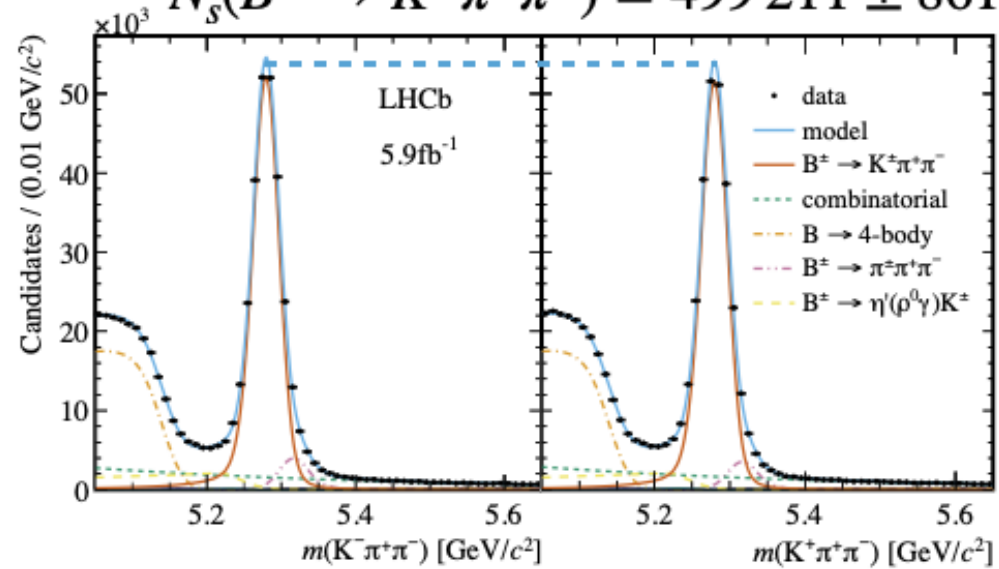
Submitted to Phys. Rev. D

# Fit results

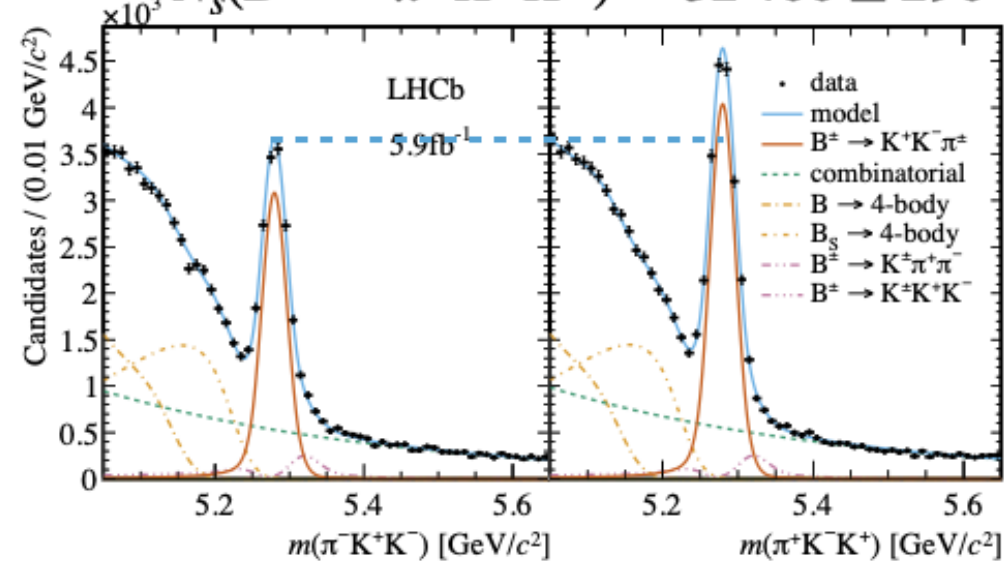
$$N_s(B^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = 100\,999 \pm 527$$



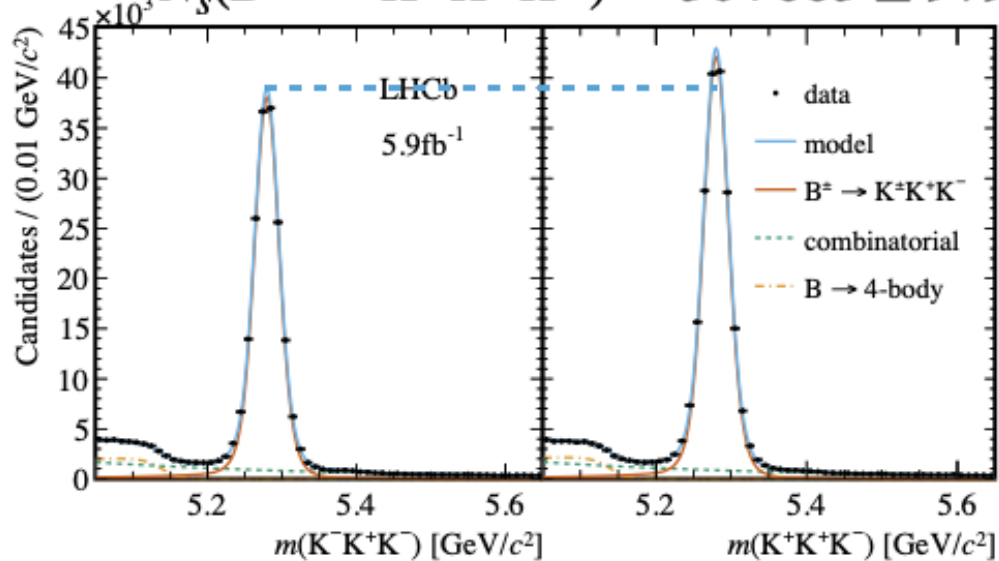
$$N_s(B^\pm \rightarrow K^\pm \pi^+ \pi^-) = 499\,211 \pm 861$$



$$N_s(B^\pm \rightarrow \pi^\pm K^+ K^-) = 32\,466 \pm 296$$

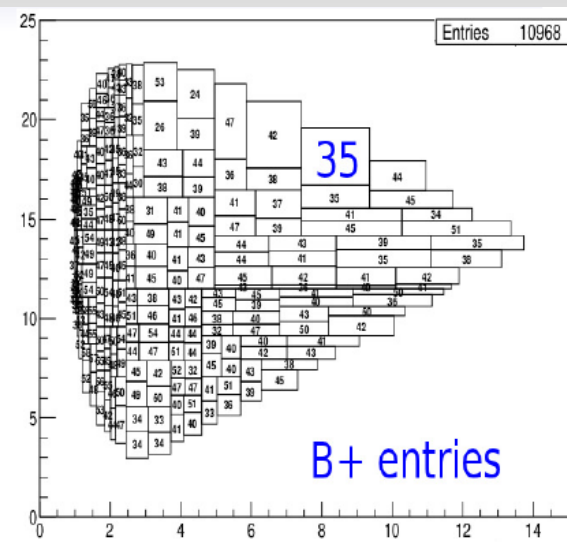
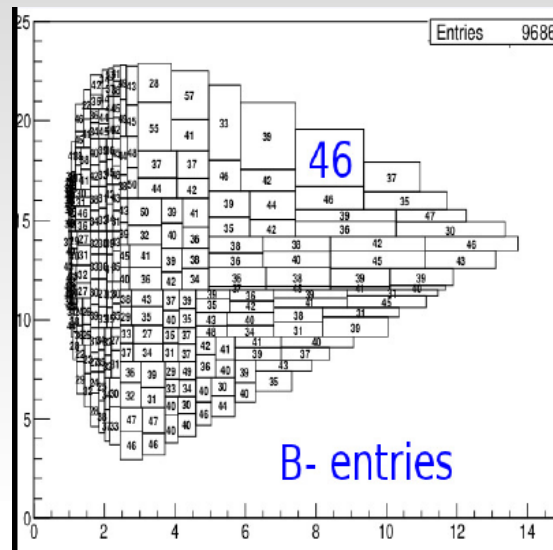
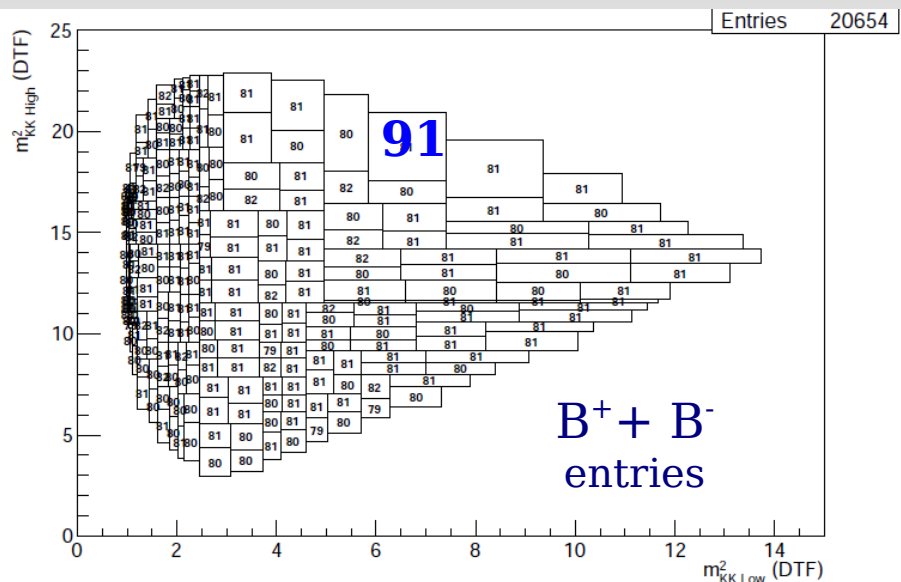


$$N_s(B^\pm \rightarrow K^\pm K^+ K^-) = 364\,883 \pm 979$$

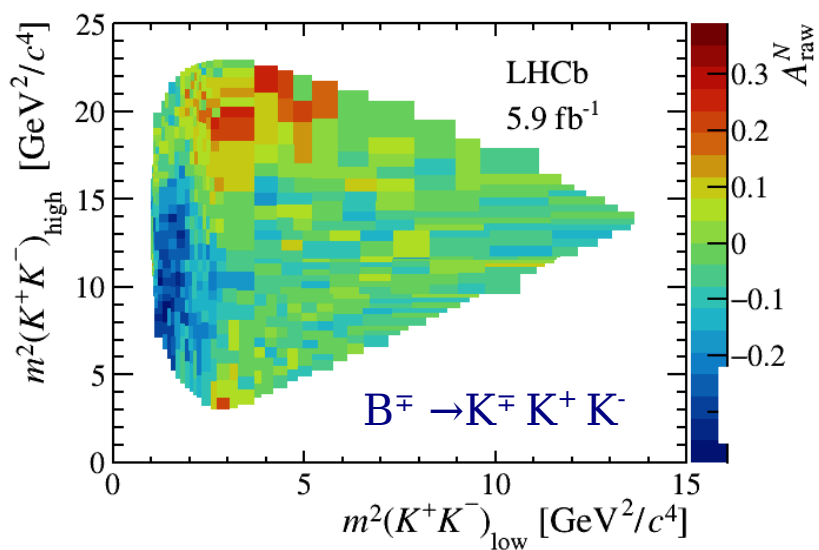


# B<sup>+</sup> - B<sup>-</sup> Dalitz differences

## M<sup>2</sup><sub>K+K-</sub> Vs M<sup>2</sup><sub>K+K-</sub> phase space distribution



## Dalitz Map of the CP violation



## Simetrical Dalitz

If M<sup>2</sup><sub>K+K1-</sub> > M<sup>2</sup><sub>K+K2-</sub>

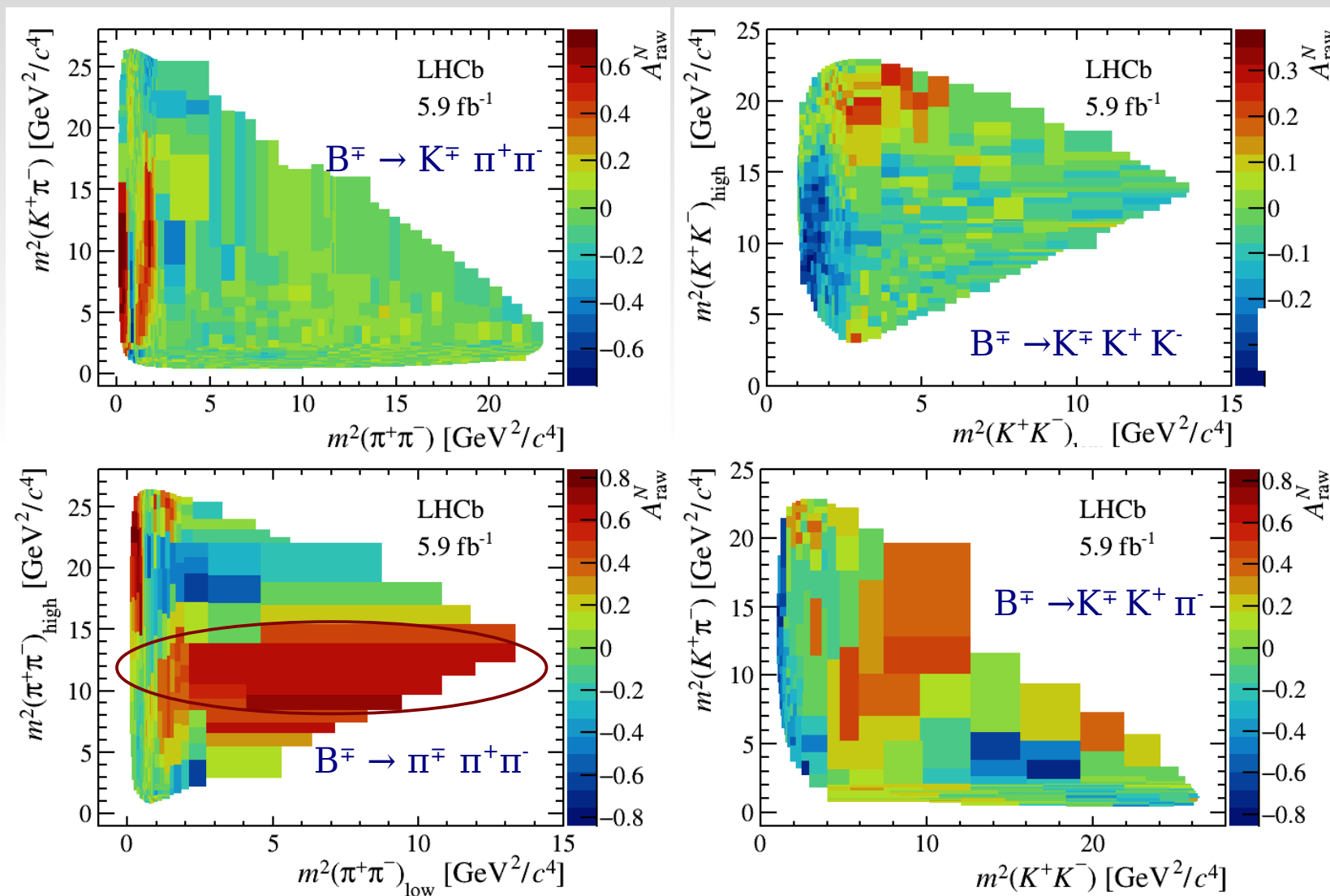
M<sup>2</sup><sub>K+K1-</sub> = M<sup>2</sup><sub>K+K-high</sub>

and

M<sup>2</sup><sub>K+K1-</sub> = M<sup>2</sup><sub>K+K-low</sub>

Otherwise .....

# CP Dalitz distribution for the four $B^\mp \rightarrow h^\mp h^+ h^-$ channels



The CKM weak phase  $\gamma$  is a constant

**Hadronic final state interaction phase changing in the phase**



## Review

Direct  $CP$  violation in beauty and charm hadron decaysIgnacio Bediaga<sup>a</sup>, Carla Göbel<sup>b,\*</sup><sup>a</sup> Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca, CEP 22290-180 Rio de Janeiro, Brazil<sup>b</sup> Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente 225, Gávea, CEP 22451-900 Rio de Janeiro, Brazil

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## ABSTRACT

Since the discovery of  $CP$  violation more than 5 decades ago, this phenomenon is still attracting a lot of interest. Among the many fascinating aspects of this subject, this review is dedicated to direct  $CP$  violation in non-leptonic decays. The advances within the last decade have been enormous, driven by the increasingly large samples of  $b$ - and  $c$ -hadron decays, and have led to very interesting results such as large  $CP$  asymmetries in charmless  $B$  decays and the observation of direct  $CP$  violation in the charm sector. We address the quest for understanding the origin of strong phases, the importance of final state interactions and the relation with  $CPT$  symmetry, and different approaches to measure direct  $CP$  violation in these decays. The main experimental results and their implications are then discussed.

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# ~~CP~~ in charge meson decays: hadronic FSI contributions to the charmless B decays

Wolfenstein ( Phys.Rev. D43 (1991) 151-156)

Simplified formulation: P particle decay in a family of only two final states  $\alpha$  e  $\beta$



Weak CKM phase  $\gamma$  change sign with charge conjugate operation

**Strong phase coming from hadronic final state interaction.**

# Sakharov model keep the CPT Invariance

- ◆ CPT invariance  $\Rightarrow$  Same lifetime and same mass to particle and anti-particle.

$$\text{Lifetime } \tau = 1 / \Gamma_{\text{total}} = 1 / \bar{\Gamma}_{\text{total}}$$

$$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots$$

$$\bar{\Gamma}_{\text{total}} = \bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \dots$$

- ◆ CP violation  $\Rightarrow \Gamma_1 > \bar{\Gamma}_1$ .

- ◆ CPT conservation:

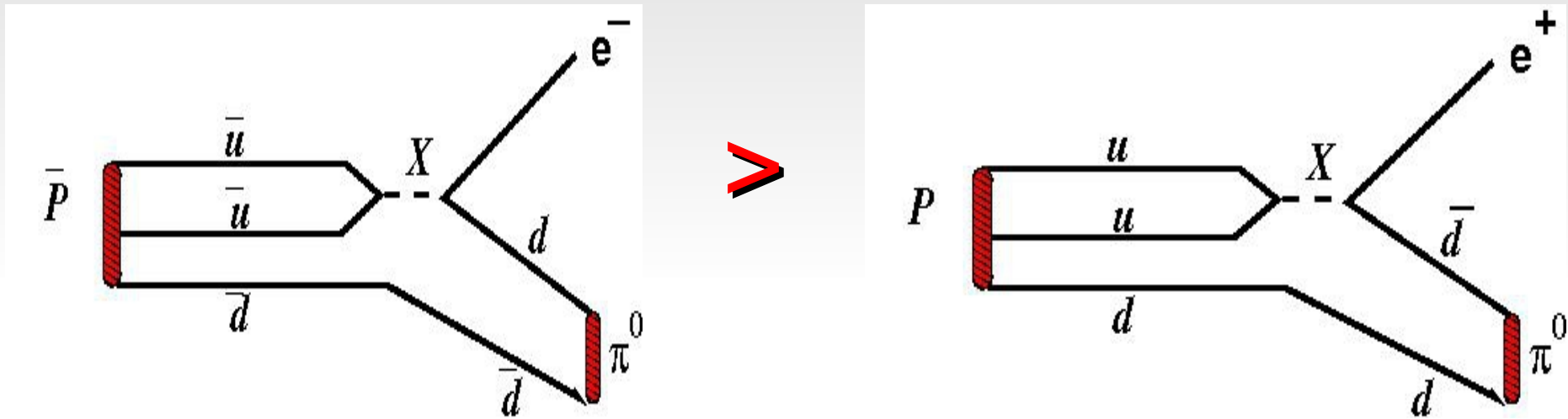
$$\Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots < \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \dots$$

**In a exact proportion.**

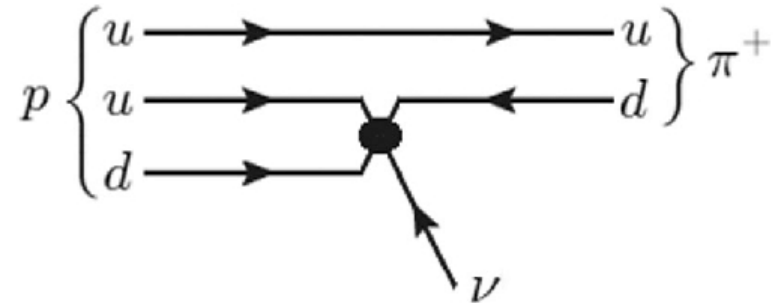
- ◆ **This imply a necessary communication between CP violation channels.**
- ◆ **In strong interaction it can be produced by re-scattering.**

# Direct CP violation

Anti-proton decay rate bigger than proton decay.



It is necessary another proton decay channel, where the CP violation of the anti-proton is bigger than the proton



The Sakharov proposed apparently is non-feasible if one keep CPT

# *Apparent solution*

James Claine lectures at Les Houches Summer School

arXiv:hep-ph/0609145

**Channel with direct CP violation**

$$\Gamma(X \rightarrow qq) \neq \Gamma(\bar{X} \rightarrow \bar{q}\bar{q})$$

**Channel with direct CP violation**

$$\Gamma(X \rightarrow Y) \neq \Gamma(\bar{X} \rightarrow \bar{Y})$$

**Y with a different baryon number than qq**

$$\Gamma(X \rightarrow qq) + \Gamma(X \rightarrow Y) = \Gamma(\bar{X} \rightarrow \bar{q}\bar{q}) + \Gamma(\bar{X} \rightarrow \bar{Y})$$

**In a exact proportion.**

**This imply a necessary communication between CP violation channels.**

**So, Y must couple with qq**

**Marshak et al,**  
**Theory of Weak interactions in particles physics:**  
**Wiley & Sons, 1969.**

2. Space-Time Properties

CPT Invariance

If CPT invariance holds, we have for any final state  $|\lambda\rangle$  into which  $K$  can decay

$$\langle \lambda^{\text{out}} | H_w^h | \bar{K} \rangle = \langle \lambda^{\text{out}} | (CPT)^{-1} H_w^h CPT | \bar{K} \rangle$$

Now we can choose the phase factors of the state vectors  $|K\rangle$  and  $|\bar{K}\rangle$  such that

$$CPT |\bar{K}\rangle = -|K\rangle \quad (6.76a)$$

and

$$CPT |\lambda^{\text{out}}\rangle = \eta_\lambda \langle \hat{\lambda}^{\text{in}} | \quad (6.76b)$$

where  $|\eta_\lambda|^2 = 1$  and  $\hat{\lambda}$  denotes the CPT conjugate state to  $\lambda$  (particles replaced by antiparticles with the same momenta but with all spins

If  $\lambda$  denotes many-particle final states, as is the case here, the application of the above result is complicated by the difficulty of classifying the final states in such a way that each has a single phase shift. Nevertheless, if we sum absolute squares on both sides of (6.77) over all final states, we obtain [33]

$$\begin{aligned} \Gamma_{\bar{K}} &= \sum_{\lambda} |\langle \lambda^{\text{out}} | H_w^h | \bar{K} \rangle|^2 \\ &= \sum_{\lambda} \sum_{\hat{\lambda}'} \sum_{\hat{\lambda}''} S_{\hat{\lambda}'' \hat{\lambda}'} S_{\hat{\lambda} \lambda} \langle \hat{\lambda}''^{\text{out}} | H_w^h | K \rangle^* \langle \hat{\lambda}^{\text{out}} | H_w^h | K \rangle \\ &= \sum_{\hat{\lambda}''} \sum_{\hat{\lambda}'} \delta_{\hat{\lambda}'' \hat{\lambda}'} \langle \hat{\lambda}''^{\text{out}} | H_w^h | K \rangle^* \langle \hat{\lambda}^{\text{out}} | H_w^h | K \rangle \\ &= \sum_{\hat{\lambda}''} |\langle \hat{\lambda}''^{\text{out}} | H_w^h | K \rangle|^2 = \Gamma_K \end{aligned} \quad (6.78)$$

It is not necessary [34] to sum over all final states. Summation is necessary over only such states as would be mixed together in forming  $S$  matrix eigenstates. Now for  $K \rightarrow 3\pi$  decay, parity conservation forbids any strong or electromagnetic transitions between a total  $J = 0$  three-pion state and a  $J = 0$  two-pion state; nor is either connected to leptonic final states except by the weak interaction. Thus, neglecting electromagnetism, two-pionic, three-pionic, and leptonic rates of  $K$  should be separately equal [34, 35] to the corresponding rates of  $\bar{K}$ .

(i)  $3\pi$  modes of charged kaons. The above considerations show that CPT invariance implies [33, 34] (neglecting the electromagnetic interaction)

$$\begin{aligned} \Gamma(K^+ \rightarrow 3\pi) &\equiv \Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-) + \Gamma(K^+ \rightarrow \pi^0 \pi^0 \pi^+) \\ &= \Gamma(K^- \rightarrow \pi^- \pi^- \pi^+) + \Gamma(K^- \rightarrow \pi^0 \pi^0 \pi^-) \equiv \Gamma(K^- \rightarrow 3\pi) \end{aligned} \quad (6.79)$$

reversed). Then

$$\begin{aligned} \langle \lambda^{\text{out}} | H_w^h | \bar{K} \rangle &= -\eta_\lambda \langle K | H_w^h | \hat{\lambda}^{\text{in}} \rangle \\ &= -\eta_\lambda \sum_{\hat{\lambda}''} \langle K | H_w^h | \hat{\lambda}''^{\text{out}} \rangle \langle \hat{\lambda}''^{\text{out}} | \hat{\lambda}^{\text{in}} \rangle \\ &= -\eta_\lambda \sum_{\hat{\lambda}''} S_{\hat{\lambda}'' \hat{\lambda}} \langle K | H_w^h | \hat{\lambda}''^{\text{out}} \rangle \end{aligned}$$

where  $S_{\hat{\lambda} \hat{\lambda}''} = e^{2i\delta} S_{\hat{\lambda} \hat{\lambda}''}$  is the  $S$  matrix element for the final states. Thus

$$\langle \lambda^{\text{out}} | H_w^h | \bar{K} \rangle = -\eta_\lambda \sum_{\hat{\lambda}''} S_{\hat{\lambda} \hat{\lambda}''} \langle \hat{\lambda}''^{\text{out}} | H_w^h | K \rangle^* \quad (6.77)$$

# Simplified version of the Marshak et al, demonstration

I.B., T. Frederico and O. Lourenço PRD 89, 094013 (2014)

## III. *CPT* INVARIANCE IN A DECAY

To define our notation and the framework for implementing the *CPT* constraint in *B* meson decays, we follow closely Refs. [22,23]. A hadron state  $|h\rangle$  transforms under *CPT* as  $CPT|h\rangle = \chi|\bar{h}\rangle$ , where  $\bar{h}$  is the charge conjugate state and  $\chi$  a phase. The weak and strong Hamiltonians conserve *CPT*, therefore  $(CPT)^{-1}H_w CPT = H_w$  and  $(CPT)^{-1}H_s CPT = H_s$ . The weak matrix element for the hadron decay is  $\langle\lambda_{\text{out}}|H_w|h\rangle$ , where  $\lambda_{\text{out}}$  includes the distortion from the strong force due to the final state interaction. The requirement of *CPT* invariance is fulfilled for the matrix element when

$$\langle\lambda_{\text{out}}|H_w|h\rangle = \chi_h \chi_\lambda \langle\bar{\lambda}_{\text{in}}|H_w|\bar{h}\rangle^*. \quad (1)$$

Inserting the completeness of the strongly interacting states, eigenstates of  $H_s$ , and using hermiticity of  $H_w$ , one gets

$$\langle\lambda_{\text{out}}|H_w|h\rangle = \chi_h \chi_\lambda \sum_{\bar{\lambda}'} S_{\bar{\lambda}', \bar{\lambda}} \langle\bar{\lambda}'_{\text{out}}|H_w|\bar{h}\rangle^*, \quad (2)$$

where the S-matrix element is  $S_{\bar{\lambda}', \bar{\lambda}} = \langle\bar{\lambda}'_{\text{out}}|\bar{\lambda}_{\text{in}}\rangle$ .

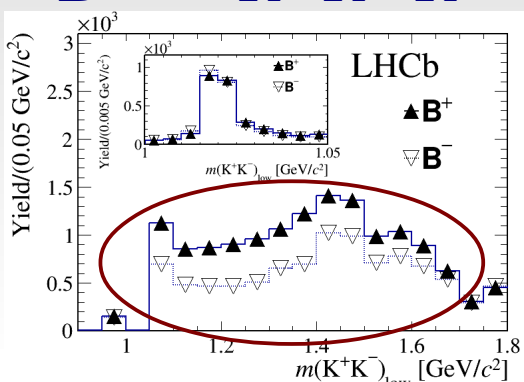
The sum of partial decays width of the hadron decay and the correspondent sum for the charge conjugate should be identical, which follows from Eq. (2)

$$\begin{aligned} \sum_{\lambda} |\langle\lambda_{\text{out}}|H_w|h\rangle|^2 &= \sum_{\bar{\lambda}} \left| \sum_{\bar{\lambda}'} S_{\bar{\lambda}', \bar{\lambda}}^* \langle\bar{\lambda}'_{\text{out}}|H_w|\bar{h}\rangle \right|^2 \\ &= \sum_{\bar{\lambda}} |\langle\bar{\lambda}_{\text{out}}|H_w|\bar{h}\rangle|^2, \end{aligned} \quad (3)$$

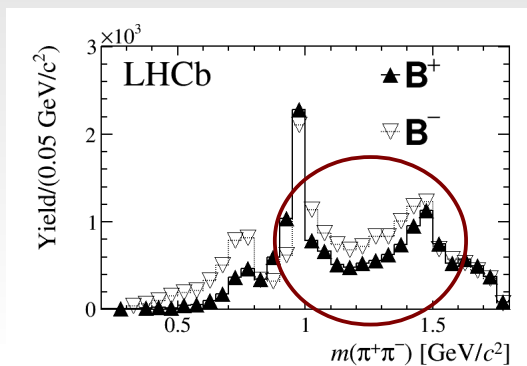
and note that besides the *CPT* constraint we have also used the hermiticity of the weak Hamiltonian.

Thanks Mario!!!!

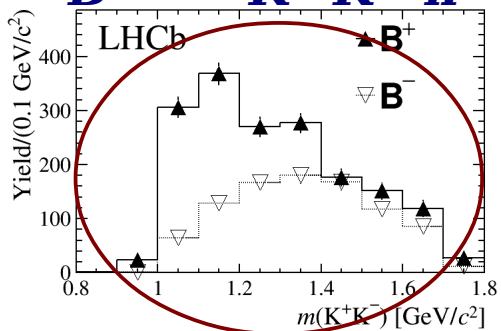
## $B^\mp \rightarrow K^\mp K^+ K^-$



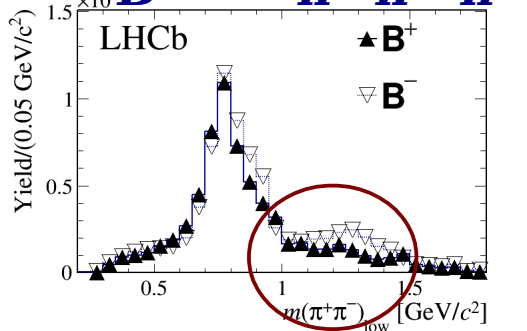
## $B^\mp \rightarrow K^\mp \pi^+ \pi^-$



## $B^\mp \rightarrow K^\mp K^+ \pi^-$



## $B^\mp \rightarrow \pi^\mp \pi^+ \pi^-$



- positive CP in  $B^\mp \rightarrow K^\mp \pi^+ \pi^-$  and  $B^\mp \rightarrow \pi^\mp \pi^+ \pi^-$
- negative in  $B^\mp \rightarrow \pi^\mp K^+ K^-$  and  $B^\mp \rightarrow K^\mp K^+ K^-$
- Same invariant mass: between 1 to 1.6 GeV

Cohen, D et al PRD 22 (1980) 2595

