Bouncing cosmological models

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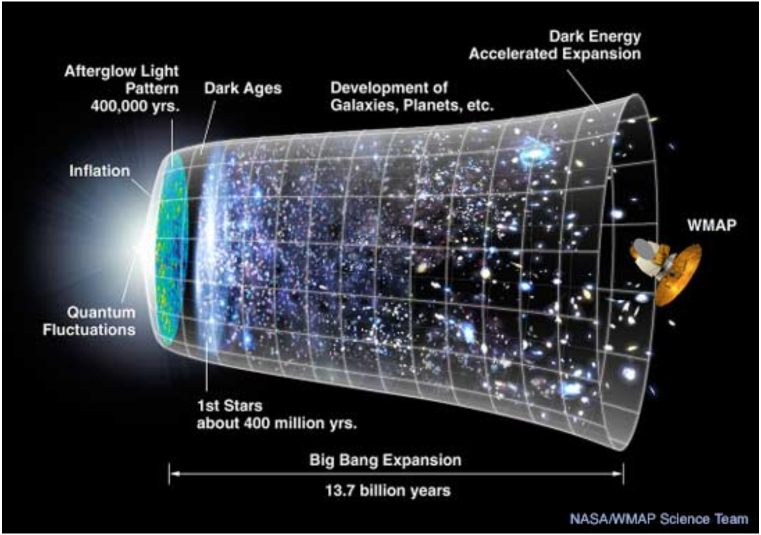

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

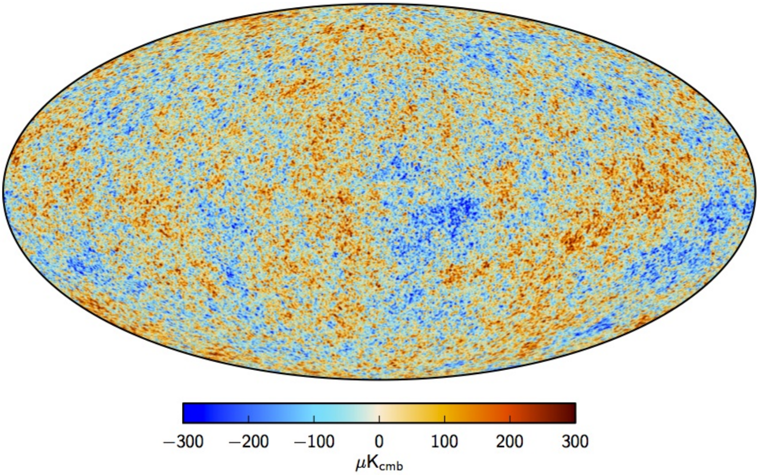
$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

> 60 ~ 55 7.5 0.5

CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation in the synchronous gauge with some additional conditions fixing it completely:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0,\ g^{(a)}_{,l}\, e_m^{l(a)} = 0,\ e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$P_{\mathcal{R}}(k),\ \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k} \equiv n_s(k) - 1,\ r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

Both $|n_s - 1|$ and r are small during slow-roll inflation.

New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$ (note that $(1 - n_s) N_H \sim 2$).

The most recent upper limits on r

1. BICEP/Keck Collaboration: P. A. R. Ade et al., Phys. Rev. Lett. 127, 151301 (2021); arXiv:2110.00483:

$r_{0.05} < 0.036$ at the 95% C.L.

2. M. Tristram et al., Phys. Rev. D 105, 083524 (2022); arXiv:2112.07961:

$r_{0.05} < 0.032$ at the 95% C.L.

For comparison, in the chaotic inflationary model $V(\varphi) \propto |\varphi|^n$, $r = \frac{4n}{N}$, $1 - n_s = \frac{n+2}{2N}$. The r upper bound gives $n < 0.5$ for $N_{0.05} = (55 - 60)$, but then $1 - n_s \leq 0.022$. Thus, this model is disfavoured by observational data.

The target prediction for r in the 3 simplest (one-parametric) inflationary models having $n_s - 1 = -\frac{2}{N}$ (the $R + R^2$, Higgs and combined Higgs-R^2 models) is

$$r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004$$

Kinematic origin of scalar perturbations

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different in different points of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\mathcal{R}(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B 117, 175 (1982) in the case of one-field inflation.

Visualizing small differences in the number of e-folds

Duration of inflation in terms of e-folds was finite for all points inside our past light cone. For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\mathcal{R}(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1, P_{\mathcal{R}} = P_0$,

$$\ell(\ell+1)\,\langle(\Delta T/T_\gamma)^2_{lm}\rangle = \frac{2\pi}{25}P_0$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\delta N \sim 5 \times 10^{-5}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, like in the minimal (one-parametric) inflationary models, $\delta t \sim 5 t_{Pl}$!

Planck time intervals are seen by the naked eye!

Before inflation

The two simplest possibilities occurring in classical (possibly modified) gravity already.

1. Quasi-isotropic bounce of the scale factor with bounded curvature not exceeding that during inflation.
2. Generic anisotropic and inhomogeneous singularity with curvature much exceeding that during inflation.

A specific intermediate case: de Sitter 'Genesis': beginning from the exact contracting full de Sitter space-time at $t \to -\infty$ (AS, PLB 91, 99 (1980)).
Requires adding an additional term

$$R_i^l R_l^k - \frac{2}{3} R R_i^k - \frac{1}{2}\delta_i^k R_{lm} R^{lm} + \frac{1}{4}\delta_i^k R^2$$

to the rhs of the gravitational field equations. Not generic.
May not be the 'ultimate' solution: a quantum system may not spend an infinite time in an unstable state.

Other more speculative possibilities

1. Creation of inflation "from nothing" (Grishchuk and Zeldovich, 1981).
One possibility among infinite number of others.

2. Our Universe was not an individual entity before inflationary stage, it was a part of some "Superuniverse" ("Multiverse" in modern terminology) (AS, Quantum Gravity, 1981).

3. More generally, any process may be responsible for the formation of inflationary stage in our Universe, that was called "creation from anything" in AS and Ya. B. Zeldovich, Sov. Sci. Rev. 1988.
Possible relation to de Sitter entropy.

Isotropic bounce with positive spatial curvature

Does not require modified gravity. Rather natural before inflationary stage since even a very small positive spatial curvature at present becomes important sufficiently early during inflation.

The simplest model: closed FLRW universe filled by a massive scalar field (AS, Sov. Astron. Lett. 4, 82 (1978)). Also the 'slow roll' approximation presently used in all viable inflationary models was first introduced in this paper (as 'slow climb' before bounce, $t > t_-$, and 'slow roll' after bounce $t < t_+$).

$$\ln \frac{a}{a_\pm} = -\frac{m^2(t-t_\pm)^2}{6}, \;\; \phi = -\sqrt{\frac{2}{3}}\,\frac{m(t-t_\pm)}{\kappa}\; sgn\, H$$

$$\kappa^2 = 8\pi G, \;\; m|t-t_\pm| \gg 1, \;\; \kappa|\phi| \gg 1, \;\; ma_\pm \gg 1, \;\; ma\kappa|\phi| \gg 1$$

Generic, but probability of a bounce is small for a large initial size of a universe $W \sim 1/ma_-$. It is difficult to reach inflation from a low curvature state.

Isotropic bounce with zero spatial curvature in scalar-tensor gravity

D. Polarski, A. A. Starobinsky, Y. Verbin. J. Cosm. Astropart. Phys. 2022, 052 (2022); arXiv:2111.07319.
In contrast to GR, scalar-tensor gravity admits breaking of weak and null energy conditions, so isotropic bounce is possible even in the absence of spatial curvature.

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - U(\Phi) - \frac{\xi}{2}R\Phi^2\right)$$

Bouncing solutions have been found for polynomial $U(\phi)$ negative in some range but bounded from below. However, bounce in scale factor does not guarantee bounce in curvature.

In all solutions either the Hubble function $H(t)$ becomes divergent at some finite moment of time before the bounce, or the effective gravitational constant $G_{eff} = G/(1 - \xi\kappa^2\Phi^2)$ becomes negative around the bounce.

As was shown in AS, Sov. Astron. Lett. 7, 36 (1981), in such solutions, arbitrarily small anisotropic perturbations diverge in the point there $G_{eff}^{-1} = 0$ and this results in the formation of generic anisotropic and inhomogeneous singularity at this moment preventing the transition to the region where G_{eff} is negative.
Generic structure of singularity:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i} a_l^{(i)} a_m^{(i)} dx^l dx^m, \ 0 < s < 1, \ u = s(2 - s)$$

where $p_i < 1, \ s = \sum_i p_i, \ u = \sum_i p_i^2$ and $a_l^{(i)}, \ p_i$ are functions of $\mathbf{r}$.

Isotropic bounce with negative spatial curvature in scalar-tensor gravity

In this case, isotropic bounce is possible, too, and even with positive potentials of the scalar field, see e.g. V. M. Frolov, A. A. Grib and V. M. Mostpanenko, Phys. Lett. A 65, 282 (1978); V. N. Melnikov and S. V. Orlov, Phys. Lett. A 70, 263 (1979). However, all such solutions have $G_{eff}^{-1} = 0$ in some point, as was shown in AS, Sov. Astron. Lett. 7, 36 (1981). So, the same problem as with spatially flat solutions arises.

Conclusions

- With a positive spatial curvature, generic bounce of an isotropic homogeneous universe is possible. Moreover, the assumption of homogeneity can be omitted: it is sufficient that spatial curvature is positive in some finite region of space only. Then only a small part of a collapsing universe bounces.
- In scalar-tensor gravity with $U(\phi)$ negative in some range but bounded from below, spatially flat isotropic bouncing solutions with a finite Hubble function $H(t)$ are possible even in the absence of spatial curvature, but they require G_{eff} becoming negative around the bounce and, thus, are unstable with respect to the formation of generic and inhomogeneous curvature singularity at the moment when $G_{eff}^{-1} = 0$. Thus, they are unphysical.
- The same problem arises in FLRW models with negative spatial curvature.

WARMEST CONGRATULATIONS,

BEST WISHES

AND NEXT SUCCESSES TO MARIO!