Asymptotically Safe Quantum Gravity

Antonio D. Pereira

Universidade Federal Fluminense & Radboud University Nijmegen

XVIII Brazilian School of Cosmology and Gravitation

Recorded Lectures…

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…in Portuguese

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Cursos - Introdução ao grupo de renormalização funcional (Antônio D. Pereira)

20 vídeos • 919 visualizações • Última atualização em 4 de jun. de 2020

Este curso foi dado em 2016 para alunos de pós-

MOSTRAR MAIS

GravBR INSCRITO

Cursos - Gravidade quântica assintoticamente segura

23 vídeos • 571 visualizações • Última atualização em 6 de jun. de 2020

Dr. Anderson Tomaz e Dr. Gustavo P. de Brito

Curso oferecido pelo Dr. Anderson Tomaz e o estudante de doutorado Gustavo P. de Brito nas Atividades Formativas de Verão no ano de 2018 (no CBPF). O curso consiste em uma introdução à abordagem para a quantização da interação gravitacional conhecida como "Gravidade quântica assintoticamente segura" (Asymptotically safe quantum gravity).

Functional Renormalization Group

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Starting point: single scalar field

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generating functional of one-particle irreducible correlation functions

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$$\Delta S_k = \frac{1}{2}\int \mathrm{d}^d x\, \phi(x)\, \mathscr{R}_k(-\partial^2)\phi(x)$$

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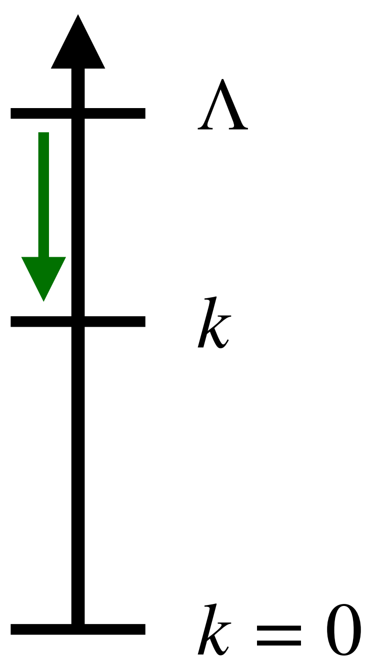

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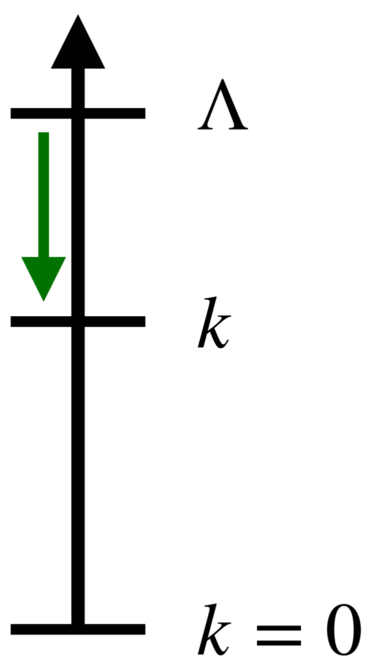

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in flat space: Fourier modes

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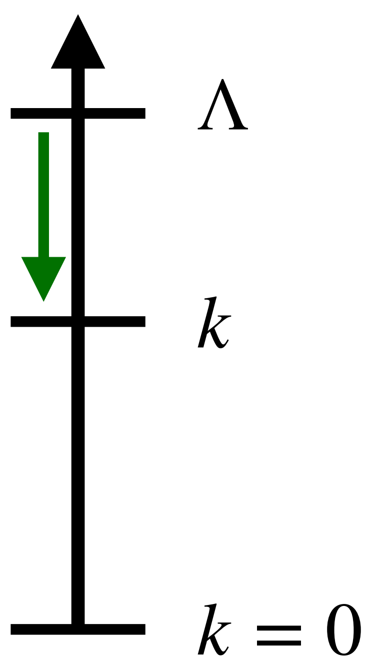

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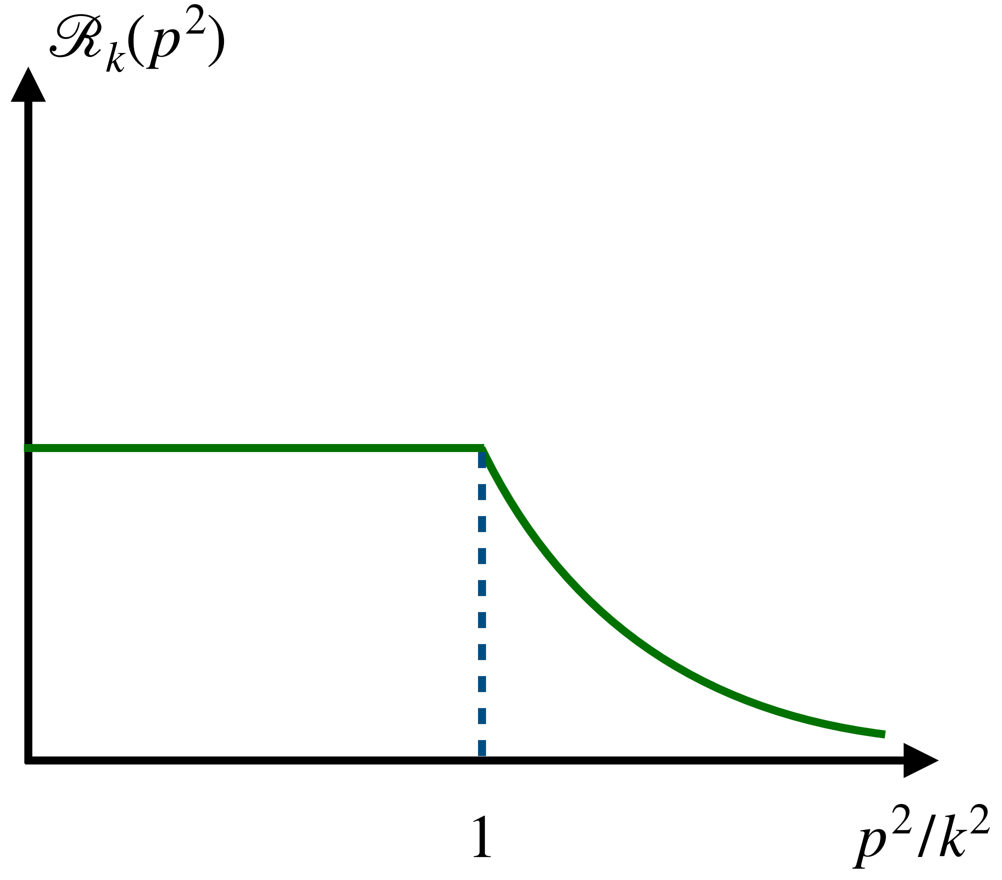
in flat space: Fourier modes

$$\phi(x) = \int_p \mathrm{e}^{ix\cdot p} \, \tilde{\phi}(p)$$

essentially

$$\mathscr{R}_k(p^2) \to \begin{cases} k^2 \,, & p^2 < k^2 \\ 0 \,, & p^2 > k^2 \end{cases}$$


$\mathscr{R}_k(p^2)$
1
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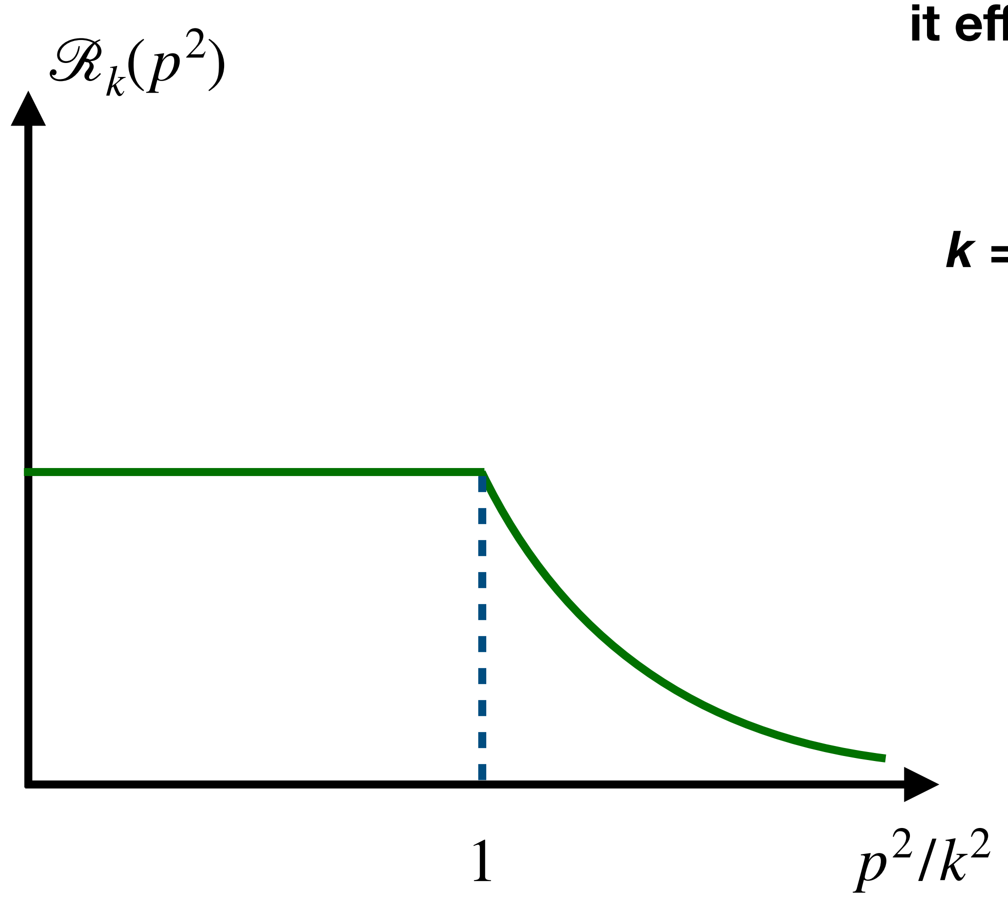
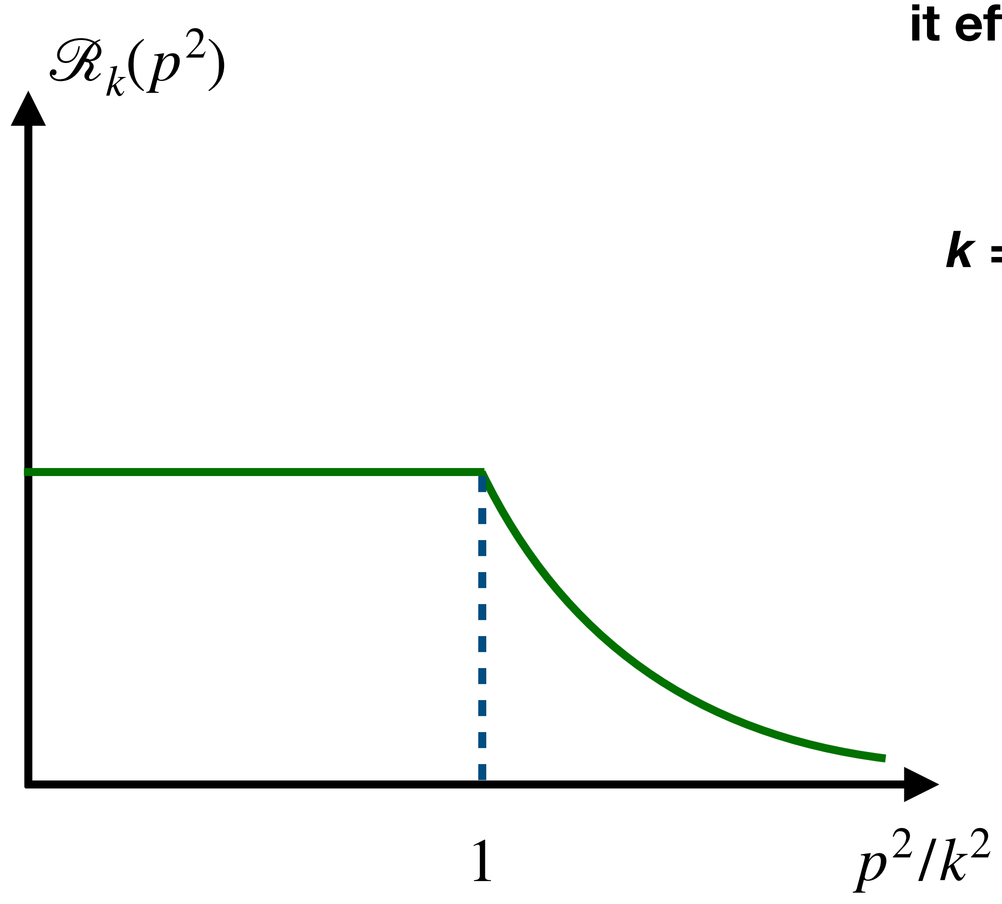


it effectively implements the suppression of “slow modes”

***k = 0 : complete* functional integration**

$$W_k[J] = \ln Z_k[J]$$



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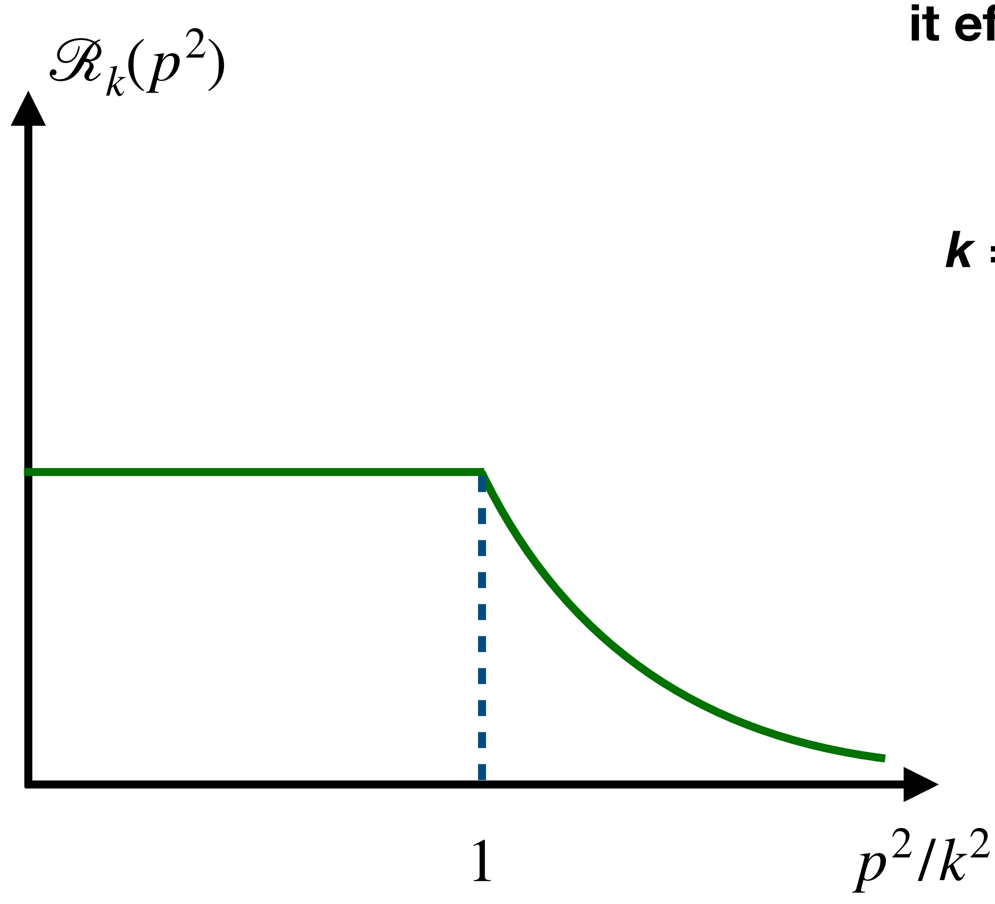

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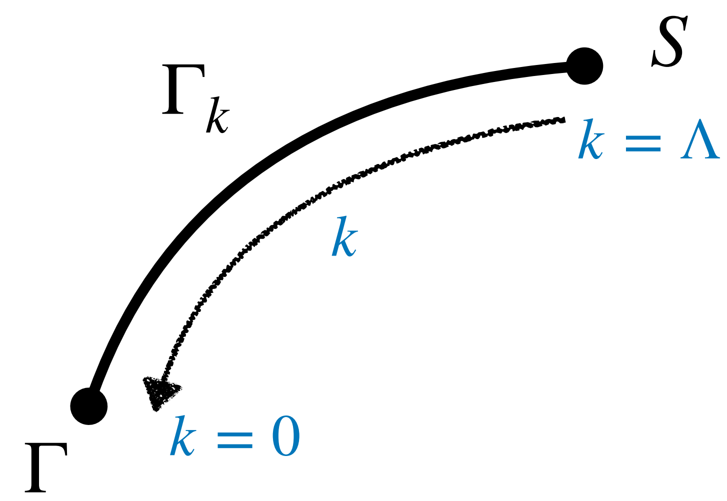
finally

$$\Gamma_k[\varphi] \equiv \bar{\Gamma}_k[\varphi] - \frac{1}{2}\int \mathrm{d}^d x \; \varphi(x)\mathscr{R}_k(-\partial^2)\varphi(x)$$

effective average action (EAA)

Properties:

interpolates between full effective action and the “classical” one



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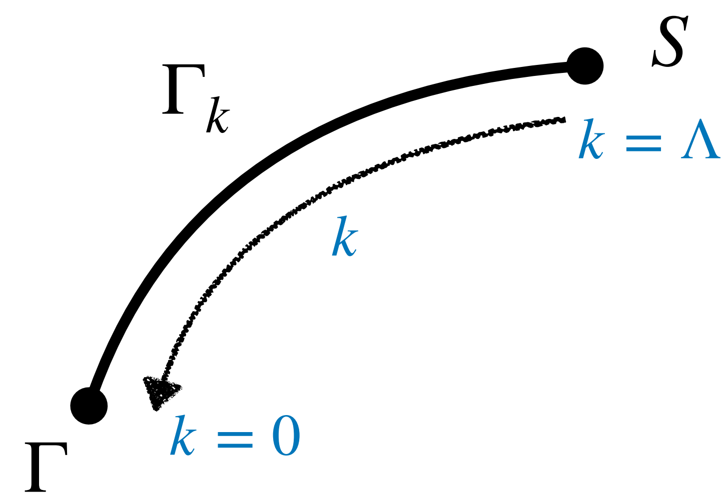
satisfies an exact flow equation



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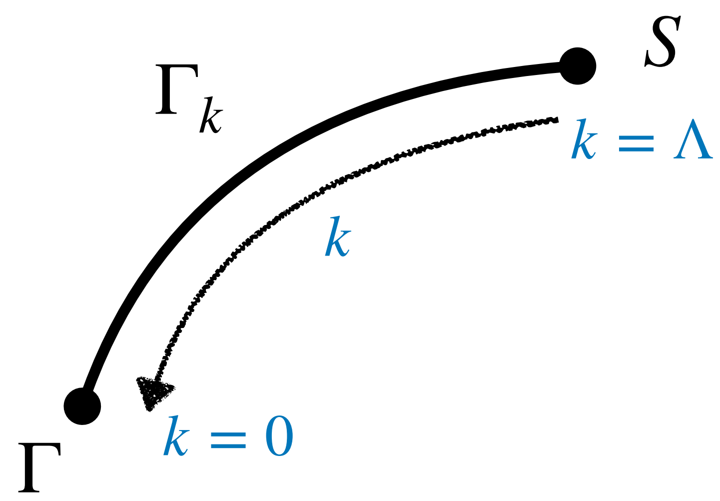

satisfies an exact flow equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + \mathscr{R}_k \right)^{-1} \partial_t \mathscr{R}_k \right]$$

$$\partial_t \equiv k \partial_k$$

(exact) flow equation

Wetterich equation

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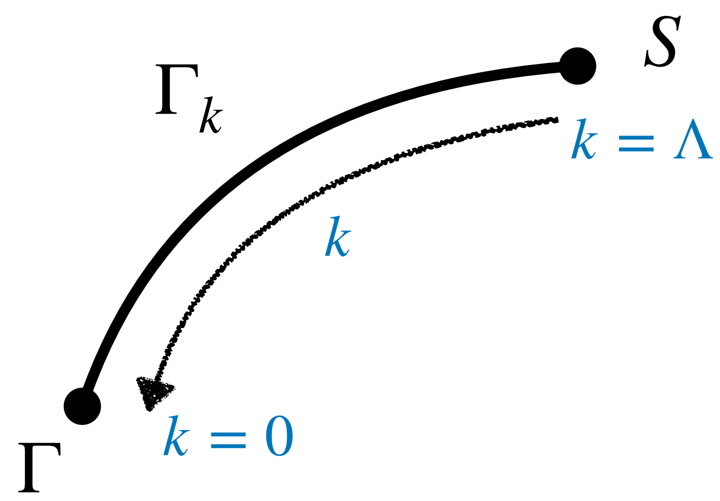

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conversion of functional integral into functional differential equation → solving the flow equation = solving the functional integral

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space of all functionals of the field which are compatible with the symmetries of the theory

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$$\bar{g}_i = k^{d_i} g_i$$

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suitable projection rule for the Wetterich equation

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→ **extraction of beta functions**

suitable projection rule for the Wetterich equation

Approximations are necessary, but we don't need to use a perturbative scheme!

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$$\beta_i(\mathbf{g}^*) = 0\,, \quad i = 1,\ldots,\infty$$

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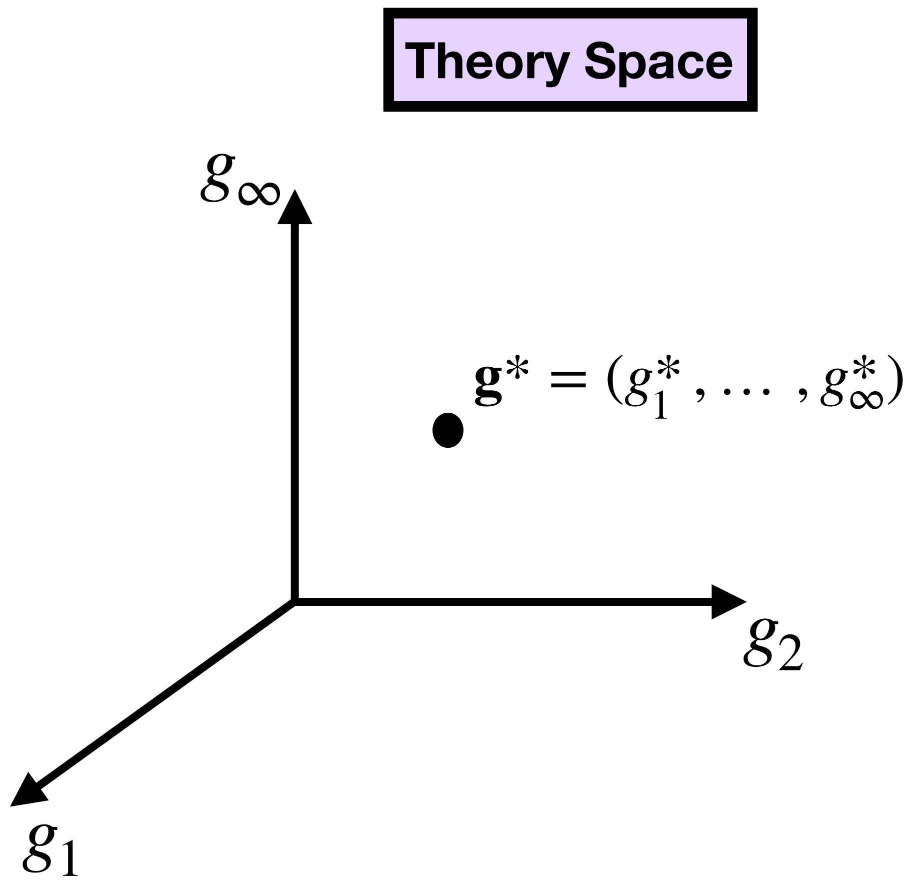

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$$\partial_t(g_i - g_i^*) = \sum_j \frac{\partial \beta_i}{\partial g_j}(g_j - g_j^*)$$

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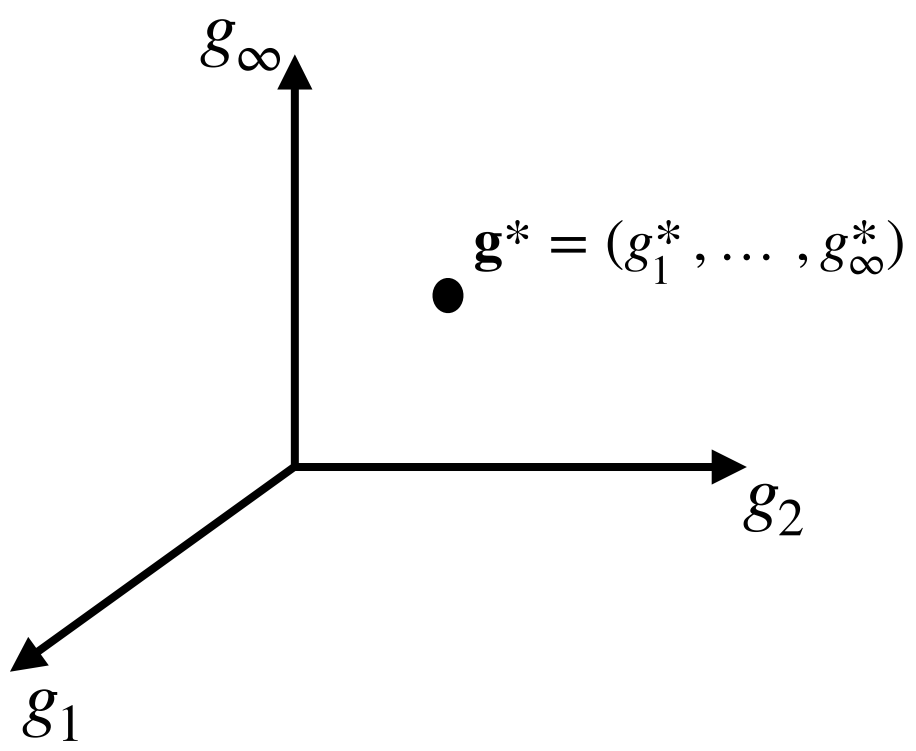

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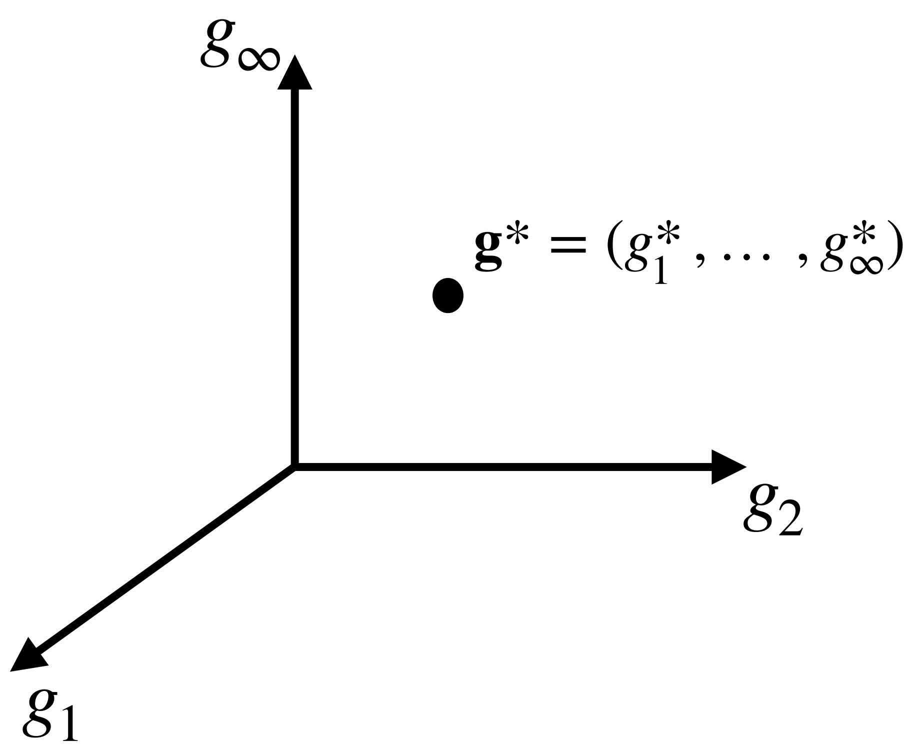

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$$\partial_t z_i = \lambda_i\, z_i$$

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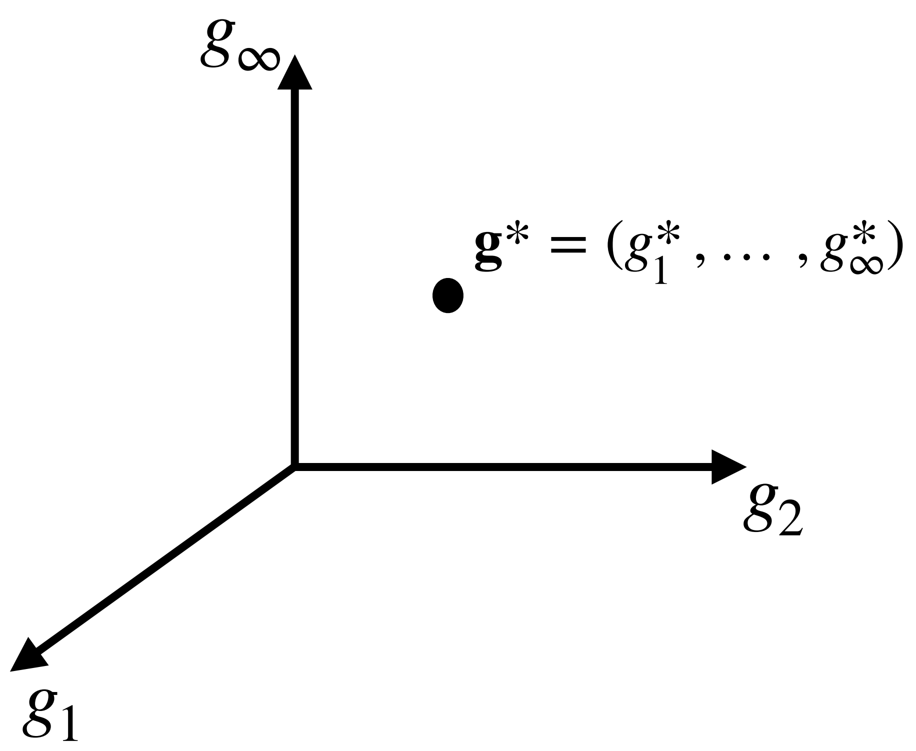

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$$z_i(t) = C_i \left(\frac{k}{k_0}\right)^{-\theta_i} \qquad \textbf{w/} \qquad \theta_i = -\lambda_i$$

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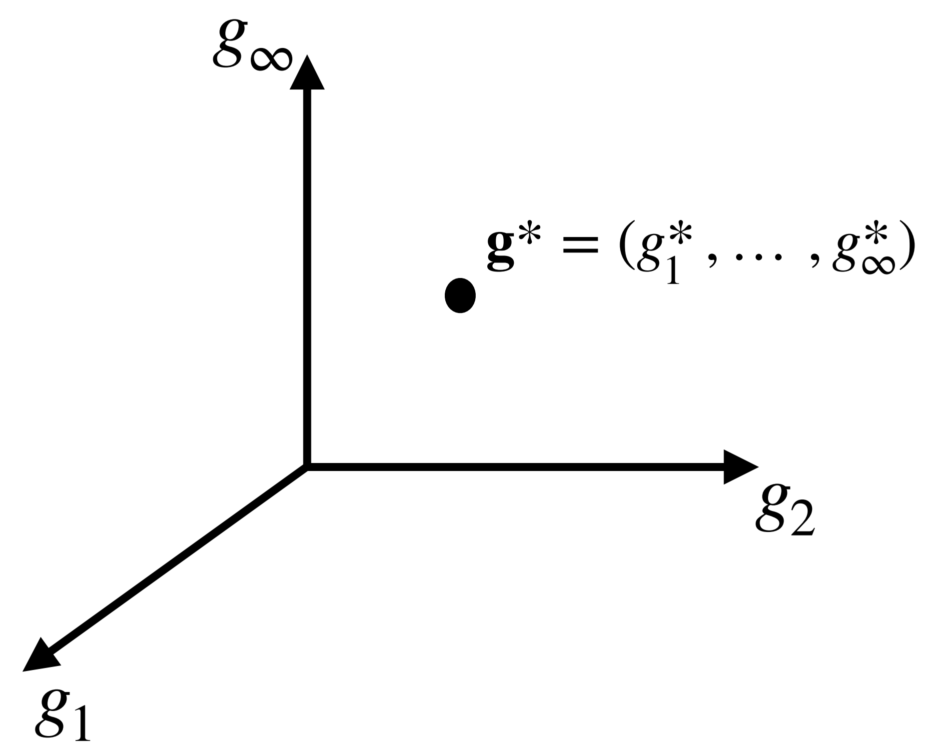


In order to hit the fixed point:

$\theta_i < 0$ → z_i **grows towards de UV**

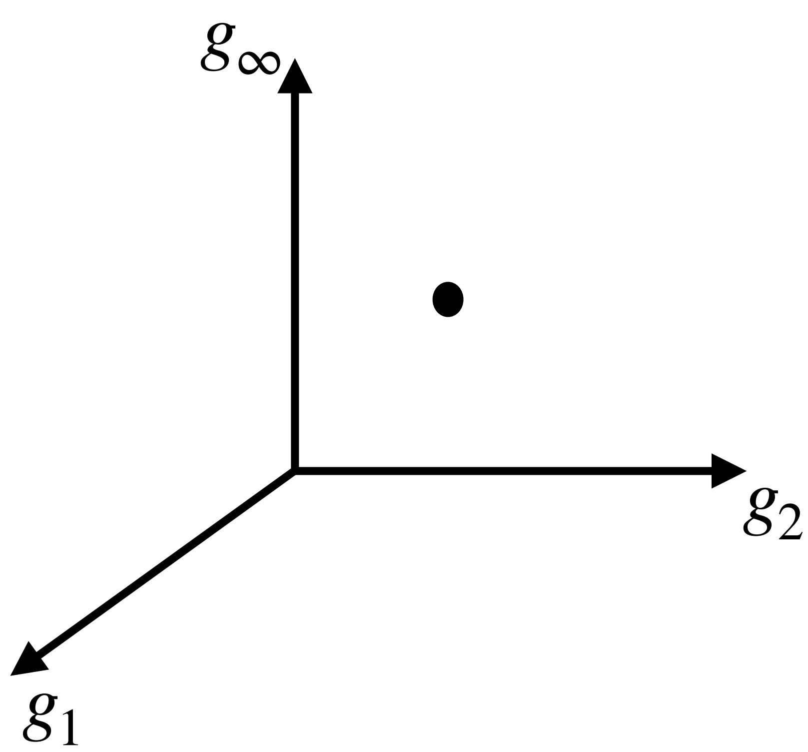
$C_i = 0$ **irrelevant direction**

$\theta_i > 0$ → z_i **decreases towards de UV**

C_i **free parameter**

relevant direction

Predictivity requires that the number of relevant directions is finite



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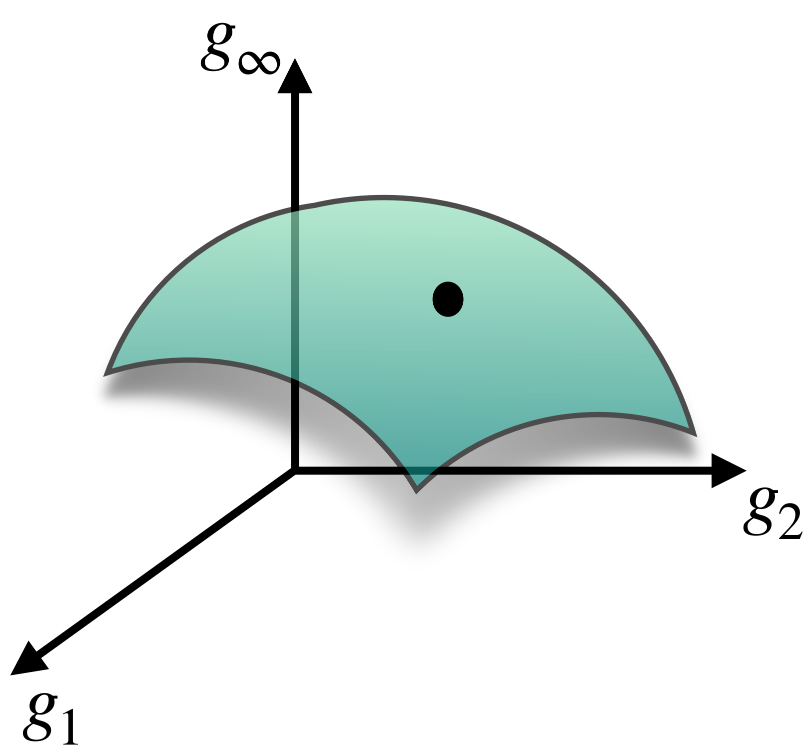

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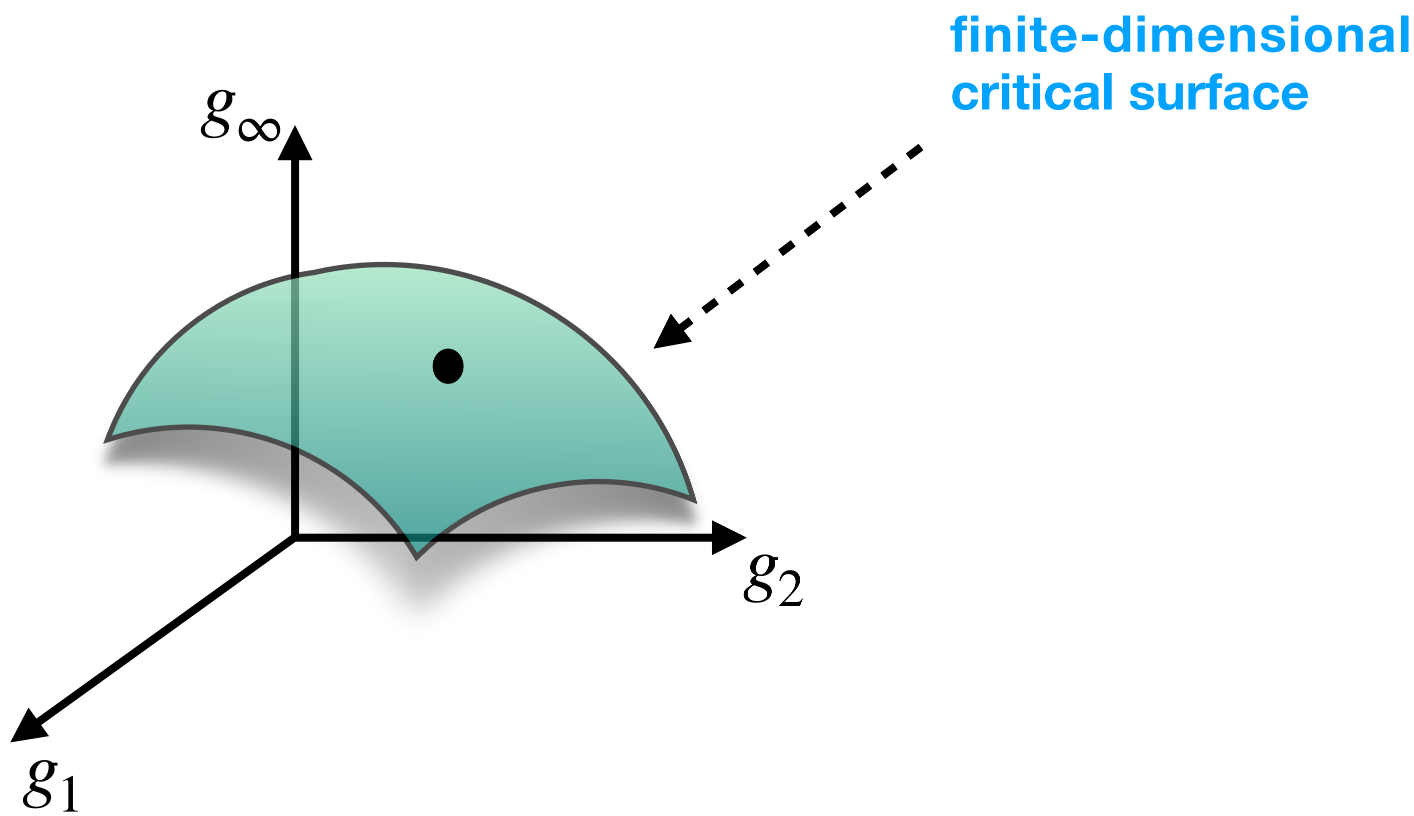

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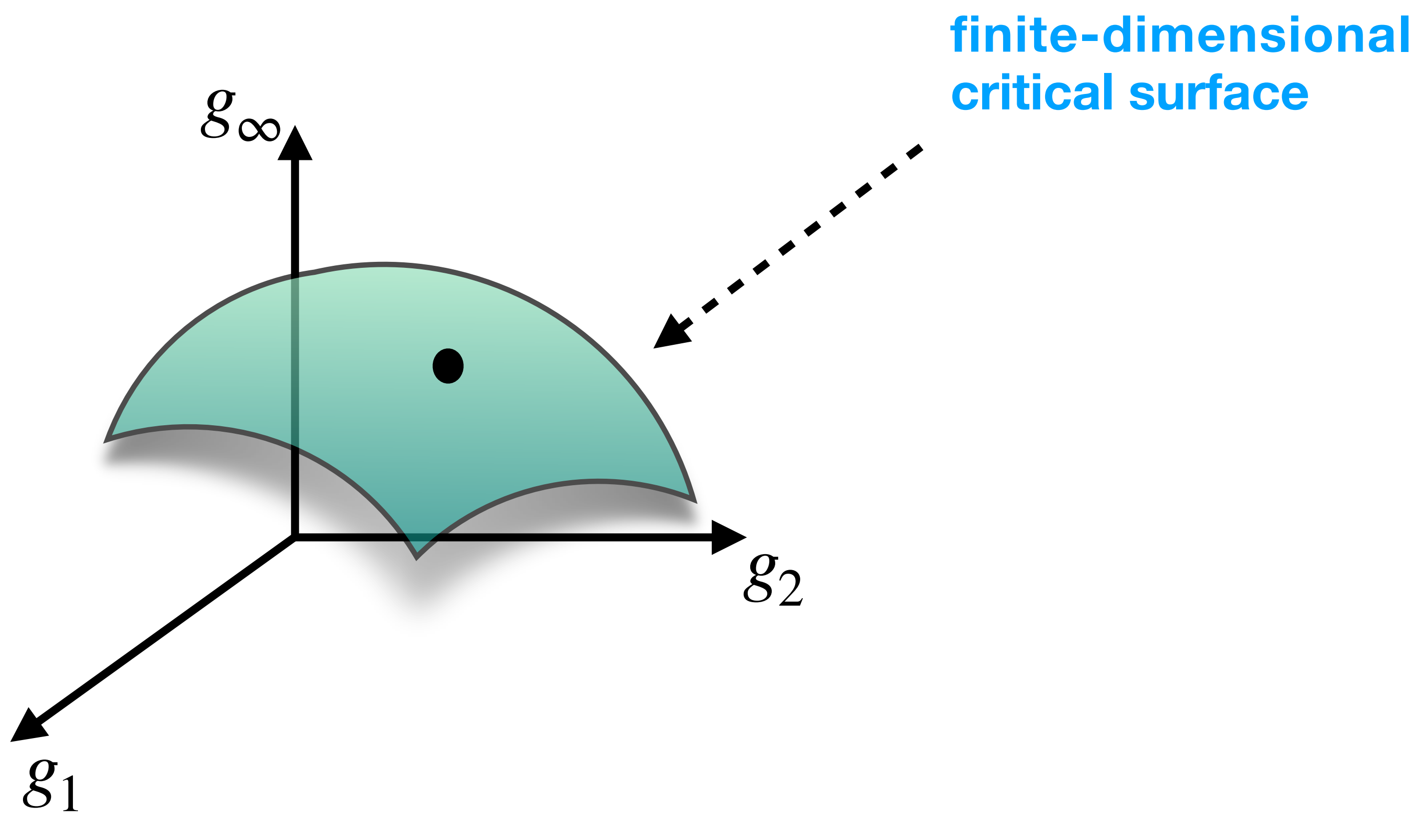

Asymptotic Safety:

Existence of a renormalization-group fixed point;

Fixed point features finitely many relevant directions;

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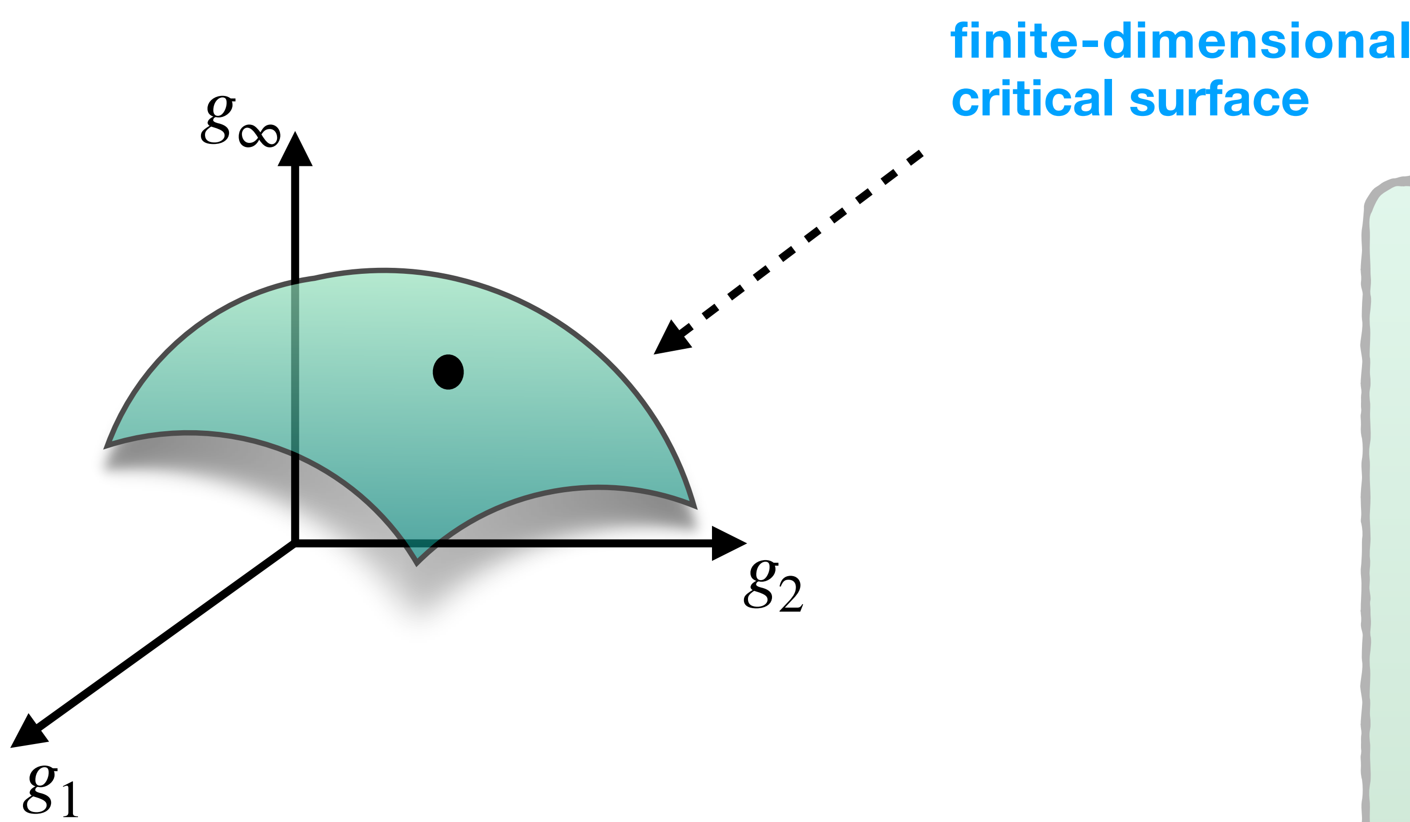

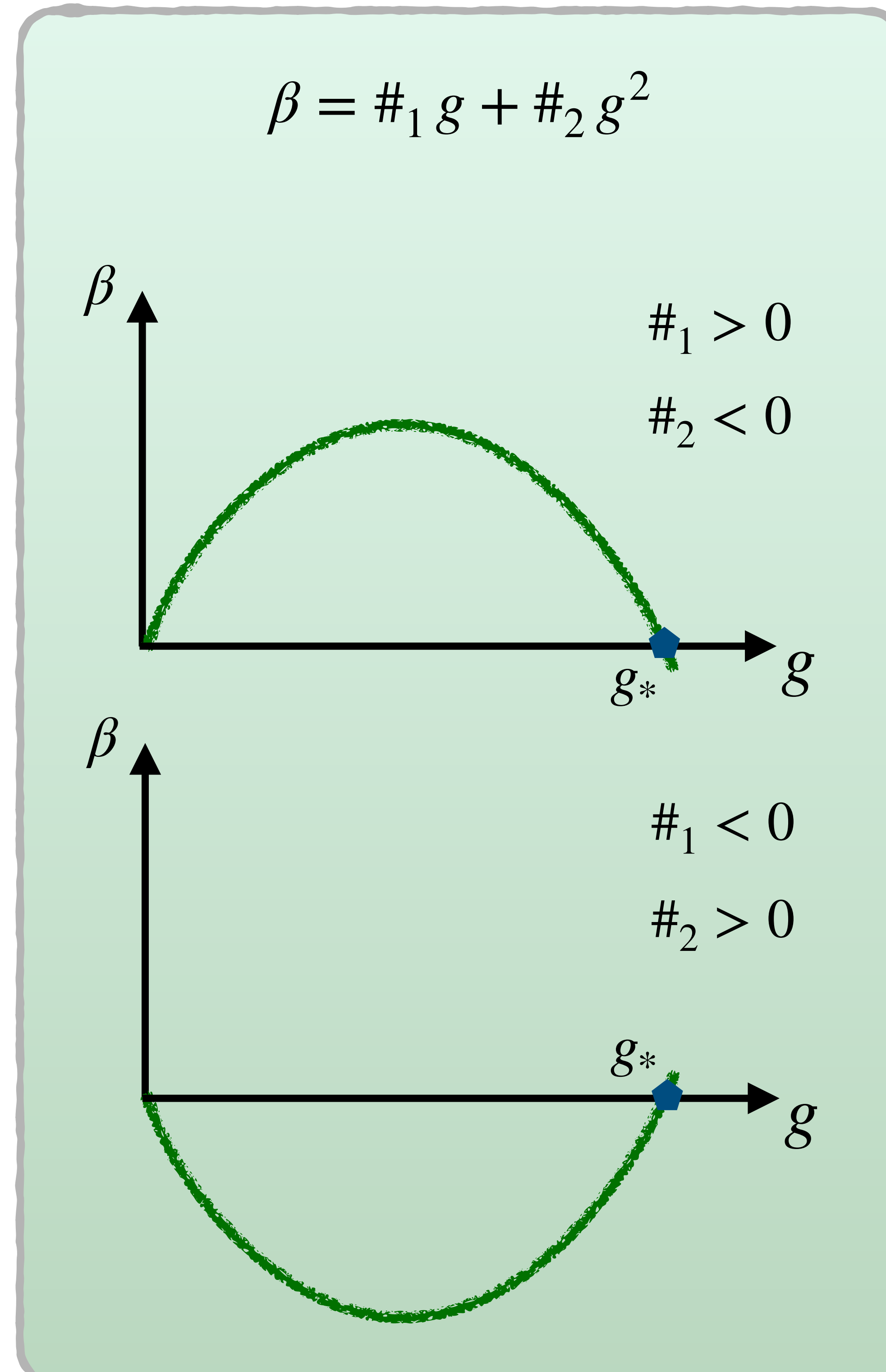

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Homework: Scalar Field Theory

$$\Gamma_k[\varphi] = \int \mathrm{d}^{\mathrm{d}}\mathrm{x} \left(\frac{Z_\varphi}{2} \partial^\mu \varphi \partial_\mu \varphi + V_k(\varphi^2) \right)$$

$$V_k(\varphi^2) = \sum_{i=1}^{N} Z_\varphi^i \frac{\bar{g}_{2i}}{(2i)!} \varphi^{2i}$$

Hints:

Take the wave function renormalization to be a constant as a first approx.

Choose:

$$\mathscr{R}_k(z) = Z_\varphi\,(k^2 - z)\,\theta(k^2 - z)$$

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CNPq

FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

Asymptotically Safe Quantum Gravity

- the technical side -

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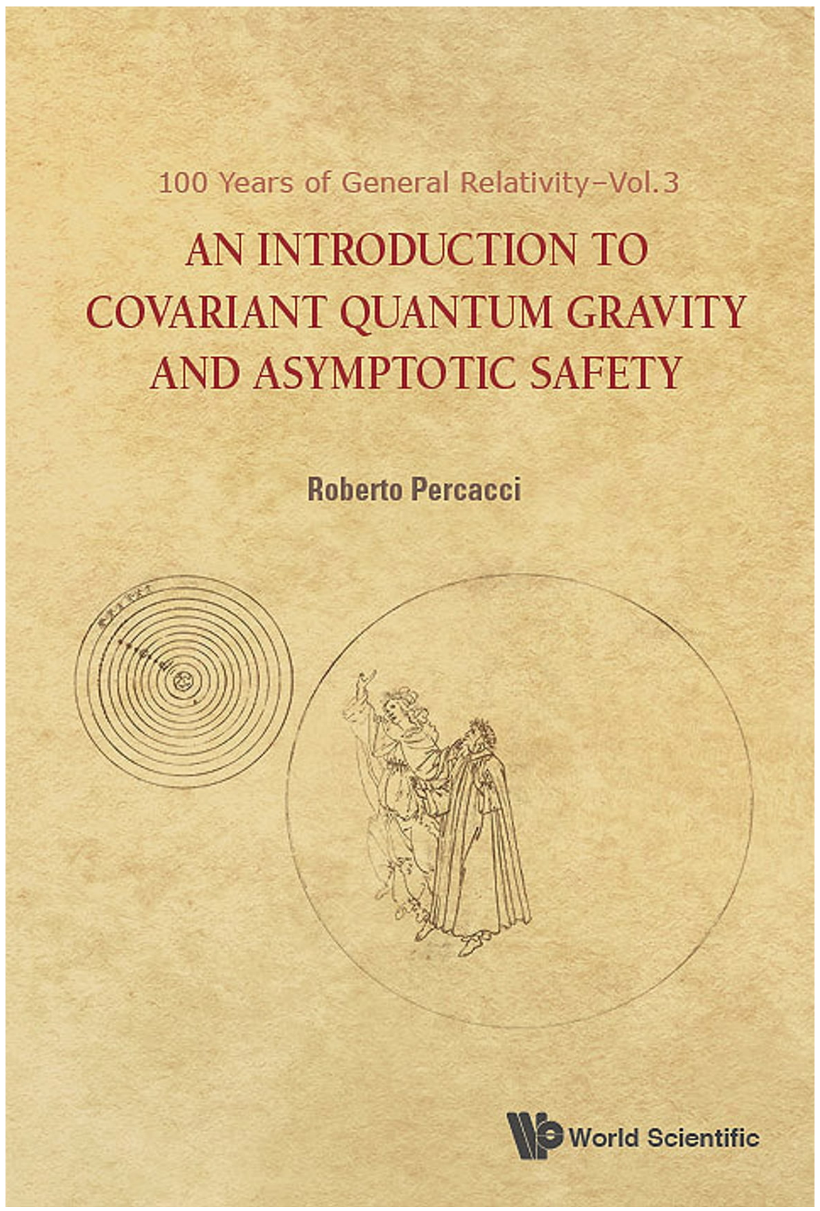
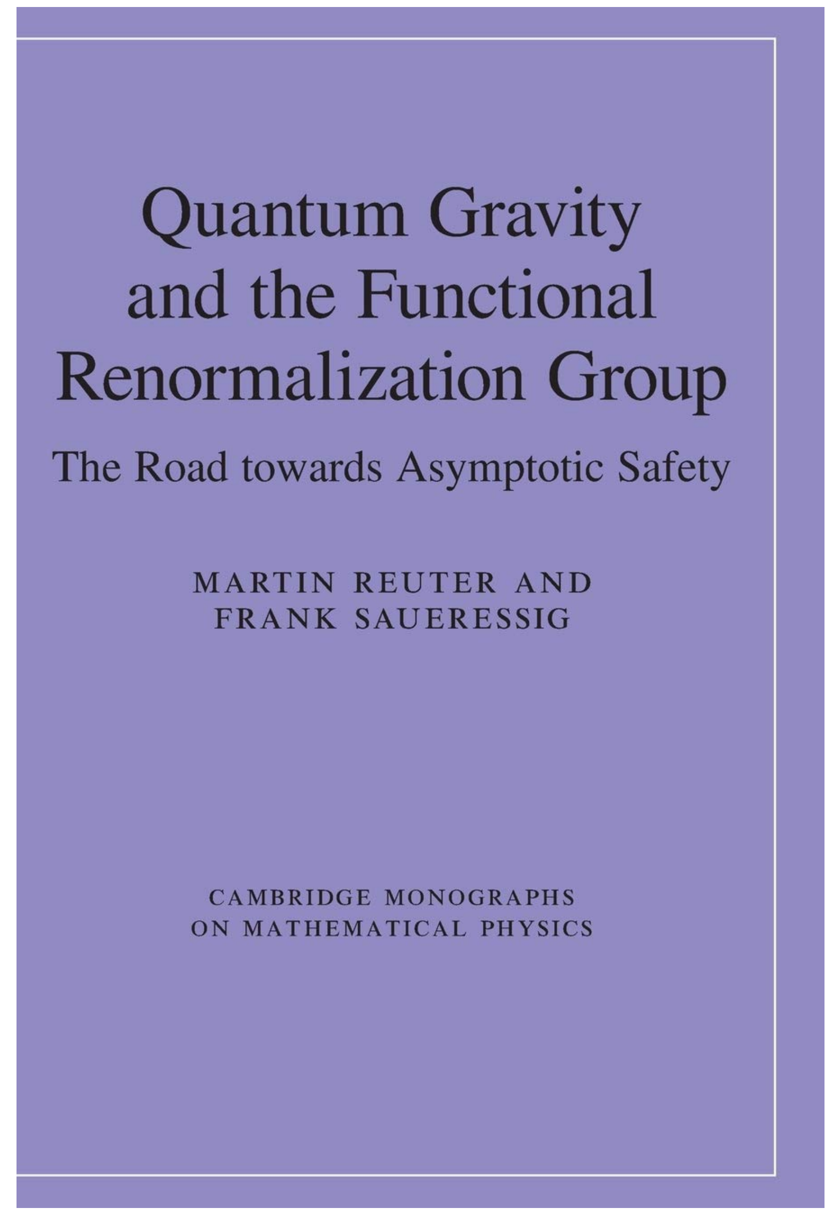
- the technical side -





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No background to set a scale: *background field method*

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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No background to set a scale: *background field method*

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

The spectrum of the Laplacian of the background metric defines a scale

background independence is encoded in split symmetry

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} + \epsilon_{\mu\nu}$$
$$h_{\mu\nu} \to h_{\mu\nu} - \epsilon_{\mu\nu}$$

Asymptotically Safe Quantum Gravity

- the technical side -

$$Z = \int \mathscr{D} g_{\mu\nu} \, \mathrm{e}^{-S[g_{\mu\nu}]}$$

No background to set a scale: *background field method*

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The spectrum of the Laplacian of the background metric defines a scale

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Introduction of a gauge fixing term:

Faddeev-Popov procedure

Gauge fixing

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Regulators break split symmetry and (quantum) gauge invariance!

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very little is known about the Gribov problem in quantum gravity and how it can affect background independence

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field independent: does not contribute to the flow

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From the previous discussion, we see that the cosmological constant drops out

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Perturbative calculations: asymptotic freedom ← $d = 2$ $\quad \beta = F_k(g)$

Predictive Power

The existence of the UV-fixed point imposes severe constraints on the RG-flow

Quantum Scale Symmetry

↓

Finitely many free parameters

We have evidence for the necessity of 3 relevant directions

The theory is highly predictive

This is a hint for a fixed point that is not deeply non-perturbative

$G_k = k^{2-d} g_k$ $\qquad$ $\beta = (d-2) g_k + F_k(g)$ $\qquad$ $d = 4$ $\qquad$ $\beta = 2 g_k + F_k(g)$

Perturbative calculations: asymptotic freedom ← $d = 2$ $\qquad$ $\beta = F_k(g)$

Asymptotic Safety can be established perturbatively ← $d = 2 + \epsilon$

This leads Weinberg to conjecture the Asymptotic Safety scenario in four dimensions

Weinberg proposes the asymptotic safety scenario in a special edition in celebration to Einstein's centenary

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Reuter fixed point

Birth of AS in its modern incarnation

Towards phenomenology

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→ A first-principle analysis requires a well controlled knowledge of the effective action

→ Classical cosmological and BH solutions are RG-improved

Final words

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Is the theory unitary?

Do we have a complete RG-trajectory that emanates from the UV to our IR?

How to connect the results obtained with the FRG and other non-perturbative schemes?

Stay tuned or join us! :)